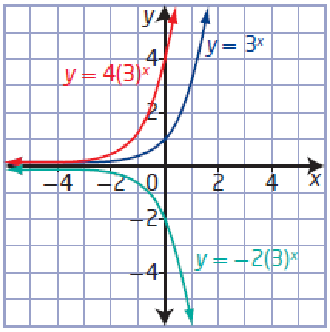
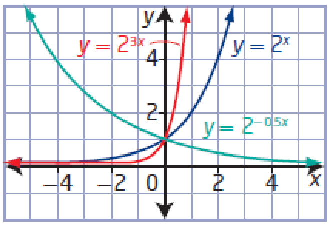
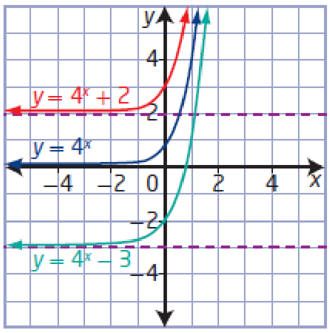
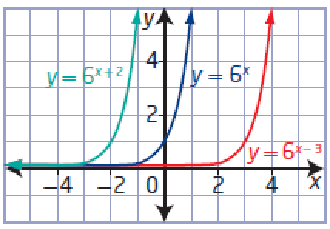
Transformations of Exponential Functions

Focus on...

- applying translations, stretches, and reflections to the graphs of exponential functions
- representing these transformations in the equations of exponential functions
- solving problems that involve exponential growth or decay

Link the Ideas

The graph of a function of the form $f(x) = a(c)^{b(x-h)} + k$ is obtained by applying transformations to the graph of the base function $y = c^x$, where $c > 0$.

<table>
<tr><th>Parameter</th><th>Transformation</th><th>Example</th></tr>
<tr><td>a</td><td>• Vertical stretch about the x-axis by a factor of $|a|$
• For $a < 0$, reflection in the x-axis
• $(x, y) \rightarrow (x, ay)$</td><td>
</td></tr>
<tr><td>b</td><td>• Horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$
• For $b < 0$, reflection in the y-axis
• $(x, y) \rightarrow \left(\frac{x}{b}, y\right)$</td><td>
</td></tr>
<tr><td>k</td><td>• Vertical translation up or down
• $(x, y) \rightarrow (x, y + k)$</td><td>
</td></tr>
<tr><td>h</td><td>• Horizontal translation left or right
• $(x, y) \rightarrow (x + h, y)$</td><td>
</td></tr>
</table>

Example 1

Apply Transformations to Sketch a Graph

Consider the base function $y = 3^x$. For each transformed function,

i) state the parameters and describe the corresponding transformations

ii) create a table to show what happens to the given points under each transformation

$y = 3^x$
$\left(-1, \frac{1}{3}\right)$
(0, 1)
(1, 3)
(2, 9)
(3, 27)

iii) sketch the graph of the base function and the transformed function

iv) describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts

a) $y = 2(3)^{x-4}$

Solution

a) i) Compare the function $y = 2(3)^{x-4}$ to $y = a(c)^{b(x-h)} + k$ to determine the values of the parameters.

- $b = 1$ corresponds to no horizontal stretch.
- $a = 2$ corresponds to a vertical stretch of factor 2. Multiply the   of the points in column 1 by
- $h = 4$ corresponds to a translation of 4 units to the right. Add   to the   coordinates of the points in column 2.
- $k = 0$ corresponds to no vertical translation.

ii) Add columns to the table representing the transformations.

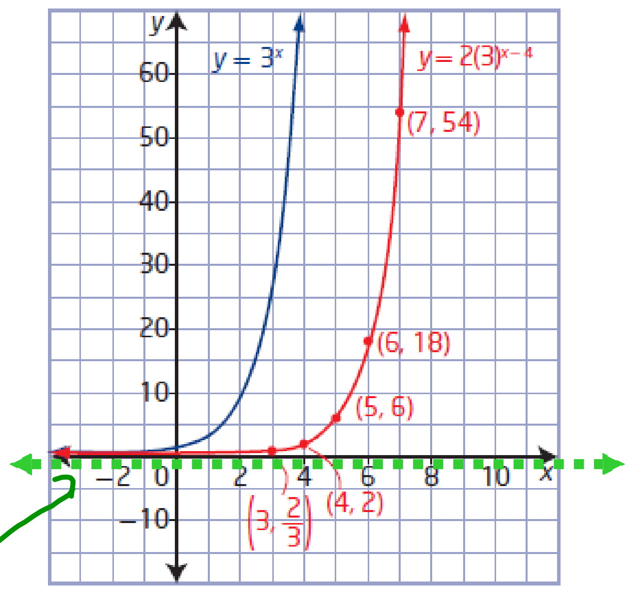
$y = 3^x$	$y = 2(3)^{x-4}$
$\left(-1, \frac{1}{3}\right)$	$\left(3, \frac{2}{3}\right)$
(0, 1)	(4, 2)
(1, 3)	(5, 6)
(2, 9)	(6, 18)
(3, 27)	(7, 54)

• y int

$(x,y) \longrightarrow \left[\frac{1}{b}(x)+h, ay+k\right]$

$(x,y) \longrightarrow \left[x+4, 2y\right]$

iii) To sketch the graph, plot the points from column 3 and draw a smooth curve through them.



y intercept $(x=0)$

$y = 2(3)^{x-4}$

$y = 2(3)^{0-4}$

$y = 2(3)^{-4}$

$y = 2\left(\frac{1}{3}\right)^4$

$y = 2\left(\frac{1}{81}\right) = \frac{2}{81}$

iv) The domain remains the same: $\{x \mid x \in R\}$.

The range also remains unchanged: $\{y \mid y > 0, y \in R\}$.

The equation of the asymptote remains as $y = 0$. (horizontal)

There is still no x-intercept, but the y-intercept changes to $\frac{2}{81}$ or approximately 0.025.

Assignment

b) $y = -\frac{1}{2}(3)^{\frac{1}{5}x} - 5$ $\qquad y = -\frac{1}{2}(3)^{\frac{1}{5}(x-0)} - 5$

(a, b, h, k)

i) state the parameters and describe the corresponding transformations

ii) create a table to show what happens to the given points under each transformation

$y = 3^x$

iii) sketch the graph of the base function and the transformed function

iv) describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts

Homework

#1-7 and #10 on page 354