Warm Up

Evaluate the following limits:

$$\lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$$

$$\lim_{x\to\pi/4} \frac{\cos x + \sin x}{1}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$$

$$\lim_{x\to 0} \frac{Arc\sin x}{x}$$

$$\lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = \boxed{1}$$

$$\lim_{x\to 0} \frac{e^x - 1}{\sin 2x}$$

$$\lim_{x\to 0} \frac{e^x}{2\cos 2x} = \boxed{\frac{1}{2}}$$

$$\lim_{x\to\infty} \frac{10x^2}{e^x}$$

$$\lim_{x\to\infty} \frac{20x}{e^x}$$

$$\lim_{x\to\infty} \frac{20}{e^x} = \boxed{0}$$

Sigma Notation

A series is the sum of a sequence. We can write a series using sigma notation.

$$\boxed{\sum_{i=1}^{n} t_i = t_1 + t_2 + t_3 + \ldots + t_n}$$

$$1+2+4+\ldots+64 = \sum_{i=1}^{7} 2^{i-1}$$

the terms form a geometric sequence with $a = 1$, $r = 2$, $t_i = 1(2)^{i-1}$

This symbol is read as "the sum of the terms of the sequence given by $t_i = 2^{i-1}$ from $i = 1$ to $i = 7$"

Example 1

Express the series 1 + 3 + 5 + 7 + 9 in sigma notation.

$a = 1$
$d = 2$

$t_n = a + (n-1)d$
$t_n = 1 + (n-1)(2)$
$t_n = 1 + 2n - 2$
$t_n = 2n - 1$
$t_i = 2i - 1$

$$\sum_{i=1}^{5} 2i - 1$$

The properties of Sigma Notation that we use in this section are summarized below:

$$\sum_{i=1}^{n} c = c + c + c + \ldots + c = nc$$

$\displaystyle\sum_{i=1}^{n} ct_i = c\sum_{i=1}^{n} t_i$, c is a constant

$$\sum_{i=1}^{n} (t_i + s_i) = \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} s_i$$

Example 2

Use the basic properties of sigma notation to express $\sum_{i=1}^{n}(3i-2)^2$ in terms of monomial summations.

$$\sum_{i=1}^{n}(9i^2-12i+4)$$

$$\sum_{i=1}^{n}9i^2+\sum_{i=1}^{n}(-12i)+\sum_{i=1}^{n}4$$

$$\boxed{9\sum_{i=1}^{n}i^2-12\sum_{i=1}^{n}i+4n}$$

The following sigma formulas will be extremely useful in the next few days when we are faced with the challenge of calculating the area under a curve.

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + 4 + \dots n = \frac{n(n+1)}{2}$$ linear

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + \dots n^2 = \frac{n(n+1)(2n+1)}{6}$$ quad

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + 4^3 + \dots n^3 = \left[\frac{n(n+1)}{2}\right]^2$$ cubic

Example 3

$$\sum_{i=1}^{n} \left(3i^2 - 2i\right)$$

$$\sum_{i=1}^{n} 3i^2 + \sum_{i=1}^{n} (-2i)$$

$$3\boxed{\sum_{i=1}^{n} i^2} - 2\boxed{\sum_{i=1}^{n} i}$$

$$\cancel{3}^{1} \cdot \boxed{\frac{(n)(n+1)(2n+1)}{\cancel{6}\,2}} - 2 \cdot \boxed{\frac{n(n+1)}{2}}$$

$$\frac{n(n+1)(2n+1) - 2n(n+1)}{2}$$ Factor

$$\frac{n(n+1)[2n+1-2]}{2}$$

$$\boxed{\frac{n(n+1)(2n-1)}{2}}$$

Example 4

$$\sum_{i=11}^{20}(2i^2-3i) = \sum_{i=1}^{20}(2i^2-3i) - \sum_{i=1}^{10}(2i^2-3i)$$

$$\sum_{i=1}^{20}(2i^2) + \sum_{i=1}^{20}(-3i) - \left[\sum_{i=1}^{10}(2i^2) + \sum_{i=1}^{10}(-3i)\right]$$

$$2\boxed{\sum_{i=1}^{20} i^2} - 3\boxed{\sum_{i=1}^{20} i} - \left[2\boxed{\sum_{i=1}^{10} i^2} - 3\boxed{\sum_{i=1}^{10} i}\right]$$

$n=20$ $\quad$ $n=20$ $\quad$ $n=10$ $\quad$ $n=10$

$$\cancel{2}^{1}\frac{n(n+1)(2n+1)}{\cancel{6}\,3} - 3\frac{n(n+1)}{2} - \cancel{2}^{1}\frac{n(n+1)(2n+1)}{\cancel{6}\,3} + 3\frac{n(n+1)}{2}$$

$$\frac{20(21)(41)}{3} - \frac{3(20)(21)}{2} - \frac{10(11)(21)}{3} + \frac{3(10)(11)}{2}$$

$$5740 - 630 - 770 + 165$$

$$\boxed{4505}$$

Homework

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