# Understanding Logarithms

$(x,y) \rightarrow (y,x)$
reflection in the line $y=x$

## Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- determining the characteristics of the graph of $y = \log_c x$, $c > 0$, $c \neq 1$
- explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

# Questions from Homework

**11.** Explain how the graph of $\frac{1}{3}(y+2) = \log_6(x-4)$ can be generated by transforming the graph of $y = \log_6 x$.

$y+2 = 3\log_6(x-4)$ Divide a+k by $\frac{1}{3}$

$y = 3\log_6(x-4) - 2$ Subtract 2 from both sides

$y = 3\log_6 1(x-4) - 2$

a=3 → A vertical stretch by a factor of 3

b=1 → No horizontal stretch.

h=4 → Translated 4 units right.

k=-2 → " 2 " down.

**5.** Identify the following characteristics of the graph of each function.

**i)** the equation of the asymptote

**ii)** the domain and range

**iii)** the $y$-intercept, to one decimal place if necessary

**iv)** the $x$-intercept, to one decimal place if necessary

**a)** $y = -5\log_3(x+3)$

**b)** $y = \log_6(4(x+9))$

**c)** $y = \log_5(x+3) - 2$

**d)** $y = -3\log_2(x+1) - 6$

b) a=1 b=4 h=-9 k=0

$(x,y) \longrightarrow \left[\frac{1}{4}x - 9,\ 1y + 0\right]$

Base: $y = \log_6 x$

D: $\{x \mid x > 0, x \in R\}$

R: $\{y \mid y \in R\}$

x int: (1,0)

y int: none

VA: x=0

For: $y = \log_6(4(x+9))$

(i) VA: $x = -9$

(ii) D: $\{x \mid x > -9, x \in R\}$

R: $\{y \mid y \in R\}$

(iii) y intercept (Let x=0)

$y = \log_6(4(x+9))$

$y = \log_6(4(0+9))$

$y = \log_6 36$

$\boxed{y=2}$ or (0,2)

(iv) x-intercept (Let y=0)

$y = \log_6(4(x+9))$

$0 = \log_6(4x+36)$ ← ans.

↑ exp ↑ base

$6^0 = 4x+36$

**General Properties of Logarithms:** * a logarithm is an exponent !

If $c > 0$ and $c \neq 1$, then...

(i) $\log_c 1 = 0$

(ii) $\log_c c^x = x$

(iii) $c^{\log_c x} = x$

**Did You Know?**

The input value for a logarithm is called an argument. For example, in the expression $\log_6 1$, the argument is 1.

(i) $\log_5 1 = 0$ (ii) $\log_2 2^3 = 3$ (iii) $7^{\log_7 49} = 49$

$5^{\log_5 10} = 10$

## Product Law of Logarithms (Page 394)

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\log_c MN = \log_c M + \log_c N$$

Ex: $\log_2 8 + \log_2 3 = \log_2(8 \times 3) = \boxed{\log_2 24}$
$\approx 4.59$

*Proof*

Let $\log_c M = x$ and $\log_c N = y$, where $M$, $N$, and $c$ are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$MN = (c^x)(c^y)$

$MN = c^{x+y}$ — Apply the product law of powers.

$\log_c MN = x + y$ — Write in logarithmic form.

$\log_c MN = \log_c M + \log_c N$ — Substitute for $x$ and $y$.

## Quotient Law of Logarithms

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$

Ex: $\log 400 - \log 4 = \log\left(\frac{400}{4}\right) = \log 100 = \boxed{2}$

*Proof*

Let $\log_c M = x$ and $\log_c N = y$, where $M$, $N$, and $c$ are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$$\frac{M}{N} = \frac{c^x}{c^y}$$

$$\frac{M}{N} = c^{x-y}$$ Apply the quotient law of powers.

$$\log_c \frac{M}{N} = x - y$$ Write in logarithmic form.

$$\log_c \frac{M}{N} = \log_c M - \log_c N$$ Substitute for $x$ and $y$.

## Power Law of Logarithms

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$\log_c M^P = P \log_c M$

How could you prove the quotient law using the product law and the power law?

*Proof*

Let $\log_c M = x$, where $M$ and $c$ are positive real numbers with $c \neq 1$.

Write the equation in exponential form as $M = c^x$.

Let $P$ be a real number.

$M = c^x$

$M^P = (c^x)^P$

$M^P = c^{xP}$ — Simplify the exponents.

$\log_c M^P = xP$ — Write in logarithmic form.

$\log_c M^P = (\log_c M)P$ — Substitute for *x*.

$\log_c M^P = P \log_c M$

The laws of logarithms can be applied to logarithmic functions, expressions, and equations.

Ex: $\log_2 \sqrt{8}$

$\log_2 8^{1/2}$

$\frac{1}{2} \log_2 8$

$\frac{1}{2}(3)$

$\frac{3}{2}$

# Homework

## Finish Exercise 2

## Example 1

### Use the Laws of Logarithms to Expand Expressions

Write each expression in terms of individual logarithms of $x$, $y$, and $z$.

**a)** $\log_5 \frac{xy}{z}$

**b)** $\log_7 \sqrt[3]{x}$

**c)** $\log_6 \frac{1}{x^2}$

**d)** $\log \frac{x^3}{y\sqrt{z}}$

## Example 2

### Use the Laws of Logarithms to Evaluate Expressions

Use the laws of logarithms to simplify and evaluate each expression.

**a)** $\log_6 8 + \log_6 9 - \log_6 2$

**b)** $\log_7 7\sqrt{7}$

**c)** $2 \log_2 12 - \left(\log_2 6 + \frac{1}{3} \log_2 27\right)$

## Example 3

### Use the Laws of Logarithms to Simplify Expressions

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

**a)** $\log_7 x^2 + \log_7 x - \dfrac{5 \log_7 x}{2}$

**b)** $\log_5 (2x - 2) - \log_5 (x^2 + 2x - 3)$

## Key Ideas

- Let $P$ be any real number, and $M$, $N$, and $c$ be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

| Name | Law | Description |
| --- | --- | --- |
| Product | $\log_c MN = \log_c M + \log_c N$ | The logarithm of a product of numbers is the sum of the logarithms of the numbers. |
| Quotient | $\log_c \frac{M}{N} = \log_c M - \log_c N$ | The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor. |
| Power | $\log_c M^P = P \log_c M$ | The logarithm of a power of a number is the exponent times the logarithm of the number. |

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

# Homework

# Do I really understand??...

a) Express the following as a single logarithm... $2\log_2 3^2 + \log_2 6 - 3\log_2 3$

b) Evaluate the following... $\log_2 (32)^{\frac{1}{3}}$

c) Express the following as a single logarithm... $\frac{1}{2}\left[(\log_5 a + 2\log_5 b) - 3\log_5 c\right]$

d) Express as a single logarithm in simplest form...

$$\frac{3}{4}\left[12(\log_b x^2 - 2\log_b x) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7}\right]$$