Warm-Up

Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \le 0$$

← where does the polynomial have negative y-values

$y = x^3 - 3x^2 - 4x + 12$ (Write as a polynomial function)

$y = (x^3 - 3x^2)(-4x + 12)$ (Factor by grouping)

$y = x^2(x-3) - 4(x-3)$

$y = (x-3)(x^2-4)$ (Factor using difference of squares)

$y = (x-3)(x-2)(x+2)$

Find the x-intercepts: $(y=0)$

$0 = (x-3)(x-2)(x+2)$

$x-3=0$ | $x-2=0$ | $x+2=0$

$x=3$ | $x=2$ | $x=-2$

$x = -2, 2, 3$

Create a number line and label your roots

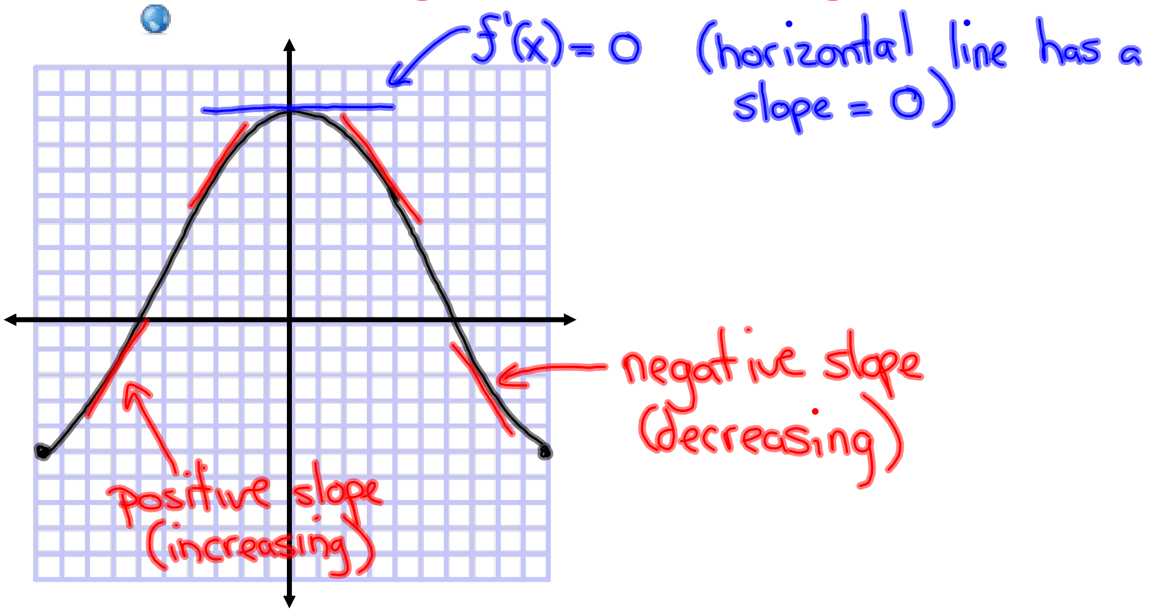
− + − +

(-3) -2 (0) 2 (2.5) 3 (4)

State the intervals that satisfy the inequality

$x \in (-\infty, -2] \cup [2, 3]$

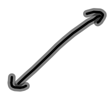
Increasing and Decreasing Functions

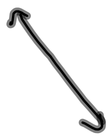


Test for Increasing and Decreasing Functions

1. If $f'(x) > 0$ for all x in an interval I, then f is increasing on I. $x \in (-10, 0)$
   ↖ positive
2. If $f'(x) < 0$ for all x in an interval I, then f is decreasing on I $x \in (0, 10)$
   ↖ negative

Recall $f'(x)$ is the slope of your tangent to the curve

- positive slope: 

- negative slope: 

Example 1

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

Solution

First we find the derivative of $f(x) = 1 - 5x + 4x^2$ and get

[redacted]

The function f will be increasing when [redacted]

Thus f will be increasing on the interval ________

Similarly,

The function f will be decreasing when [redacted]

Thus f will be decreasing on the interval ________

Example 2

Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

where is $y' > 0$

Solution

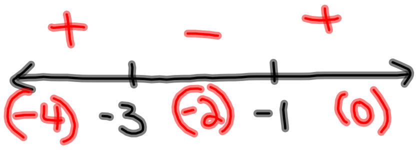
First we compute the derivative and factor it:

$y' = 3x^2 + 12x + 9$
$= 3(x^2 + 4x + 3)$
$= 3(x+1)(x+3)$

Find critical numbers
$y' = 3(x+1)(x+3)$
$0 = 3(x+1)(x+3)$
$x+1=0 \mid x+3=0$
$x=-1 \mid x=-3$

The function f will be increasing when $y' > 0$, so we have to solve the quadratic ineqaulity $(x+1)(x+3)>0$

Create a number line and label your critical #'s



State the intervals where function is increasing.

$x \in (-\infty,-3) \cup (-1,\infty)$ or increasing on $(-\infty,-3)$ and $(-1,\infty)$

Interval	(x+3)	(x+1)	$f'(x)$	f
$(-\infty,-3)$ (-4)	–	–	+	increasing
$(-3,-1)$ (-2)	+	–	–	decreasing
$(-1,\infty)$ (0)	+	+	+	increasing

Example 3

Find the intervals on which the function $f(x) = x^4 - 4x^3 - 8x^2 - 1$ is increasing and decreasing.

Solution

First we compute the derivative and factor it:

$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x-4)(x+1)$

① Find Critical Numbers ($f'(x) = 0$)

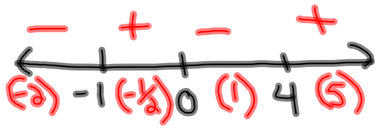
$0 = 4x(x-4)(x+1)$

$4x = 0 \mid x - 4 = 0 \mid x + 1 = 0$

$x = 0 \mid x = 4 \mid x = -1$

$x = -1, 0, 4$

② Create a number line and label your critical #'s



③ Increasing on $(-1, 0)$ + $(4, \infty)$

Decreasing on $(-\infty, -1)$ + $(0, 4)$

Interval	$4x$	$(x-4)$	$(x+1)$	$f'(x)$	
$(-\infty, -1)$ (-2)	−	−	−	−	decreasing
$(-1, 0)$ (-0.5)	−	−	+	+	increasing
$(0, 4)$ (1)	+	−	+	−	decreasing
$(4, \infty)$ (5)	+	+	+	+	increasing

Homework