Questions from Homework

Product Rule

$$F'(x) = f(x)g'(x) + f'(x)g(x)$$

Quotient Rule

$$F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

④ $f(2) = 3$, $f'(2) = 5$, $g(2) = -1$, $g'(2) = -4$

$$\left(\frac{f}{g}\right)'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$$

$$= \frac{(-1)(5) - (3)(-4)}{(-1)^2}$$

$$= \frac{-5+12}{1}$$

$$= 7$$

① d) $g(x) = \frac{x^3-1}{x^2+x+1}$

$$g'(x) = \frac{(x^2+x+1)(3x^2) - (x^3-1)(2x+1)}{[x^2+x+1]^2}$$

$$g'(x) = \frac{3x^4+3x^3+3x^2 - (2x^4+x^3-2x-1)}{[x^2+x+1]^2}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{x^4+2x^3+3x^2+2x+1}$$

$$g'(x) = 1$$

⑥ $y = \frac{x^2}{2x+5}$

* Horizontal Line
Slope = 0 $m = 0$

① Differentiate

$$y' = \frac{(2x+5)(2x) - x^2(2)}{[2x+5]^2} = \frac{4x^2+10x-2x^2}{[2x+5]^2} = \frac{2x^2+10x}{(2x+5)^2}$$

Slope of Tangent

② Take derivative and set it equal to 0

Factor $\frac{2x^2+10x}{(2x+5)^2} \overset{\text{set}}{=} \frac{0}{1}$

$2x^2+10x = 0$

$(2x)(x+5) = 0$

$2x = 0$ | $x+5 = 0$

$x = 0$ | $x = -5$

③ Find Points. (x, y)

$y = \frac{x^2}{2x+5}$

when $(x = 0)$

$y = \frac{(0)^2}{2(0)+5} = \frac{0}{5} = 0$

$(0, 0)$

when $(x = -5)$

$y = \frac{(-5)^2}{2(-5)+5} = \frac{25}{-5} = -5$

$(-5, -5)$

Questions from Homework

① e) $y = \dfrac{\sqrt{x}}{x^2+1} = \dfrac{x^{1/2}}{x^2+1}$

$$y' = \frac{(x^2+1)\left(\frac{1}{2}x^{-1/2}\right) - (x^{1/2})(2x)}{(x^2+1)^2}$$

$$= \frac{2\cdot\frac{1}{2}x^{3/2} + 2\cdot\frac{1}{2}x^{-1/2} - 2x^{3/2}}{2\cdot(x^2+1)^2}$$

$$= \frac{x^{3/2} + x^{-1/2} - 4x^{3/2}}{2(x^2+1)^2}$$

$$= \frac{\frac{1}{\sqrt{x}} - 3x^{3/2}\sqrt{x}}{2(x^2+1)^2\sqrt{x}}$$

$$= \frac{1-3x^2}{2\sqrt{x}(x^2+1)^2}$$

Chain Rule:

The Chain Rule If f and g are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))g'(x)$$

Work from the outside to the inside

Examples:

$f(x) = (5x^3 + 1)^{10}$

$f'(x) = 10(5x^3+1)^9(15x^2)$

$f'(x) = 150x^2(5x^3+1)^9$

$F(x) = \sqrt{2x^2 + 3} = (2x^2+3)^{1/2}$

$F'(x) = \frac{1}{2}(2x^2+3)^{-1/2}(4x)$

$F'(x) = 2x(2x^2+3)^{-1/2}$

$F'(x) = \frac{2x}{(2x^2+3)^{1/2}} = \frac{2x}{\sqrt{2x^2+3}}$

$h(x) = \sqrt[3]{5 - 3x^4} = (5-3x^4)^{1/3}$

$h'(x) = \frac{1}{3}(5-3x^4)^{-2/3}(-12x^3)$

$h'(x) = -4x^3(5-3x^4)^{-2/3}$

$h'(x) = \frac{-4x^3}{(5-3x^4)^{2/3}} = \frac{-4x^3}{\sqrt[3]{(5-3x^4)^2}}$

Homework

$$g(x)=9x^{-3}\left(5x^{3}-1\right)^{6}$$

$$g(x)=\frac{\left(x^{2}-5x+1\right)^{8}}{\left(1-x^{-7}\right)^{20}}$$