Questions from Homework

4. Given $f(x) = 3x^2 + 2$, $g(x) = \sqrt{x + 4}$, and $h(x) = 4x - 2$, determine each combined function and state its domain.

a) $y = (f + g)(x)$ **b)** $y = (h - g)(x)$

c) $y = (g - h)(x)$ **d)** $y = (f + h)(x)$

$f(x) = 3x^2 + 2$ D: $\{x \mid x \in R\}$

$g(x) = \sqrt{x+4}$ D: $\{x \mid x \geq -4, x \in R\}$

$x + 4 \geq 0$

$x \geq -4$

$h(x) = 4x - 2$ D: $\{x \mid x \in R\}$

a) $y = f(x) + g(x)$

$y = 3x^2 + 2 + \sqrt{x+4}$

D: $\{x \mid x \geq -4, x \in R\}$

c) $y = g(x) - h(x)$

$y = \sqrt{x+4} - (4x - 2)$

$y = \sqrt{x+4} - 4x + 2$

D: $\{x \mid x \geq -4, x \in R\}$

d) $y = f(x) + h(x)$

$y = 3x^2 + 2 + 4x - 2$

$y = 3x^2 + 4x$

D: $\{x \mid x \in R\}$

11. If $h(x) = (f - g)(x)$ and $f(x) = 5x + 2$, determine $g(x)$.

a) $h(x) = -x^2 + 5x + 3$

b) $h(x) = \sqrt{x - 4} + 5x + 2$

c) $h(x) = -3x + 11$

d) $h(x) = -2x^2 + 16x + 8$

d) $h(x) = f(x) - g(x)$

$-2x^2 + 16x + 8 = 5x + 2 - g(x)$

$g(x) = 5x + 2 + 2x^2 - 16x - 8$

$g(x) = 2x^2 - 11x - 6$

Function Operations

To combine two functions, $f(x)$ and $g(x)$, multiply or divide as follows:

Product of Functions	*Quotient of Functions*
$h(x) = f(x)g(x)$	$h(x) = \frac{f(x)}{g(x)}$
$h(x) = (f \cdot g)(x)$	$h(x) = \left(\frac{f}{g}\right)(x)$

The domain of a product of functions is the domain common to the original functions. However, the domain of a quotient of functions must take into consideration that division by zero is undefined. The domain of a quotient, $h(x) = \frac{f(x)}{g(x)}$, is further restricted for values of x where $g(x) = 0$.

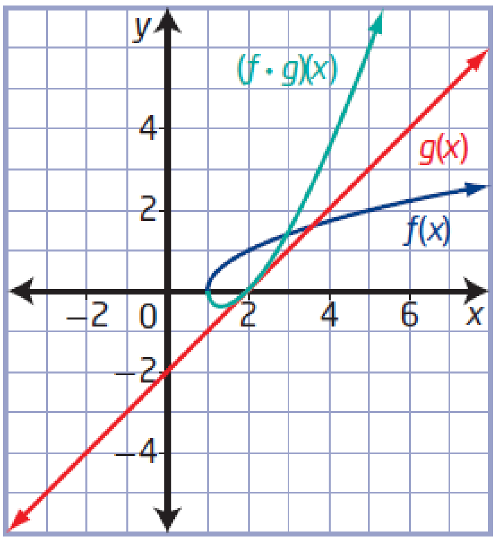
Example

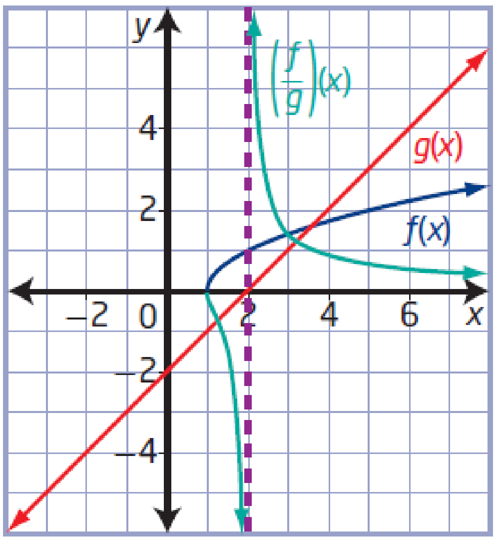
$x - 1 \geq 0$
$x \geq 1$

Consider $f(x) = \sqrt{x - 1}$ and $g(x) = x - 2$.

The domain of $f(x)$ is $\{x \mid x \geq 1, x \in \mathbb{R}\}$, and the domain of $g(x)$ is $\{x \mid x \in \mathbb{R}\}$. So, the domain of $(f \cdot g)(x)$ is $\{x \mid x \geq 1, x \in \mathbb{R}\}$, while the domain of $\left(\frac{f}{g}\right)(x)$ is $\{x \mid x \geq 1, x \neq 2, x \in \mathbb{R}\}$

$x - 2 \neq 0$
$x \neq 2$





Key Ideas Page 495

- The combined function $h(x) = (f \cdot g)(x)$ represents the product of two functions, $f(x)$ and $g(x)$.
- The combined function $h(x) = \left(\frac{f}{g}\right)(x)$ represents the quotient of two functions, $f(x)$ and $g(x)$, where $g(x) \neq 0$.
- The domain of a product or quotient of functions is the domain common to both $f(x)$ and $g(x)$. The domain of the quotient $\left(\frac{f}{g}\right)(x)$ is further restricted by excluding values where $g(x) = 0$.
- The range of a combined function can be determined using its graph.

Example 1

Determine the Product of Functions

Given $f(x) = (x + 2)^2 - 5$ and $g(x) = 3x - 4$, determine $h(x) = (f \cdot g)(x)$. State the domain and range of $h(x)$.

Solution

To determine $h(x) = (f \cdot g)(x)$, multiply the two functions.

$h(x) = (f \cdot g)(x)$

$h(x) = f(x)g(x)$ → $(x+2)(x+2) - 5$

$h(x) = ((x + 2)^2 - 5)(3x - 4)$ $x^2 + 2x + 2x + 4 - 5$

$h(x) = (x^2 + 4x - 1)(3x - 4)$

$h(x) = 3x^3 - 4x^2 + 12x^2 - 16x - 3x + 4$

$h(x) = 3x^3 + 8x^2 - 19x + 4$

How can you tell from the original functions that the product is a cubic function?

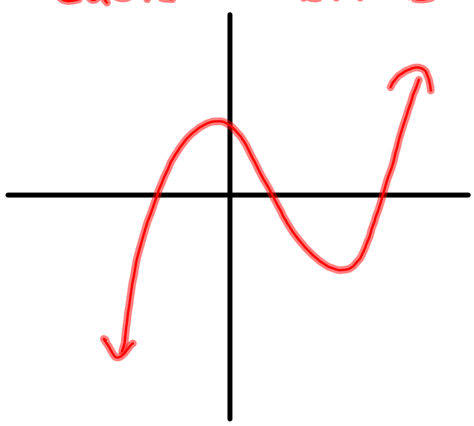
The function $f(x) = (x + 2)^2 - 5$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.

The function $g(x) = 3x - 4$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.

The domain of $h(x) = (f \cdot g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$.

Therefore, the cubic function $h(x) = 3x^3 + 8x^2 - 19x + 4$ has domain $\{x \mid x \in \mathrm{R}\}$ and range $\{y \mid y \in \mathrm{R}\}$.

Cubic Functions (S-shaped)



Example 2

Determine the Quotient of Functions

Consider the functions $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$.

a) Determine the equation of the function $h(x) = \left(\frac{g}{f}\right)(x)$.

b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.

c) State the domain and range of $h(x)$.

Solution

a) To determine $h(x) = \left(\frac{g}{f}\right)(x)$, divide the two functions.

$h(x) = \left(\frac{g}{f}\right)(x)$

$h(x) = \frac{g(x)}{f(x)}$

$h(x) =$

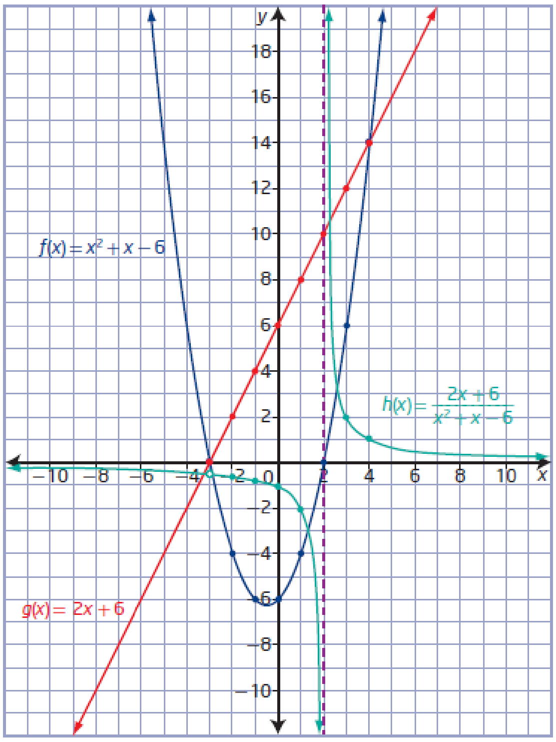
$h(x) =$ Factor.

$h(x) =$

$h(x) =$ Identify any non-permissible values.

b) Method 1: Use Paper and Pencil

x	$f(x) = x^2 + x - 6$	$g(x) = 2x + 6$	$h(x) = \frac{2}{x-2}, x \neq -3, 2$
−3	0	0	
−2	−4	2	
−1	−6	4	
0	−6	6	
1	−4	8	
2	0	10	
3	6	12	
4	14	14	

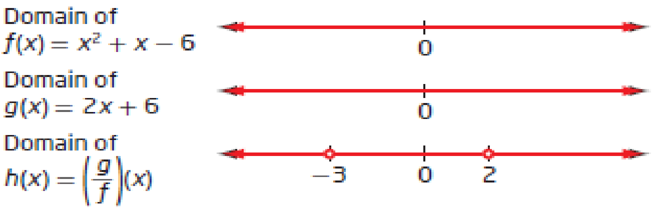


How are the y-coordinates of points on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

c) The function $f(x) = x^2 + x - 6$ is quadratic with domain
The function $g(x) = 2x + 6$ is linear with domain
The domain of $h(x) = \left(\frac{g}{f}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $f(x) = 0$.

Since the function $h(x)$ does not exist at $\left(-3, -\frac{2}{5}\right)$ and is undefined at $x = 2$, the domain is This is shown in the graph by the point of discontinuity at $\left(-3, -\frac{2}{5}\right)$ and the vertical asymptote that appears at $x = 2$.

How do you know there is a point of discontinuity and an asymptote?



The range of $h(x)$ is

Homework

finish #1-9 on page 496-497

10.2 Products and Quotients of Functions, pages 496 to 498

1. a) $h(x) = x^2 - 49$, $k(x) = \dfrac{x + 7}{x - 7}$, $x \neq 7$

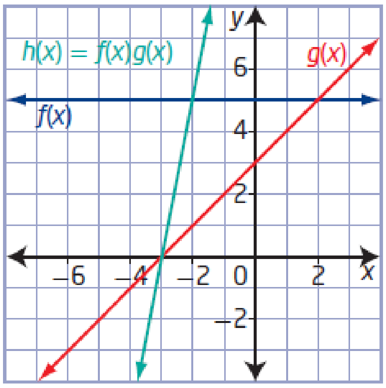
   b) $h(x) = 6x^2 + 5x - 4$, $k(x) = \dfrac{2x - 1}{3x + 4}$, $x \neq -\frac{4}{3}$

   c) $h(x) = (x + 2)\sqrt{x + 5}$, $k(x) = \dfrac{\sqrt{x + 5}}{x + 2}$, $x \geq -5$, $x \neq -2$

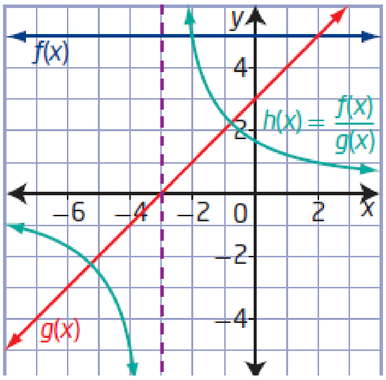
   d) $h(x) = \sqrt{-x^2 + 7x - 6}$, $k(x) = \dfrac{\sqrt{x - 1}}{\sqrt{6 - x}}$, $1 \leq x < 6$

2. a) -3 b) 0 c) -1 d) 0

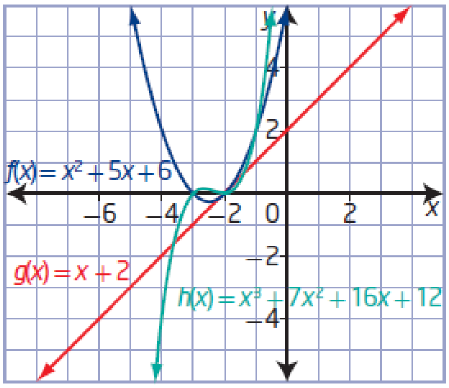
3. a)



b)

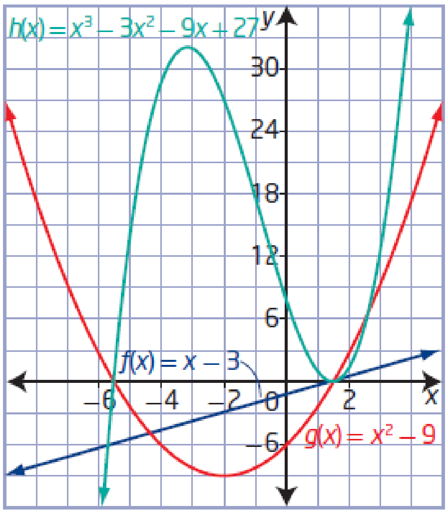


4. a) $h(x) = x^3 + 7x^2 + 16x + 12$



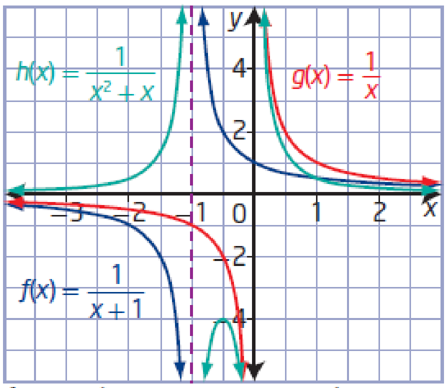
domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

b) $h(x) = x^3 - 3x^2 - 9x + 27$



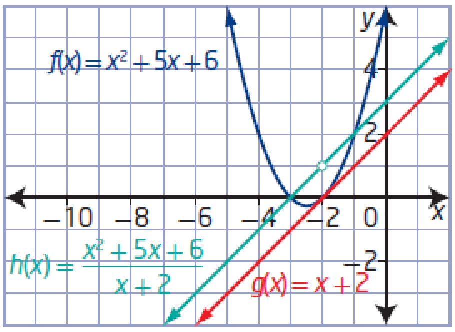
domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

c) $h(x) = \dfrac{1}{x^2 + x}$



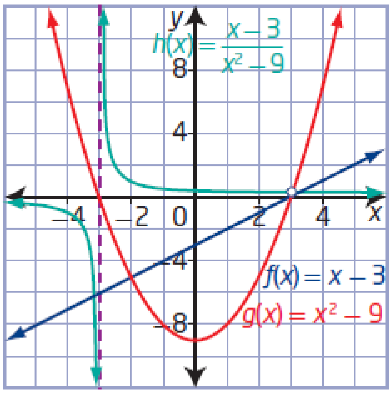
domain $\{x \mid x \neq 0, -1, x \in R\}$,
range $\{y \mid y \leq -4 \text{ or } y > 0, y \in R\}$

5. a) $h(x) = x + 3,\ x \neq -2$



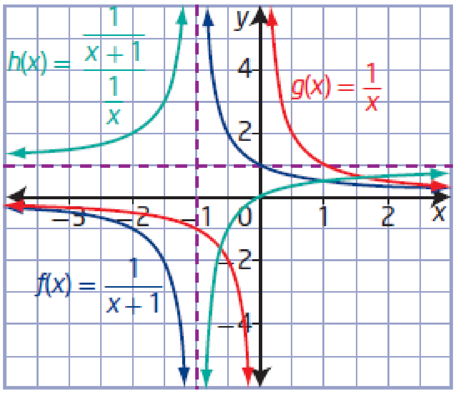
domain $\{x \mid x \neq -2, x \in \mathbb{R}\}$,
range $\{y \mid y \neq 1, y \in \mathbb{R}\}$

b) $h(x) = \dfrac{1}{x + 3},\ x \neq \pm 3$



domain $\{x \mid x \neq \pm 3, x \in \mathbb{R}\}$,
range $\left\{y \mid y \neq 0, \frac{1}{6}, y \in \mathbb{R}\right\}$

c) $h(x) = \dfrac{x}{x + 1},\ x \neq -1, 0$



domain $\{x \mid x \neq -1, 0, x \in \mathbb{R}\}$,
range $\{y \mid y \neq 0, 1, y \in \mathbb{R}\}$

6. a) $y = x^3 + 3x^2 - 10x - 24$

b) $y = \dfrac{x^2 - x - 6}{x + 4},\ x \neq -4$ **c)** $y = \dfrac{2x - 1}{x + 4},\ x \neq -4$

d) $y = \dfrac{x^2 - x - 6}{x^2 + 8x + 16},\ x \neq -4$

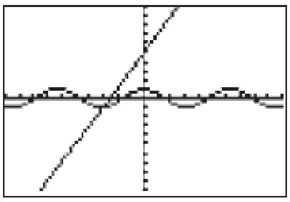
7. a) $g(x) = 3$ **b)** $g(x) = -x$

c) $g(x) = \sqrt{x}$ **d)** $g(x) = 5x - 6$

8. a) $g(x) = x + 7$ **b)** $g(x) = \sqrt{x + 6}$

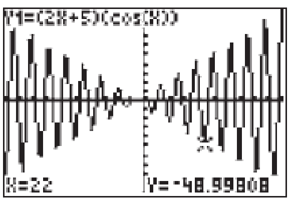
c) $g(x) = 2$ **d)** $g(x) = 3x^2 + 26x - 9$

9. a)



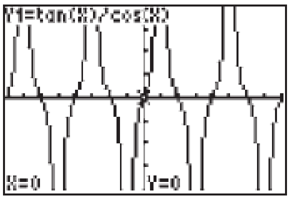
$f(x)$:
domain $\{x \mid x \in \mathbb{R}\}$,
range $\{y \mid y \in \mathbb{R}\}$
$g(x)$: domain $\{x \mid x \in \mathbb{R}\}$,
range
$\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$

b)



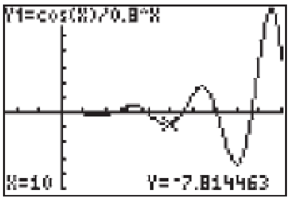
domain $\{x \mid x \in \mathbb{R}\}$,
range $\{y \mid y \in \mathbb{R}\}$

10. a)



domain $\{x \mid x \neq (2n - 1)\frac{\pi}{2}, n \in I, x \in R\}$, range $\{y \mid y \in R\}$

b)



domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

11. a) $y = \frac{f(x)}{g(x)}$ **b)** $y = f(x)f(x)$

c) The graphs of $y = \frac{\sin x}{\cos x}$ and $y = \tan x$ appear to be the same. The graphs of $y = 1 - \cos^2 x$ and $y = \sin^2 x$ appear to be the same.