Warm Up

Differentiate:

$\sec^{-1}\left((\tan\sqrt{x})^2\right)$

$$(1)\ f(x) = e^{\log\sqrt{x}} - 3^{\cos^{-1}(\ln x^5)} + Arc\sec\left(\tan^2\sqrt{x}\right)$$

$$f'(x) = e^{\log\sqrt{x}}\left(\frac{1}{\sqrt{x}\ln 10}\right)\left(\frac{1}{2\sqrt{x}}\right) - 3^{\cos^{-1}(\ln x^5)}(\ln 3)\left(\frac{-1}{\sqrt{1-(\ln x^5)^2}}\right)\left(\frac{5x^4}{x^5}\right) + \left(\frac{1}{\tan^2\sqrt{x}\sqrt{(\tan^2\sqrt{x})^2-1}}\right)(2)(\tan\sqrt{x})(\sec^2\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)$$

Problems with homework?

Antiderivatives

Up to this point in our study of calculus, we have been concerned primarily with the problem:
Given a function, find its derivative.

Many important applications of calculus involve the inverse problem:
Given the derivative, find the original function.

"The basic problem of differentiation is: given the path of a moving point, to calculate its velocity, or given a curve, to calculate its slope. The basic problem of integration is the inverse: given the velocity of a moving point at every instant, to calculate its path, or given the slope of a curve at each of its points, to calculate the curve."

For example, suppose we are given the following derivatives: $f'(x) = 2$, $g'(x) = 3x^2$ $h'(t) = 4t$

Our goal is to determine f(x), g(x), and h(x) that have the respective derivatives given above. If we make some educated guesses, what would these functions be????

$f(x) = 2x + C$ $g(x) = x^3 + C$ $h(t) = 2t^2 + C$

This operation of determining the original function from its derivative is the inverse operation of differentiation and we call it antidifferentiation.

Definition: A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

"F(x) is an antiderivative of f(x)"

It should be emphasized that if F(x) is an antiderivative of f(x), then F(x) + C (C is any constant) is also an antiderivative of f(x).

Antiderivatives

$$f(x) \xrightarrow{\text{Differentiate}} f'(x)$$

$$f(x) \xleftarrow{\text{Antidifferentiate}} f'(x)$$

Definition A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example:

$$F(x) = \frac{x^2}{2} \quad \rightleftarrows \quad F'(x) = x$$

What about $F(x) = \frac{x^2}{2} + 2$?

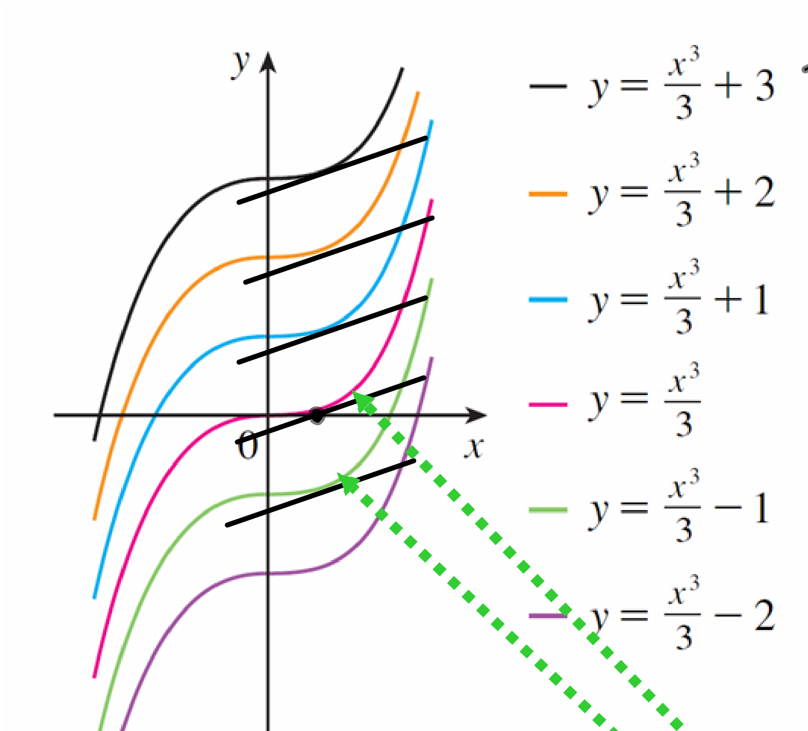
1 Theorem If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$\boxed{F(x) + C}$$

where C is an arbitrary constant.

General antiderivatives are considered a family of curves...

Here is an example of a family of general antiderivatives:



Constant

$$y = \frac{x^3}{3} + C$$

FIGURE 1

Notice the slopes of the tangents on each curve at the same *x*-coordinate. (same slope)

Antidifferentiation Rules...

Constants:

Determine the antiderivative of any constant

$f'(x) = 3$ $f(x) = -4$

$f(x) = 3x + C$ $F(x) = -4x + C$

Rule:

$$f(x) = k \quad \Rightarrow \quad F(x) = kx + C$$

Bring exponent down (multiply it by the constant) rewrite your base and then subtract one from the original exponent

Power Law:

How will we put the power rule in reverse?

ie. If $f'(x) = 6x^2$ what is $f(x)$? $= \frac{6x^{2+1}}{2+1} = 2x^3 + C$

Flip your brain into reverse...what will be the rule used to antidifferentiate power rules?

Rule:

$$f(x) = kx^n \quad \Rightarrow \quad F(x) = \frac{k}{n+1}x^{n+1} + C$$

Add one to the exponent and divide by this NEW exponent

Determine the general antiderivative of each of the following...

(1) $f(x) = x^5 - 2x^4 - 3x^3 + \dfrac{2}{x^2} + 5x^{-3} + 5$

$f(x) = x^5 - 2x^4 - 3x^3 + 2x^{-2} + 5x^{-3} + 5$

$F(x) = \dfrac{x^6}{6} - \dfrac{2x^5}{5} - \dfrac{3x^4}{4} + \dfrac{2x^{-1}}{-1} + \dfrac{5x^{-2}}{-2} + 5x + C$

$F(x) = \dfrac{1}{6}x^6 - \dfrac{2}{5}x^5 - \dfrac{3}{4}x^4 - \dfrac{2}{x} - \dfrac{5}{2x^2} + 5x + C$

(2) $f(x) = 3\sqrt{x} - \dfrac{2}{5x^4} + \sqrt[5]{x^7} - \dfrac{6\sqrt{x}}{x^2} + e^2$

$f(x) = 3x^{1/2} - \dfrac{2}{5}x^{-4} + x^{7/5} - 6x^{1/2}x^{-2} + e^2$

$f(x) = 3x^{1/2} - \dfrac{2}{5}x^{-4} + x^{7/5} - 6x^{-3/2} + e^2$

$F(x) = \dfrac{3x^{3/2}}{3/2} \dfrac{-\frac{2}{5}x^{-3}}{-3} + \dfrac{x^{12/5}}{12/5} - \dfrac{6x^{-1/2}}{-1/2} + e^2x + C$

$F(x) = 2x^{3/2} + \dfrac{2}{15x^3} + \dfrac{5}{12}x^{12/5} + \dfrac{12}{\sqrt{x}} + e^2x + C$

- ✓ constants
- ✓ power rules
- logarithmic functions
- trigonometric functions
- exponential functions
- inverse trigonometric functions
- chain rules

Table of some of the Most General Antiderivatives

where a is a constant!

Function, f(x)	Most General Antiderivative, F(x)
a	$ax + C$
$ax^n \ (n \neq -1)$	$\frac{a}{n+1}x^{n+1} + C$
$\frac{a}{x} \ (x \neq 0)$	$a \ln\lvert x\rvert + C$
ae^{kx}	$\frac{a}{k}e^{kx} + C$
a^{kx}	$\frac{a^x}{k \ln a} + c$
$a \cos kx$	$\frac{a}{k}\sin kx + C$
$a \sin kx$	$-\frac{a}{k}\cos kx + C$
$a \sec^2 kx$	$\frac{a}{k}\tan kx + C$
$a \sec kx \tan kx$	$\frac{a}{k}\sec kx + C$
$a \csc kx \cot kx$	$-\frac{a}{k}\csc kx + C$
$a \csc^2 kx$	$-\frac{a}{k}\cot kx + C$
$\frac{a}{\sqrt{1-(kx)^2}}$	$\frac{a}{k}\sin^{-1} kx + C$
$\frac{a}{1+(kx)^2}$	$\frac{a}{k}\tan^{-1} kx + C$

Practice:

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