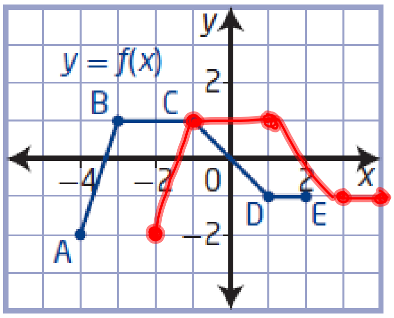
Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$	
H	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$	h = -7
H	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$	h = 3
V	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$	K = -6
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$	h = -4 K = -9
horizontal and vertical	$y = f(x-4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$	h = 4 K = -6
H+V	$y = f(x+2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$	h = -2 K = 3
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+k)$	

Questions from Homework

②

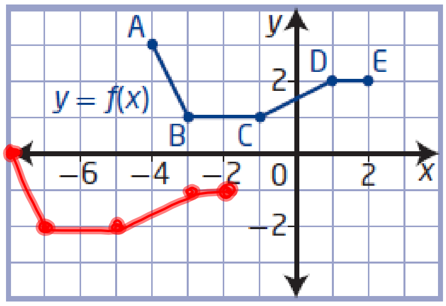


② b) $h(x) = f(x-2)$ $h=2$

$(x,y) \longrightarrow (x+2, y)$

A (-4,-2)	A' (-2,-2)
B (-3,1)	B' (-1,1)
C (-1,1)	C' (1,1)
D (1,-1)	D' (3,-1)
E (2,-1)	E' (4,-1)

④



$h=-4$ $k=-3$

a) $g(x) = f(x+4) - 3$

$(x,y) \longrightarrow (x-4, y-3)$

A(-4,3)	A' (-8,0)
B(-3,1)	B' (-7,-2)
C(-1,1)	C' (-5,-2)
D(1,2)	D' (-3,-1)
E(2,2)	E' (-2,-1)

Transformations:

New Functions From Old Functions

Translations

Stretches

Reflections

Reflections and Stretches

Focus on...

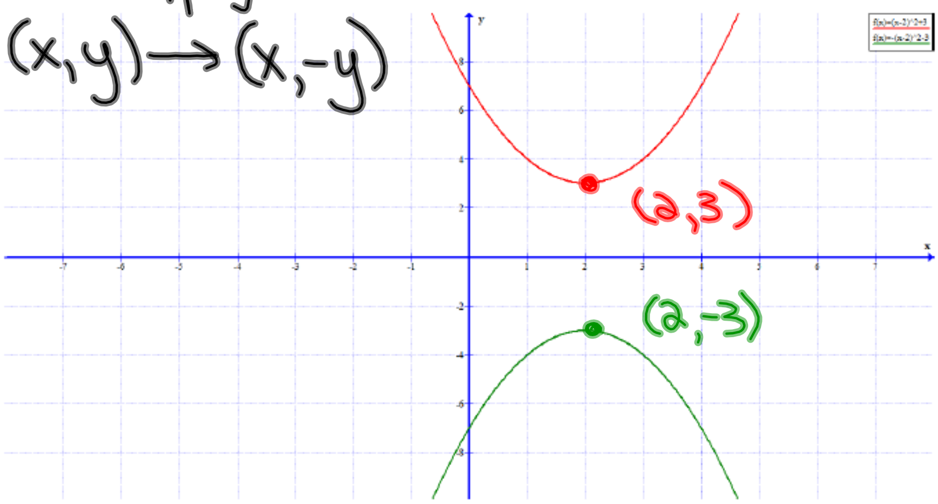
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis. (vertical reflection)

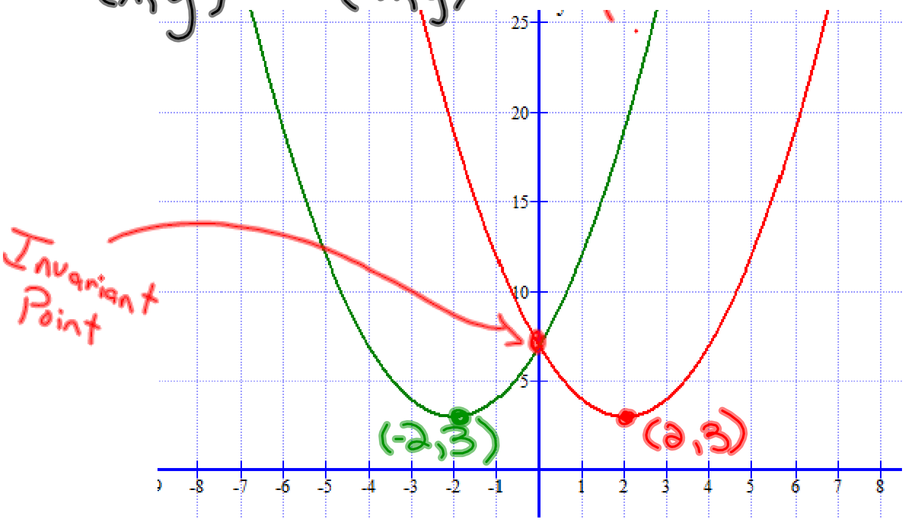
Mapping

$(x, y) \rightarrow (x, -y)$



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis. (horizontal reflection)

Mapping:
$(x, y) \rightarrow (-x, y)$



invariant point

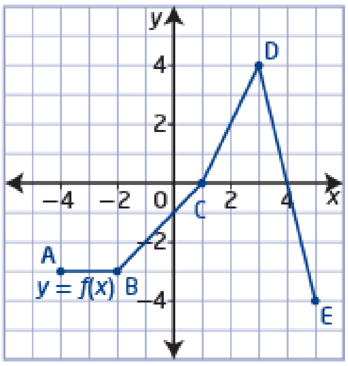
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

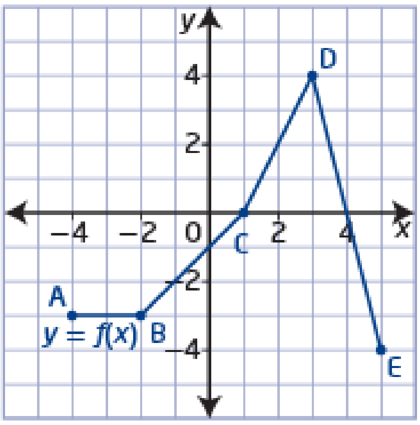
b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

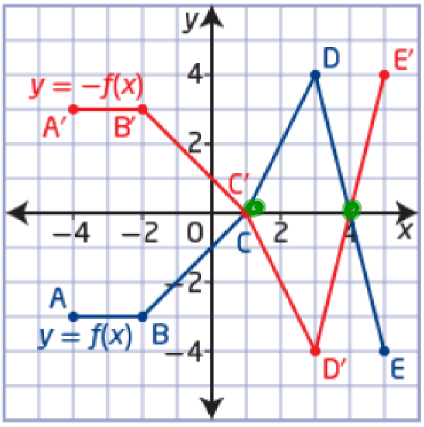
- Sketch $y = -f(x)$ on the axis below



$(x, y) \rightarrow (x, -y)$

A(-4, -3)	A'(-4, 3)
B(-2, -3)	B'(-2, 3)
C(1, 0)	C'(1, 0)
D(3, 4)	D'(3, -4)
E(5, -4)	E'(5, 4)

- Invariant Points



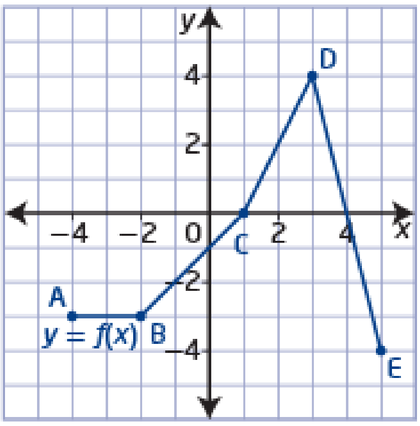
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

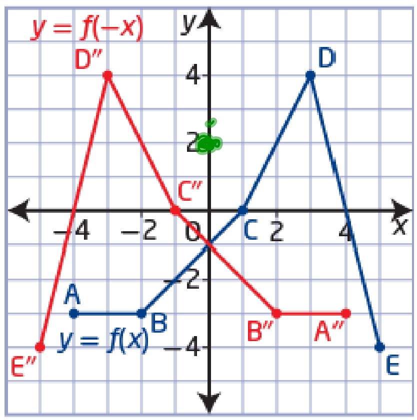
- Sketch $y = f(-x)$ on the axis below

• Invariant Point

$(x,y) \rightarrow (-x,y)$



A(-4,-3) A'(4,-3)
B(-2,-3) B'(2,-3)
C(1,0) C'(-1,0)
D(3,4) D'(-3,4)
E(5,-4) E'(-5,4)



Homework

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stretch

- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Vertical Stretch or Compression...

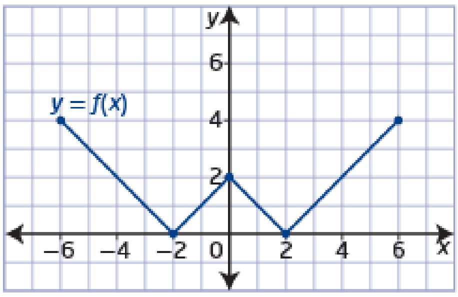
- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

Example 2

Graph $y = af(x)$

Given the graph of $y = f(x)$,

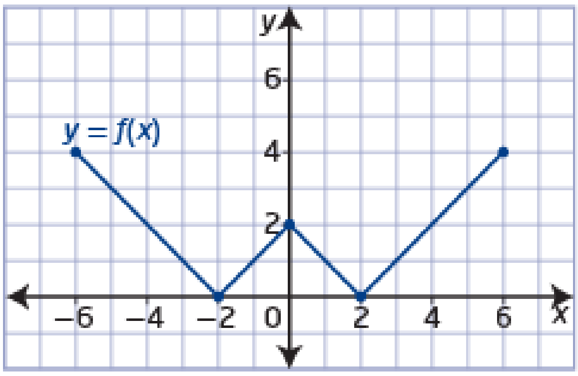
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



a) $g(x) = 2f(x)$

b) $g(x) = \frac{1}{2}f(x)$

a) $g(x) = 2f(x)$



[redacted]

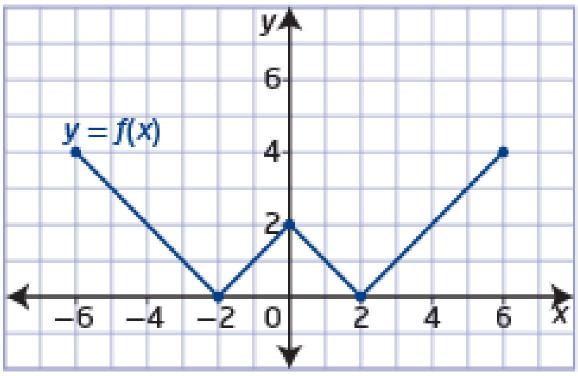
The invariant points are and

For $f(x)$, the domain is

and the range is

For $g(x)$, the domain is
and the range is

b) $g(x) = \frac{1}{2}f(x)$



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The invariant points are and

For $f(x)$, the domain is

and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...

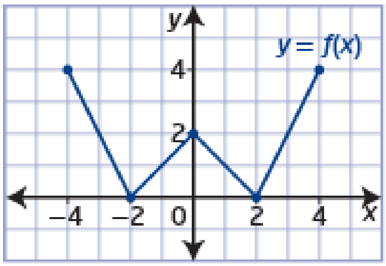
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Example 3

Graph $y = f(bx)$

Given the graph of $y = f(x)$,

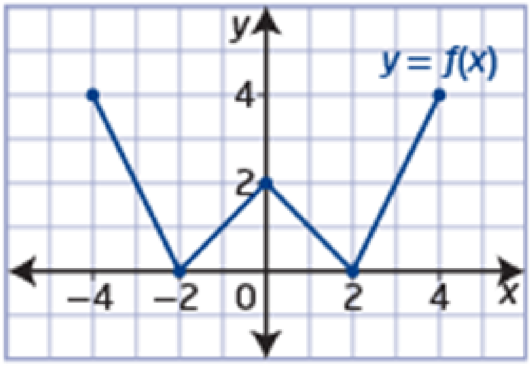
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



a) $g(x) = f(2x)$

b) $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$



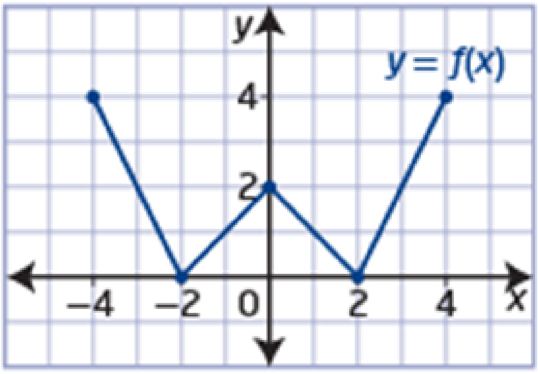
[redacted]

The invariant point is

For $f(x)$, the domain is
or and the range is
or

For $g(x)$, the domain is
or and the range is
or

b) $g(x) = f\left(\frac{1}{2}x\right)$



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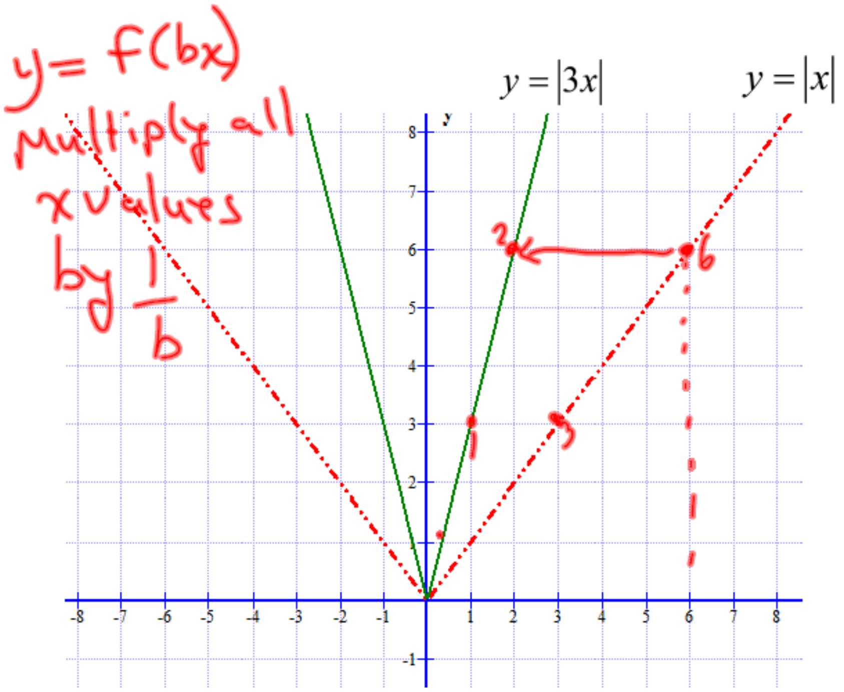
The invariant point is

For $f(x)$, the domain is
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...



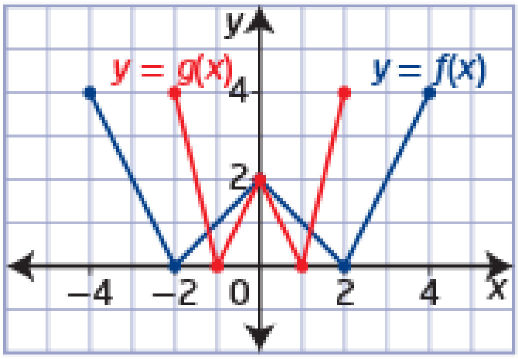
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$y = -3f(-2x) + 7$

Homework

Determine the Equation of a Translated Function:


y
y = g(x)
y = f(x)
4
2
−4
−2
0
2
4
x