Understanding Logarithms

Focus on...

- demonstrating that a logarithmic function is the inverse of an exponential function
- sketching the graph of $y = \log_c x$, c > 0, $c \ne 1$
- determining the characteristics of the graph of $y = \log_c x$, c > 0, $c \ne 1$
- · explaining the relationship between logarithms and exponents
- expressing a logarithmic function as an exponential function and vice versa
- evaluating logarithms using a variety of methods

General Properties of Logarithms:

If C > 0 and $C \ne 1$, then... (i) $\log_C 1 = 0$ (ii) $\log_C c^x = x$ (iii) $c^{\log_C x} = x$

(i)
$$\log_{c} 1 = 0$$

(ii)
$$\log_{\mathbf{c}} \mathbf{c}^{x} = x$$

(iii)
$$c^{\log_{c} x} = x$$

Did You Know?

The input value for a logarithm is called an argument. For example, in the expression log₆ 1, the argument is 1.

(1)
$$\log_5 1 = 0$$
 (11) $\log_3 3 = 3$

Product Law of Logarithms (Page 394)

The logarithm of a product of numbers can be expressed as the sum of the logarithms of the numbers.

$$\log_c MN = \log_c M + \log_c N$$

$$Proof$$

$$= \log_c M + \log_c N$$

Let $\log_c M = x$ and $\log_c N = y$, where M, N, and c are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$$\begin{split} MN &= (c^x)(c^y) \\ MN &= c^{x+y} \\ \log_c MN &= x+y \\ \log_c MN &= \log_c M + \log_c N \end{split} \qquad \text{Apply the product law of powers.}$$

Quotient Law of Logarithms

The logarithm of a quotient of numbers can be expressed as the difference of the logarithms of the dividend and the divisor.

$$\frac{\log_c \frac{M}{N} = \log_c M - \log_c N}{Proof}$$
Ex: $\log 400 - \log 4 = \log(\frac{400}{4}) = \log 100$

Let $\log_c M = x$ and $\log_c N = y$, where M, N, and c are positive real numbers with $c \neq 1$.

Write the equations in exponential form as $M = c^x$ and $N = c^y$:

$$\frac{M}{N} = \frac{c^x}{c^y}$$

$$\frac{M}{N} = c^{x-y}$$
 Apply the quotient law of powers.
$$\log_c \frac{M}{N} = x - y$$
 Write in logarithmic form.
$$\log_c \frac{M}{N} = \log_c M - \log_c N$$
 Substitute for x and y .

Power Law of Logarithms

The logarithm of a power of a number can be expressed as the exponent times the logarithm of the number.

$$\log_c M^p = P \log_c M$$

How could you prove the quotient law using the product law and the power law?

Proof

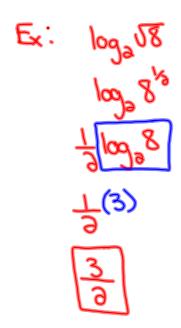
Let $\log_c M = x$, where M and c are positive real numbers with $c \neq 1$.

Write the equation in exponential form as $M = c^{x}$.

Let P be a real number.

$$M=c^x$$
 $M^p=(c^x)^p$ $M^p=c^{xp}$ Simplify the exponents. $\log_c M^p=xP$ Write in logarithmic form. $\log_c M^p=(\log_c M)P$ Substitute for x . $\log_c M^p=P\log_c M$

The laws of logarithms can be applied to logarithmic functions, expressions, and equations.



Questions from Homework

$$\Theta$$
 c) $\log_{10}(3x+5) = 3$ Logar ithmic Form

 $10^3 = 3x+5$
 $100 = 3x+5$
 $96 = 3x$
 $95 = x$

h)
$$10^{5^{\times}}$$
 = 3 Exponential Form $(\log_{10} 3) = 5^{\times}$ Logarithmic Form ans.

$$\int \log_{\varepsilon}(\log^{3}) = \times$$

9)
$$\log_{3}(\log_{3}x) = 4$$

 $\partial^{4} = \log_{3}x$
 $16 = \log_{3}x$
 $3^{6} = x$
 $43.046.721 = x$

9)
$$\log_{3}(\log_{3}x) = 4$$

e) $2^{1-x} = 3$
 $2^{4} = \log_{3}x$
 $16 = \log_{3}x$
 $x = 1 - \log_{3}x$
 $x = 1 - \log_{3}x$
 $x = 1 - \log_{3}x$

Example 1

Use the Laws of Logarithms to Expand Expressions

Write each expression in terms of individual logarithms of x, y, and z.

a)
$$\log_5 \frac{xy}{z} = \log_5 x + \log_5 y - \log_5 z$$

b)
$$\log_7 \sqrt[3]{x} = \log_7 x^{1/3} = \frac{1}{3} \log_7 x$$

b)
$$\log_7 \sqrt[3]{x} = \log_7 x^{1/3} = \frac{1}{3}\log_7 x$$

c) $\log_6 \frac{1}{x^2} = \log_6 1 - \log_6 x^3 = 0 - 2\log_6 x = -2\log_6 x$

d)
$$\log \frac{X^3}{y\sqrt{Z}}$$

$$= \log x^3 - [\log y + \log z^3]$$

$$= 3\log x - \log y - \frac{1}{5}\log z$$

Example 2

Use the Laws of Logarithms to Evaluate Expressions

Use the laws of logarithms to simplify and evaluate each expression.

- a) $\log_6 8 + \log_6 9 \log_6 2$
- **b)** $\log_7 7\sqrt{7}$
- c) $2 \log_2 12 (\log_2 6 + \frac{1}{3} \log_2 27)$

Example 3

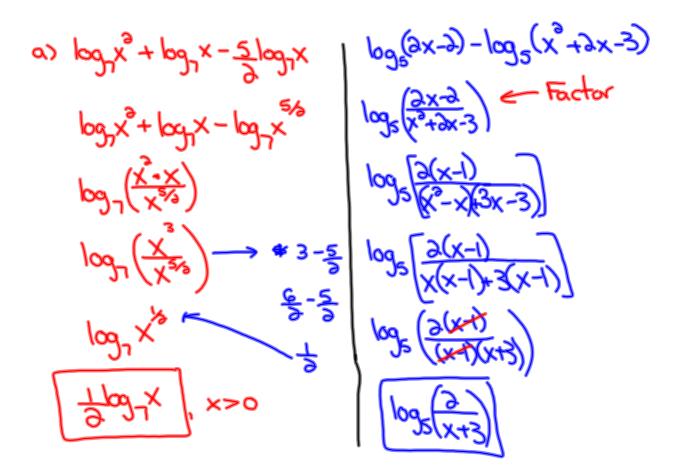
—0

Use the Laws of Logarithms to Simplify Expressions

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a)
$$\log_7 x^2 + \log_7 x - \frac{5 \log_7 x}{2}$$

b)
$$\log_5 (2x-2) - \log_5 (x^2 + 2x - 3)$$



For the original expression to be defined, both logarithmic terms must be defined.

$$2x-2>0 \qquad x^2+2x-3>0 \qquad \text{What other methods could} \\ 2x>2 \qquad (x+3)(x-1)>0 \qquad \text{you have used to solve this} \\ x>1 \quad \text{and} \quad x<-3 \text{ or } x>1$$

The conditions x > 1 and x < -3 or x > 1 are both satisfied when x > 1.

Hence, the variable x needs to be restricted to x>1 for the original expression to be defined and then written as a single logarithm.

Therefore,
$$\log_5 (2x-2) - \log_5 (x^2 + 2x - 3) = \log_5 \frac{2}{x+3}, x > 1.$$

Key Ideas

• Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \frac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^p = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

Many quantities in science are measured using a logarithmic scale. Two
commonly used logarithmic scales are the decibel scale and the pH scale.

Homework

Finish Exercise 3

Do I really understand??...

- a) Express the following as a single logarithm... $2 \log_2 3^2 + \log_2 6 3 \log_2 3$
- b) Evaluate the following... $\log_2(32)^{\frac{1}{3}}$
- c) Express the following as a single logarithm... $\frac{1}{2} [(\log_5 a + 2\log_5 b) 3\log_5 c]$
- d) Express as a single logarithm in simplest form...

$$\frac{3}{4} \left[12 (\log_b x^2 - 2\log_b x) + 8\log_b \sqrt{x} - 4\log_b \frac{1}{x^7} \right]$$