Inverse of a Relation

f^{-1} is read f "inverse" ← not considered an exponent

An inverse function is a second function which undoes the work of the first one.

1. Introduction

Suppose we have a function f that takes x to y, so that

$$f(x) = y.$$

An inverse function, which we call f^{-1}, is another function that takes y back to x. S

$$f^{-1}(y) = x.$$

For f^{-1} to be an inverse of f, this needs to work for every x that f acts upon.

Inverse of a Relation

The inverse of a relation is found by interchanging the x-coordinates and y-coordinates of the ordered pairs of the relation. In other words, for every ordered pair (x, y) of a relation, there is an ordered pair (y, x) on the inverse of the relation. This means that the graphs of a relation and its inverse are reflections of each other in the line $y = x$.

$(x, y) \rightarrow (y, x)$ In plain English....the x and y coordinates will just switch places

The inverse of a function $y = f(x)$ may be written in the form $x = f(y)$. The inverse of a function is not necessarily a function. When the inverse of f is itself a function, it is denoted as f^{-1} and read as "f inverse." When the inverse of a function is not a function, it may be possible to restrict the domain to obtain an inverse function for a portion of the original function.

The inverse of a function reverses the processes represented by that function. Functions $f(x)$ and $g(x)$ are inverses of each other if the operations of $f(x)$ reverse all the operations of $g(x)$ in the opposite order and the operations of $g(x)$ reverse all the operations of $f(x)$ in the opposite order.

For example, $f(x) = 2x + 1$ multiplies the input value by 2 and then adds 1. The inverse function subtracts 1 from the input value and then divides by 2. The inverse function is $f^{-1}(x) = \frac{x-1}{2}$.

Example 1

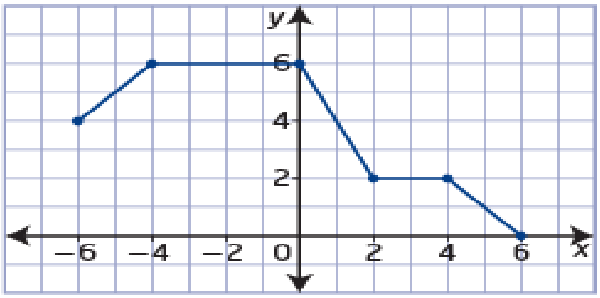
Graph an Inverse

Consider the graph of the relation shown.

a) Sketch the graph of the inverse relation.

b) State the domain and range of the relation and its inverse.

c) Determine whether the relation and its inverse are functions.



Solution

a) To graph the inverse relation, interchange the x-coordinates and y-coordinates of key points on the graph of the relation.

Points on the Relation	Points on the Inverse Relation
(−6, 4)	
(−4, 6)	
(0, 6)	
(2, 2)	
(4, 2)	
(6, 0)	

[redacted]

The graphs are reflections of each other in the line $y = x$. The points on the graph of the relation are related to the points on the graph of the inverse relation by the mapping $(x, y) \rightarrow (y, x)$.

What points are invariant after a reflection in the line $y = x$?

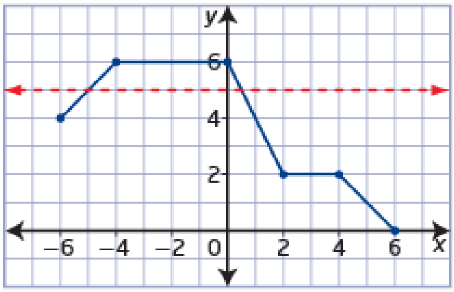
b) State the domain and range of the relation and its inverse.

	Domain	Range
Relation		
Inverse Relation		

The domain of the relation becomes the range of the inverse relation and the range of the relation becomes the domain of the inverse relation.

In plain English....the x and y coordinates will just switch places

c) Determine whether the relation and its inverse are functions.



horizontal line test

- a test used to determine if the graph of an inverse relation will be a function
- if it is possible for a horizontal line to intersect the graph of a relation more than once, then the inverse of the relation is not a function

The inverse relation is not a function of x because it fails the vertical line test. There is more than one value of y in the range for at least one value of x in the domain. You can confirm this by using the **horizontal line test** on the graph of the original relation.

What if given the function algebraically?

Determine algebraically the equation of the inverse of each function.

a) $f(x) = 3x - 6$

b) $f(x) = \frac{1}{2}x + 5$

c) $f(x) = \frac{1}{3}(x + 12)$

d) $f(x) = \frac{8x + 12}{4}$

Practice Problems...

Pages 51 - 55
#2, 3, 5, 6, 8, 9, 11, 15, 18, 20, 21