Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a = 3$
$b = 1$
$h = 0$
$k = 0$

$(x, y) \rightarrow (x, 3y)$
$(-2, 5) \rightarrow (-2, 15)$

(2) $y = f\left(-\frac{1}{3}x\right)$

$a = 1$
$b = \frac{-1}{3}$
$h = 0$
$k = 0$

$(x, y) \rightarrow (-3x, y)$
$(-2, 5) \rightarrow (6, 5)$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right] - 3$

$a = 4$
$b = \frac{1}{2}$
$h = -5$
$k = -3$

$(x, y) \rightarrow (2x - 5, 4y - 3)$
$(-2, 5) \rightarrow (-9, 17)$

Factor

(4) $y - 5 = -2f(-2x + 6)$

$y = -2f(-2(x-3)) + 5$

$a = -2$
$b = -2$
$h = 3$
$k = 5$

$(x, y) \rightarrow \left(-\frac{1}{2}x + 3, -2y + 5\right)$
$(-2, 5) \rightarrow (4, -5)$

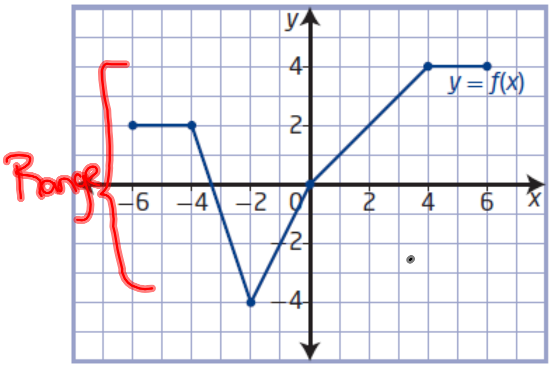
Summary of Transformations...

Transformations of the graphs of functions		
$f(x)+c$	shift $f(x)$ up c units	} vertical translation → k
$f(x)-c$	shift $f(x)$ down c units	
$f(x+c)$	shift $f(x)$ left c units	} horizontal translation → h
$f(x-c)$	shift $f(x)$ right c units	
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal reflection
$-f(x)$	reflect $f(x)$ about the x-axis	vertical reflection
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ Multiply the y values by c	vertical stretch → a
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ Divide the x values by c	horizontal stretch → b

Multiply x by the reciprocal of b

Questions from Homework

6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a=2$



$(x,y) \rightarrow (x, 2y)$

D: $\{x \mid -6 \le x \le 6, x \in R\}$

or $[-6,6]$

R: $\{y \mid -8 \le y \le 8, y \in R\}$

or $[-8,8]$

2. a) Copy and complete the table of values for the given functions.

$a=3$ $\qquad$ $a=\frac{1}{3}$

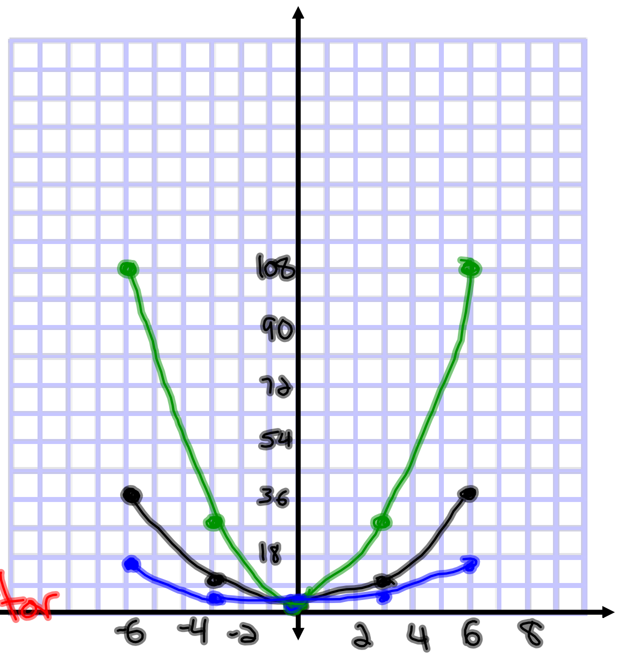
x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

stretched vertically by a factor of 3

$(x,y) \rightarrow (x,3y)$

compressed vertically by a factor of $\frac{1}{3}$

$(x,y) \rightarrow (x,\frac{1}{3}y)$



Transformations:

$g(x) = -3f(4(x-4)) - 10$

$a = -3$
$b = 4$
$h = 4$
$k = -10$

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

a) y axis
b) $\frac{1}{4}$
c) x axis
d) 3
e) x axis
f) 4
g) 10

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right) + k$$

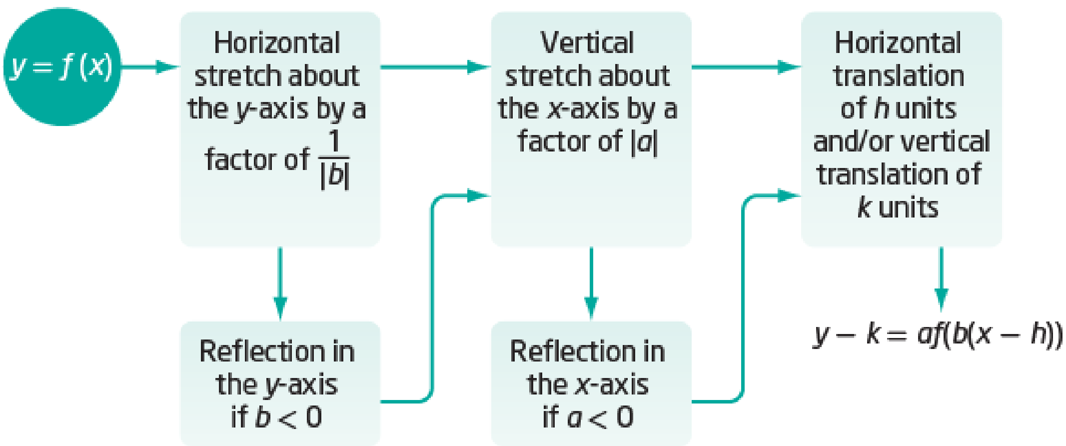
Mapping Rule: $$(x,y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



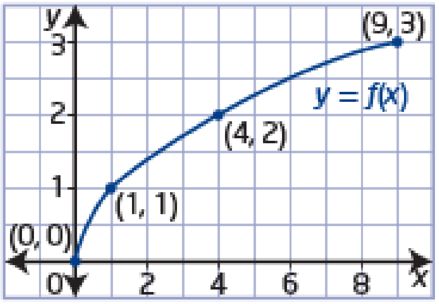
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

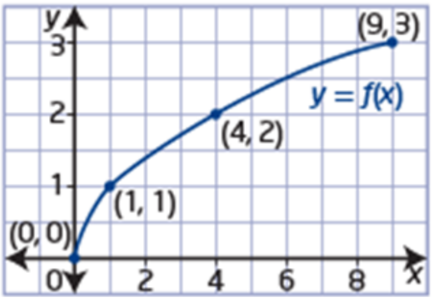
b) $y = f(3x + 6)$



a) $y = 3f(2x)$ $a = 3$ $b = 2$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

Base: $y = \sqrt{x}$



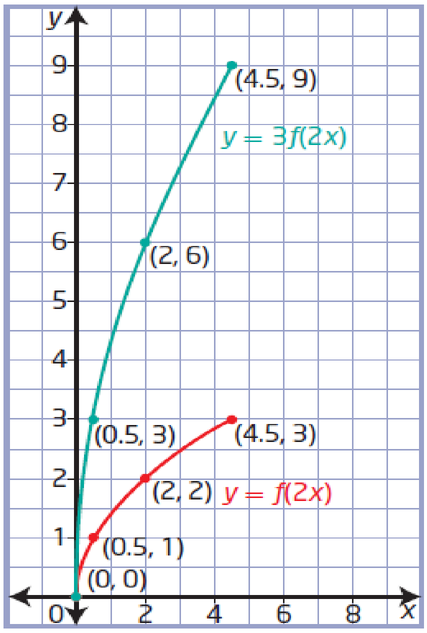
$(x, y) \rightarrow (\frac{1}{2}x, 3y)$

$(0,0) \rightarrow (0,0)$

$(1,1) \rightarrow (\frac{1}{2}, 3)$

$(4,2) \rightarrow (2, 6)$

$(9,3) \rightarrow (\frac{9}{2}, 9)$

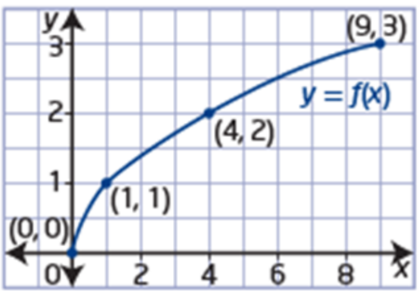


Factor

b) $y = f(3x + 6)$ $a = 1$ $b = 3$ $h = -2$ $k = 0$

$y = f(3(x+2))$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.



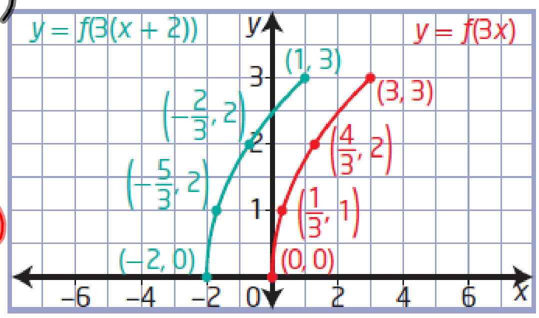
$(x,y) \rightarrow \left(\frac{1}{3}x - 2, y\right)$

$(0,0) \rightarrow (-2, 0)$

$(1,1) \rightarrow (-\frac{5}{3}, 1)$

$(4,2) \rightarrow (-\frac{2}{3}, 2)$

$(9,3) \rightarrow (1, 3)$



Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

$\frac{1}{b}$ HSF is the reciprocal of b

Function	Reflections	Vertical Stretch Factor (a)	Horizontal Stretch Factor ($\frac{1}{b}$)	Vertical Translation (k)	Horizontal Translation (h)
$y - 4 = f(x - 5)$	—	—	—	4	5
$y + 5 = 2f(3x)$	—	2	$\frac{1}{3}$	-5	—
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	—	$\frac{1}{2}$	2	—	4
$y + 2 = -3f(2(x + 2))$	x	3	$\frac{1}{2}$	-2	-2

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

d) $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$ Factor First

$y = -2f\left(-\frac{2}{3}(x + 9)\right) + 4$

$a = -2$ $b = -\frac{2}{3}$ $h = -9$ $k = 4$

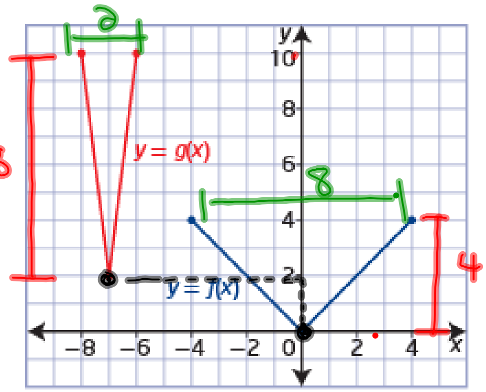
$(x, y) \longrightarrow \left(-\frac{3}{2}x - 9, -2y + 4\right)$

$\boxed{(-12, 18) \longrightarrow (9, -32)}$

Example 3 (Question 4 + 10 pg 39 + 40)

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Vertical Stretch Factor: $\frac{8}{4} = 2$ $a = 2$
(Compare Range $\frac{\text{New}}{\text{old}}$)

② Horizontal Stretch Factor: $\frac{2}{8} = \frac{1}{4}$ $b = 4$
(Compare Domain $\frac{\text{New}}{\text{old}}$)

③ Reflections: None

④ Vertical Translation: Up 2 $k = 2$

⑤ Horizontal Translation: Left 7 $h = -7$

⑥ Equation: $g(x) = 2f(4(x+7)) + 2$

* Check using Key Points:

$(x, y) \longrightarrow (\frac{1}{4}x - 7, 2y + 2)$

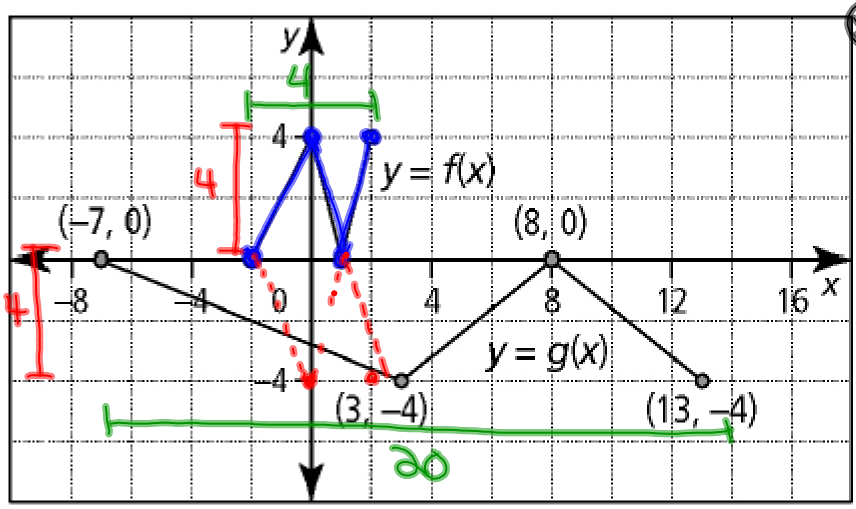
$(-4, 4) \longrightarrow (-8, 10)$

$(0, 0) \longrightarrow (-7, 2)$

$(4, 4) \longrightarrow (-6, 10)$

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

$y=-f\left(\frac{1}{5}(x-3)\right)$



$(x,y) \rightarrow (5x+3, -y+0)$

(-2,0)	(-7,0)
(0,4)	(3,-4)
(1,0)	(8,0)
(2,4)	(13,-4)

① VSF: $\frac{4}{4} = 1$ $a=1$

② HSF: $\frac{20}{4} = 5$ $b=\frac{1}{5}$

(-a)

③ Reflection: Vertical in x-axis $y=-f(x)$

④ VT (Pick a point on x-axis because y=0): None $k=0$

$(-2,0) \rightarrow (-7,0)$

⑤ HT (Pick a point on y-axis because x=0): Right 3 $h=3$

$(0,4) \rightarrow (3,-4)$

⑥ Equation: $y={}^{-}f\left(\frac{1}{5}(x-3)\right)+0$

$y={}^{-}f\left(\frac{1}{5}(x-3)\right)$

Homework

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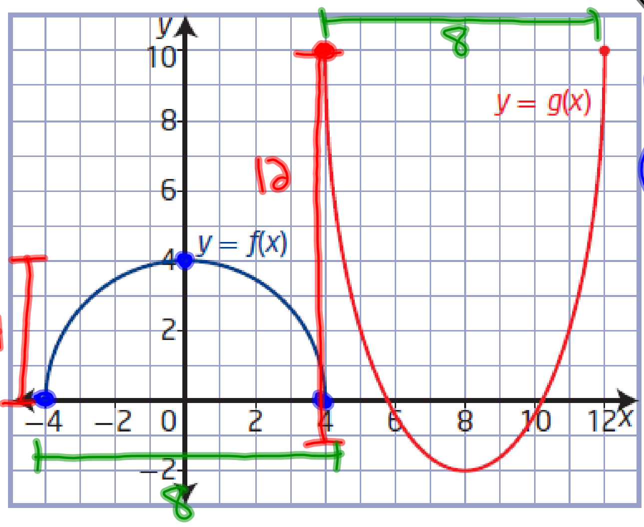
Plus 7, 8, 9 (a, c, e) and 10

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

a)



$(x,y) \rightarrow (1x+8, -3y+10)$

$(-4,0) \rightarrow (4,10)$

$(0,4) \rightarrow (8,-2)$

$(4,0) \rightarrow (12,10)$

① VSF: $\frac{12}{4} = 3$ $(a=3)$

② HSF: $\frac{8}{8} = 1$ $(b=1)$ ← reciprocal of 1 is 1

③ Reflection: Vertical reflection in the x-axis $(-a)$

④ VT: $(-4,0) \rightarrow (4,10)$ Up 10 → $k=10$

⑤ HT: $(0,4) \rightarrow (8,-2)$ Right 8 → $h=8$

⑥ Equation: $y = -3f(1(x-8)) + 10$

$y = -3f(x-8) + 10$

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

(parabola)

$a=2 \quad b=3$

$h=2 \quad k=-4$

Complete the Square:

$y = 2x^2+x+1$ (Bring the 1 over)

$y-1 = 2x^2+x$ (Factor out 2)

$y-1 = 2(x^2+\frac{1}{2}x)$ (Half it + Square it)

$\left[\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right]^2 = \frac{1}{16}$

$y-1+\frac{1}{8} = 2(x^2+\frac{1}{2}x+\frac{1}{16})$ (Balance Left Side)

$y-\frac{8}{8}+\frac{1}{8} = 2(x+\frac{1}{4})^2$ (Keep the sign and use half the middle term)

$y-\frac{7}{8} = 2(x+\frac{1}{4})^2$ (Bring the $-\frac{7}{8}$ over)

$y = 2(x+\frac{1}{4})^2+\frac{7}{8}$

$a=2 \quad b=3$

$h=2 \quad k=-4$

$y = 2^2(3(x+\frac{1}{4}-2))^2+\frac{7}{8}-\frac{4}{1}$

$y = 4(3(x-\frac{7}{4}))^2-\frac{25}{8}$