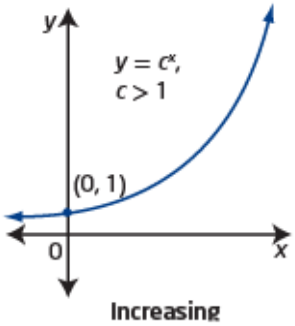
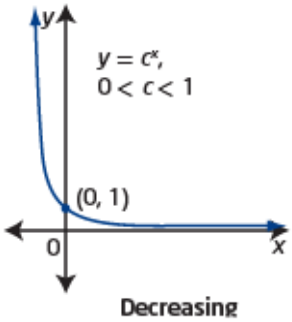
# Exponential Functions

The graph of an **exponential function**, such as $y = c^x$, is increasing for $c > 1$, decreasing for $0 < c < 1$, and neither increasing nor decreasing for $c = 1$. From the graph, you can determine characteristics such as domain and range, any intercepts, and any asymptotes.



Increasing



Decreasing

**exponential function**

- a function of the form $y = c^x$, where $c$ is a constant ($c > 0$) and $x$ is a variable

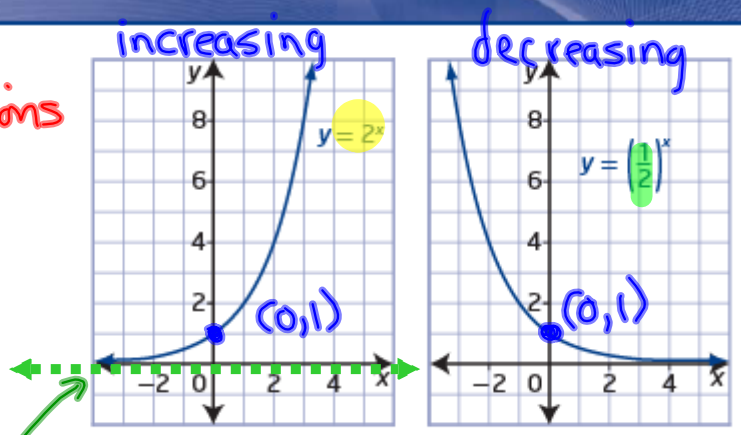
Why is the definition of an exponential function restricted to positive values of $c$?

**Did You Know?**

Any letter can be used to represent the base in an exponential function. Some other common forms are $y = a^x$ and $y = b^x$. In this chapter, you will use the letter $c$. This is to avoid any confusion with the transformation parameters, $a$, $b$, $h$, and $k$, that you will apply in Section 7.2.

## Key Ideas Page 342

- An exponential function of the form $y = c^x$, $c > 0$, no transformations
  - is increasing for $c > 1$
  - is decreasing for $0 < c < 1$
  - is neither increasing nor decreasing for $c = 1$
  - has a domain of $\{x \mid x \in R\}$
  - has a range of $\{y \mid y > 0, y \in R\}$
  - has a $y$-intercept of 1 (0,1)
  - has no $x$-intercept
  - has a horizontal asymptote at $y = 0$



## Example 1

### Analyse the Graph of an Exponential Function

Graph each exponential function. Then identify the following:

- the domain and range
- the $x$-intercept and $y$-intercept, if they exist
- whether the graph represents an increasing or a decreasing function
- the equation of the horizontal asymptote

**a)** $y = 4^x$

**b)** $f(x) = \left(\frac{1}{2}\right)^x$

## Solution

**a) Method 1: Use Paper and Pencil**

Use a table of values to graph the function.

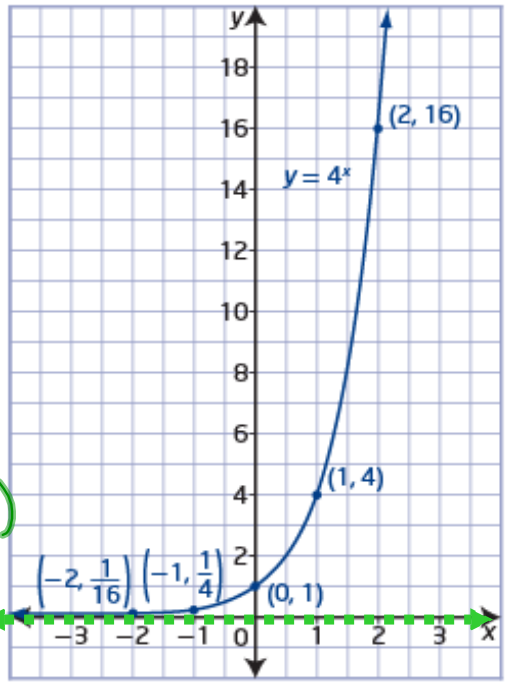
Select integral values of $x$ that make it easy to calculate the corresponding values of $y$ for $y = 4^x$.

$y=4^x$

| $x$ | $y$ |
|---|---|
| $-2$ | $\frac{1}{16}$ |
| $-1$ | $\frac{1}{4}$ |
| 0 | 1 |
| 1 | 4 |
| 2 | 16 |

- D: $\{x \mid x \in R\}$
- R: $\{y \mid y>0, y \in R\}$
- x-int: none
- y-int: (0,1)

  $y=1$
- Increasing ($C=4$)
- HA: $y=0$



**b) Method 1: Use Paper and Pencil**

Use a table of values to graph the function.

Select integral values of $x$ that make it easy to calculate the corresponding values of $y$ for $f(x) = \left(\frac{1}{2}\right)^x$.

$c = \frac{1}{2}$

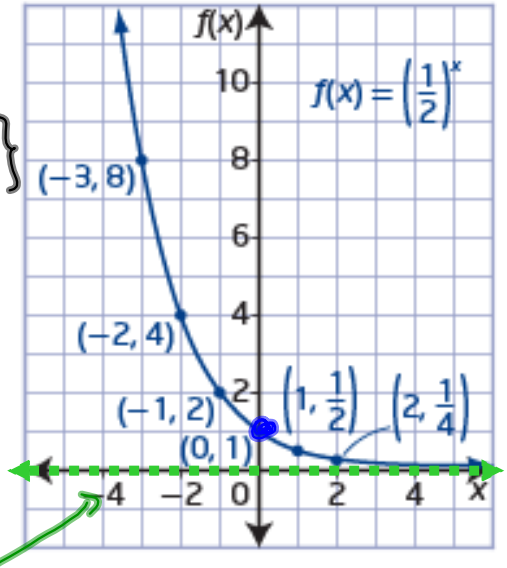
$0 < c < 1$

| $x$ | $f(x)$ |
|---|---|
| −3 | 8 |
| −2 | 4 |
| −1 | 2 |
| 0 | 1 |
| 1 | $\frac{1}{2}$ |
| 2 | $\frac{1}{4}$ |

- D: $\{x \mid x \in \mathbb{R}\}$
- R: $\{y \mid y > 0, y \in \mathbb{R}\}$
- x-int: none
- y-int: (0, 1)

$y = 1$

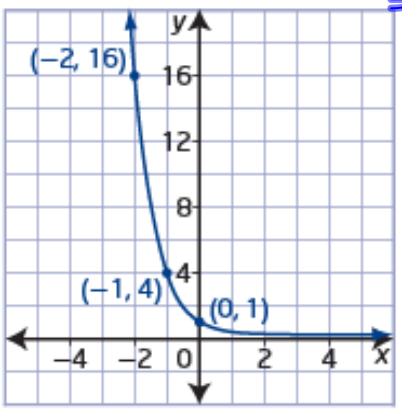
- Decreasing $(c = \frac{1}{2})$
- HA: $y = 0$



## Example 2

### Write the Exponential Function Given Its Graph

What function of the form $y = c^x$ can be used to describe the graph shown?




Trying to solve for the Base: What is c?

**Solution**

Look for a pattern in the ordered pairs from the graph.

| x | y | |
|---|---|---|
| −2 | 16 | $\times \frac{1}{4}$ |
| −1 | 4 | $\times \frac{1}{4}$ |
| 0 | 1 | |

16, 4, 1 → form a geometric sequence.

As the value of $x$ increases by 1 unit, the value of $y$ decreases by a factor of $\frac{1}{4}$. Therefore, for this function, $c = \frac{1}{4}$.

Choose a point other than (0, 1) to substitute into the function $y = \left(\frac{1}{4}\right)^x$ to verify that the function is correct. Try the point (−2, 16).

Why should you not use the point (0, 1) to verify that the function is correct?

(0,1) is common to all exponential functions of the form $y = c^x$, $c > 0$

Check:

Left Side Right Side

$y = \left(\frac{1}{4}\right)^x$

16 $\left(\frac{1}{4}\right)^{-2}$

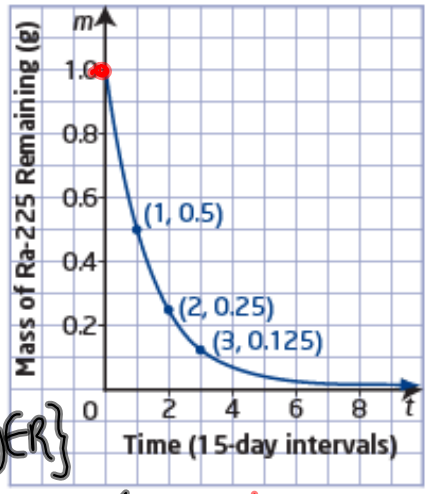
$(4)^2$

16 ✓

## Example 3

### Application of an Exponential Function

A radioactive sample of radium (Ra-225) has a **half-life** of 15 days. The mass, $m$, in grams, of Ra-225 remaining over time, $t$, in 15-day intervals, can be modelled using the exponential graph shown.




**a)** What is the initial mass of Ra-225 in the sample? What value does the mass of Ra-225 remaining approach as time passes? 1g → 0g

**b)** What are the domain and range of this function? $\{x \mid x \in R\}$ & $\{y \mid y > 0, y \in R\}$

**c)** Write the exponential decay model that relates the mass of Ra-225 remaining to time, in 15-day intervals. $y = 1\left(\frac{1}{2}\right)^x$ initial amount

**d)** Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass.

$\frac{1}{30}$ of $1 = \frac{1}{30}$

$y = \left(\frac{1}{2}\right)^x$

$\frac{1}{30} = \left(\frac{1}{2}\right)^x$ Get common base

$\left(\frac{1}{2}\right)^{4.9} = \left(\frac{1}{2}\right)^x$ $\log(1/30) \div \log(1/2)$

$x = 4.9 \times 15 = 73.5$ days

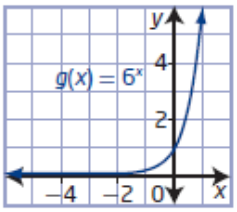
↑ x represents the number of 15 day intervals

# Homework

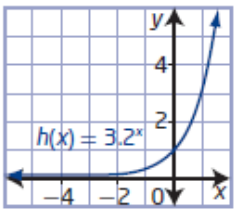
## #1-8 on page 343

## 7.1 Characteristics of Exponential Functions, pages 342 to 345

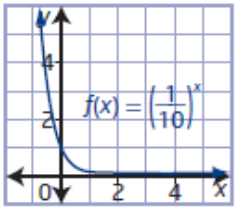
1. **a)** No, the variable is not the exponent.
   **b)** Yes, the base is greater than 0 and the variable is the exponent.
   **c)** No, the variable is not the exponent.
   **d)** Yes, the base is greater than 0 and the variable is the exponent.
2. **a)** $f(x) = 4^x$ **b)** $g(x) = \left(\frac{1}{4}\right)^x$
   **c)** $x = 0$, which is the $y$-intercept
3. **a)** B **b)** C **c)** A
4. **a)** $f(x) = 3^x$ **b)** $f(x) = \left(\frac{1}{5}\right)^x$

5. **a)** 

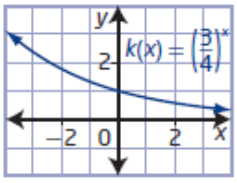
domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, $y$-intercept 1, function increasing, horizontal asymptote $y = 0$

**b)** 

domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, $y$-intercept 1, function increasing, horizontal asymptote $y = 0$

**c)** 

domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, $y$-intercept 1, function decreasing, horizontal asymptote $y = 0$

**d)** 

domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, $y$-intercept 1, function decreasing, horizontal asymptote $y = 0$

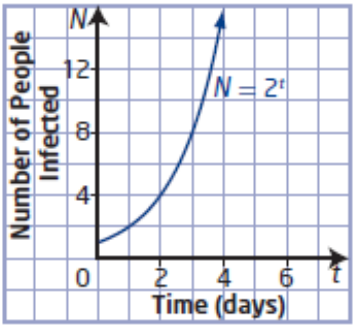
**6. a)** $c > 1$; number of bacteria increases over time

**b)** $0 < c < 1$; amount of actinium-225 decreases over time

**c)** $0 < c < 1$; amount of light decreases with depth

**d)** $c > 1$; number of insects increases over time

**7. a)**

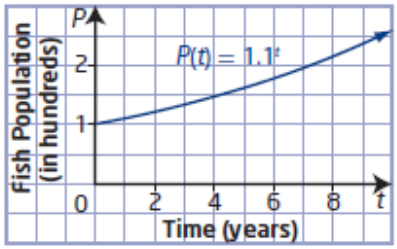



The function $N = 2^t$ is exponential since the base is greater than zero and the variable $t$ is an exponent.

**b)** **i)** 1 person **ii)** 2 people
**iii)** 16 people **iv)** 1024 people

**8. a)** If the population increases by 10% each year, the population becomes 110% of the previous year's population. So, the growth rate is 110% or 1.1 written as a decimal.

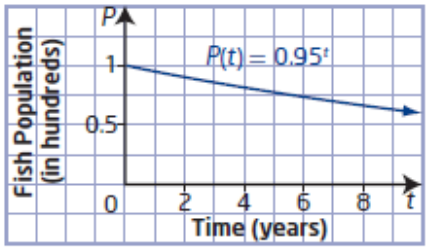
**b)**




domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$ and range $\{P \mid P \geq 100, P \in \mathrm{R}\}$

**c)** The base of the exponent would become 100% − 5% or 95%, written as 0.95 in decimal form.

**d)**




domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$ and range $\{P \mid 0 < P \leq 100, P \in \mathrm{R}\}$