Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a = 3$ $(x,y) \rightarrow (x, 3y)$

$b = 1$

$h = 0$ $(-2,5) \rightarrow (-2, 15)$

$k = 0$

(2) $y = f\left(-\dfrac{1}{3}x\right)$

$a = 1$ $(x,y) \rightarrow (-3x, y)$

$b = -\dfrac{1}{3}$

$h = 0$ $(-2,5) \rightarrow (6,5)$

$k = 0$

(3) $y = 4f\left[\dfrac{1}{2}(x+5)\right] - 3$

$a = 4$ $(x,y) \rightarrow (2x - 5, 4y - 3)$

$b = \dfrac{1}{2}$ $(-2,5) \rightarrow (-9, 17)$

$h = -5$

$k = -3$

(4) $y - 5 = -2f(-2x + 6)$

Factor

$y = -2f(-2(x-3)) + 5$

$a = -2$ $(x,y) \rightarrow \left(-\dfrac{1}{2}x + 3, -2y + 5\right)$

$b = -2$ $(-2,5) \rightarrow (4, -5)$

$h = 3$

$k = 5$

1

Summary of Transformations...

Transformations of the graphs of functions

$f(x) + c$	shift $f(x)$ up c units
$f(x) - c$	shift $f(x)$ down c units
$f(x + c)$	shift $f(x)$ left c units
$f(x - c)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ Multiply the y values by c
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ Divide the x values by c

} vertical translation → K

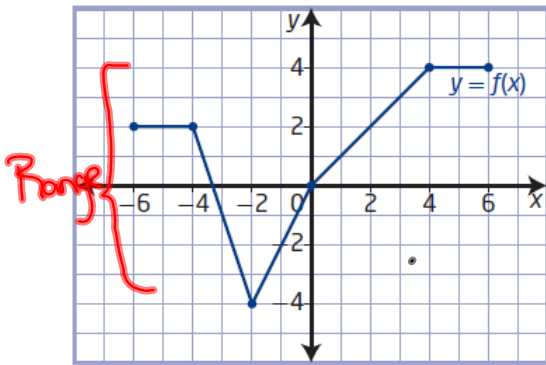} horizontal translation → h

horizontal reflection

vertical reflection

vertical stretch → a

horizontal stretch → b

Multiply x by the reciprocal of b

Questions from Homework

6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2.

$a = 2$

$(x,y) \longrightarrow (x, 2y)$

$D: \{x \mid -6 \leq x \leq 6, x \in R\}$

or $[-6, 6]$

$R: \{y \mid -8 \leq y \leq 8, y \in R\}$

or $[-8, 8]$

Range

2. a) Copy and complete the table of values for the given functions.

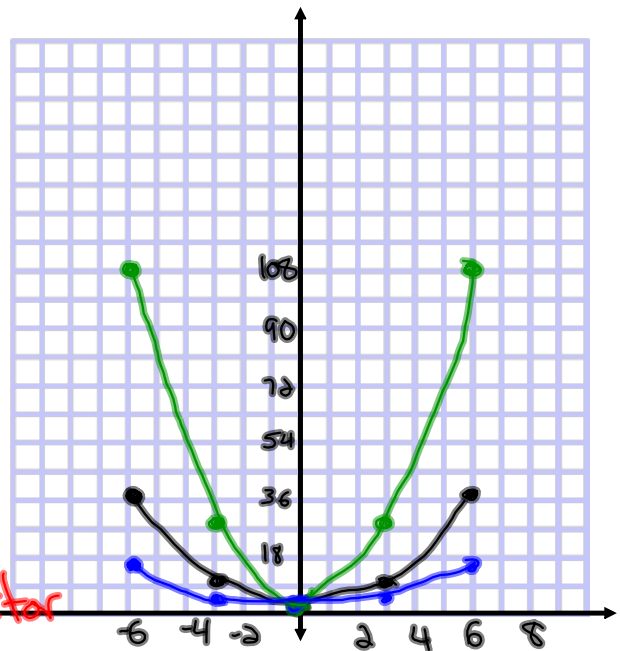$a = 3$ $a = \frac{1}{3}$

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

stretched vertically by a factor of 3

compressed vertically by a factor of $\frac{1}{3}$

$(x,y) \rightarrow (x, 3y)$ $(x,y) \rightarrow (x, \frac{1}{3}y)$

108
90
72
54
36
18

Transformations:

$$g(x) = -3f(4(x-4)) - 10$$

$a = 3$
$b = 4$
$h = 4$
$k = -10$

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the [a] by a factor of [b]. It is vertically stretched about the [c] by a factor of [d]. It is reflected in the [e], and then translated [f] units to the right and [g] units down.

a) y axis
b) ¼
c) x axis
d) 3
e) x axis
f) 4
g) 10

Transformations:

$$y = f(x) \xrightarrow{\hspace{4cm}} y = af\big(b(x-h)\big) + k$$

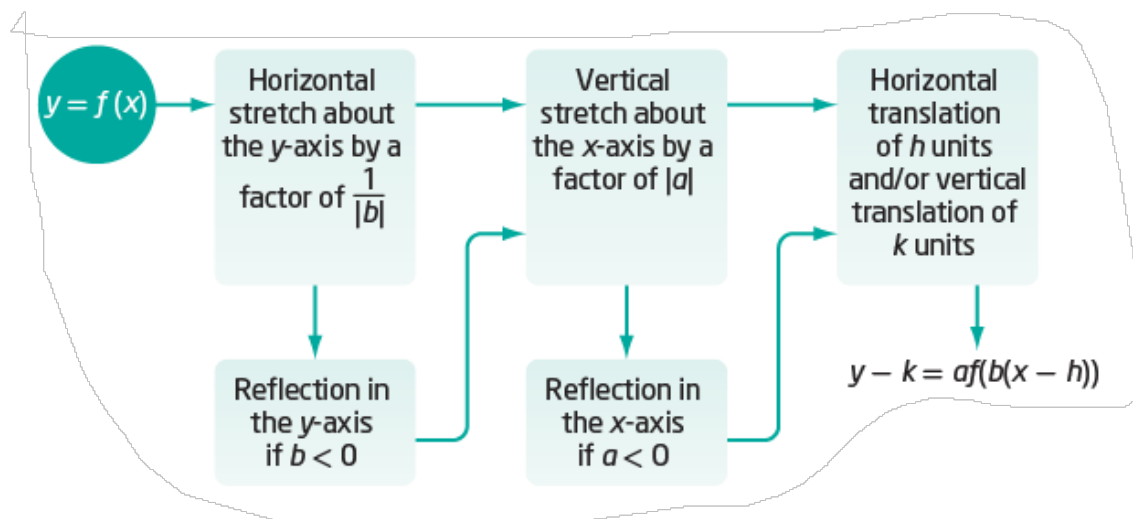**Mapping Rule:** $\qquad (x,y) \to \left(\dfrac{1}{b}x + h, ay + k \right)$

Important note for sketching...

Transformations should be applied in following order:
1. **Reflections**
2. **Stretches**
3. **Translations**

Remember....RST

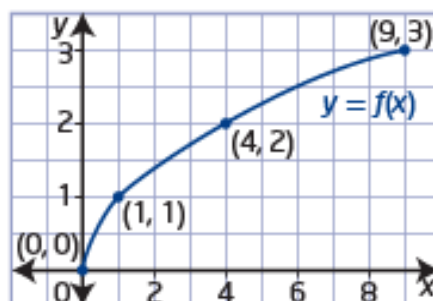| $y = f(x)$	Horizontal stretch about the y-axis by a factor of $\dfrac{1}{\lvert b \rvert}$	Vertical stretch about the x-axis by a factor of $\lvert a \rvert$	Horizontal translation of h units and/or vertical translation of k units
	Reflection in the y-axis if $b < 0$	Reflection in the x-axis if $a < 0$	$y - k = af(b(x - h))$

Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.
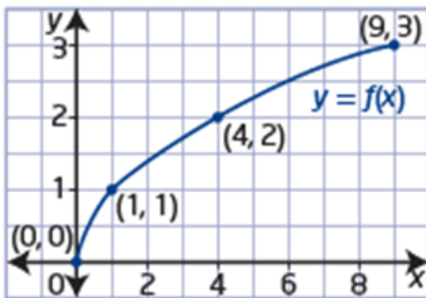
a) $y = 3f(2x)$

b) $y = f(3x + 6)$

a) $y = 3f(2x)$ $a = 3$ $b = 2$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.
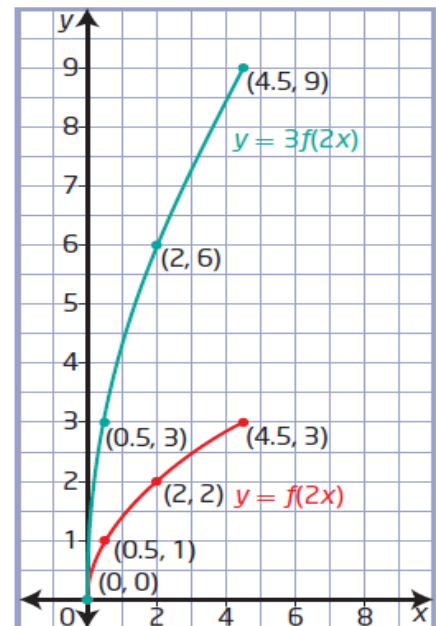
Base: $y = \sqrt{x}$

$(x,y) \rightarrow (\frac{1}{2}x, 3y)$

$(0,0) \rightarrow (0,0)$

$(1,1) \rightarrow (\frac{1}{2}, 3)$
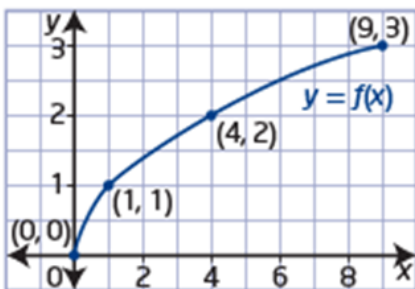
$(4,2) \rightarrow (2, 6)$

$(9,3) \rightarrow (\frac{9}{2}, 9)$
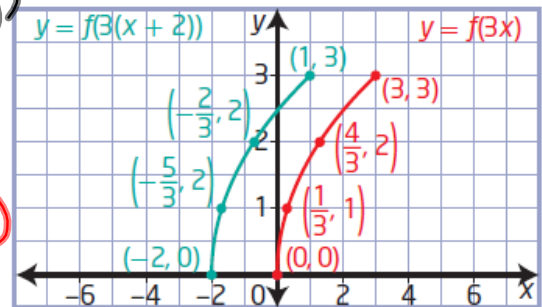
✓ Factor

b) $y = f(3x + 6)$ $a = 1$ $b = 3$ $h = -2$ $k = 0$

$y = f(3(x+2))$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x, y) \rightarrow \left(\frac{1}{3}x - 2, y\right)$

$(0, 0) \rightarrow (-2, 0)$

$(1, 1) \rightarrow \left(-\frac{5}{3}, 1\right)$

$(4, 2) \rightarrow \left(-\frac{2}{3}, 2\right)$
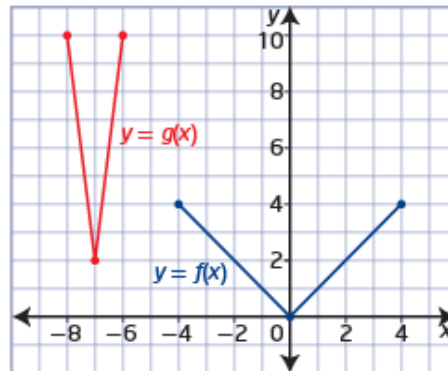
$(9, 3) \rightarrow (1, 3)$

Homework

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Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.
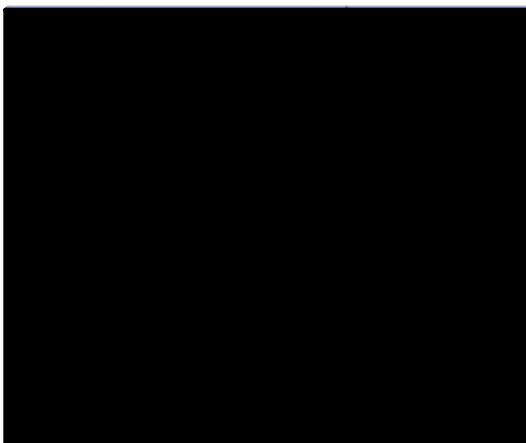
Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is ██████████████

████████████████████

How could you use the mapping

$$(x, y) \rightarrow \left(\frac{1}{b}x + h, \, ay + k\right) \text{ to}$$

verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.