Questions from Homework

⑥a) $f(x) = \sqrt{x} - \sqrt{1-x} = x^{1/2} - (1-x)^{1/2}$

$F(x) = \frac{2}{3}x^{3/2} - \frac{2}{3}(1-x)^{3/2}(-1) + C$

$F(x) = \frac{2}{3}x^{3/2} + \frac{2}{3}(1-x)^{3/2} + C$

⑥b) $f(x) = \frac{1}{x} - \frac{1}{1-x}$

$F(x) = 1\ln|x| - 1\ln|1-x|(-1) + C$

$F(x) = \ln|x| + \ln|1-x| + C$

$F(x) = \ln(x-x^2) + C$

Warm Up

Determine the general antiderivative of the following:

$$f(x) = 2x^2 - x + 7$$

$F(x) = \frac{2x^3}{3} - \frac{x^2}{2} + 7x + C$

$$f(x) = \cos x - \sin x$$

$F(x) = \sin x - (-\cos x) + C$

$F(x) = \sin x + \cos x + C$

$$f(x) = -3e^{-x} + 6e^{2x}$$

$F(x) = \frac{-3e^{-x}}{-1} + \frac{6e^{2x}}{2} + C$

$F(x) = 3e^{-x} + 3e^{2x} + C$

$$f(x) = \frac{2}{x^2} - \frac{5}{x} + x \quad = 2x^{-2} - \frac{5}{x} + x$$

$F(x) = \frac{2x^{-1}}{-1} - 5\ln|x| + \frac{x^2}{2} + C$

$F(x) = \frac{-2}{x} - 5\ln|x| + \frac{x^2}{2} + C$

Differential Equations

An equation that involves the derivative of a function is called a differential equation:

As discussed previously, in applications of calculus it is very common to have a situation where it is required to find a function, given knowledge about its derivatives.

Find all functions g such that:

$$g'(x) = 4\sin x - 3x^5 + 6\sqrt[4]{x^3}$$

$$f(x) = -4\cos x - \frac{x^6}{2} + \frac{24}{7}x^{7/4} + C$$

$g'(x) = 4\sin x - 3x^5 + 6x^{3/4}$

$g(x) = -4\cos x - \frac{3x^6}{6} + \frac{6x^{7/4}}{7/4} + C$

$g(x) = -4\cos x - \frac{x^6}{2} + \frac{24x^{7/4}}{7} + C$

Identifying a unique solution for an antiderivative

Examples:

Determine the function with the given derivative whose graph satisfies the initial condition provided.

Find f if given f '(x): and f (0) = - 2

$$f'(x) = e^x + \frac{20}{1+x^2}$$

$f(x) = e^x + \frac{20}{1}\tan^{-1}x + C$

if $f(0) = 2$

$f(x) = e^x + 20\tan^{-1}x + C$

$-2 = e^0 + 20\tan^{-1}(0) + C$

$-2 = 1 + 20(0) + C$

$-3 = C$

$$f(x) = e^x + 20\tan^{-1}x - 3$$

Find f if given f ''(x): and f (0) = 4, and f (1) = 1

$$f''(x) = 12x^2 + 6x - 4$$

$f'(x) = \frac{12x^3}{3} + \frac{6x^2}{2} - \frac{4x^1}{1} + C$

$f'(x) = 4x^3 + 3x^2 - 4x + C$

$f(x) = \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + Cx + D$

$f(x) = x^4 + x^3 - 2x^2 + Cx + D$

if $f(0) = 4$

$4 = (0)^4 + (0)^3 - 2(0)^2 + C(0) + D$

$4 = D$

if $f(1) = 1$

$1 = (1)^4 + (1)^3 - 2(1)^2 + C(1) + 4$

$1 = 1 + 1 - 2 + C + 4$

$-3 = C$

$$f(x) = x^4 + x^3 - 2x^2 - 3x + 4$$

Practice Problems...

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