eview – Counting Methods

___ 1. A combination lock opens with the correct three-digit code. Each wheel rotates through the digits 1 to 8. Suppose each digit can be used only once in a code. How many different codes are possible when repetition is not allowed?

A. 21
B. 63
C. 256
D. 336

___ 2. The lunch special at a sandwich bar offers you a choice of 6 sandwiches, 4 salads, 6 drinks, and 3 desserts. How many different meals are possible if you choose one item from each category?

A. 432
B. 576
C. 646
D. 720

___ 3. Evaluate. $8! + 1!$

A. 40 321
B. 5041
C. 40 123
D. 16 777 217

___ 4. Evaluate. $(3!)^2$

A. 8
B. 9
C. 18
D. 36

___ 5. Identify the expression that is equivalent to the following:

$$\frac{n!}{(n-2)!} + n$$

A. n
B. $-n$
C. n^2
D. n^3

A. n
B. $-n$
C. n^2
D. n^3

___ 6. Solve for n, where $n \in \mathrm{I}$.

$$2\left(\frac{(n+1)!}{n!}\right) = 40$$

A. 10
B. 19
C. 20
D. 39

___ 7. How many different permutations can be created when 7 people line up to buy movie tickets?
A. 49
B. 128
C. 720
D. 5040

___ 8. Evaluate. ${}_3P_1$
A. 1
B. 2
C. 3
D. 6

___ 9. Suppose a word is any string of letters. How many three-letter words can you make from the letters in REGINA if you do not repeat any letters in the word?
A. 20
B. 16
C. 216
D. 120

10. ☺

11. How many ways can 7 friends stand in a row for a photograph if Sheng always stands beside his girlfriend?

A. 1440
B. 5040
C. 360
D. 720

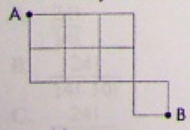
12. Evaluate. $\frac{6!}{2! \cdot 2!}$

A. 48
B. 72
C. 140
D. 180

13. How many different arrangements can be made using all the letters in CANADA?

A. 120
B. 180
C. 360
D. 720

14. How many different routes are there from A to B, if you only travel south or east?



A. 10
B. 20
C. 40
D. 8

___ **15.** There are 14 members of a student council. How many ways can 4 of the members be chosen to serve on the dance committee?

A. 1001
B. 2002
C. 6006
D. 24 024

___ **16.** Evaluate. $\binom{4}{1}$

A. 0
B. 1
C. 4
D. 16

___ **17.** Evaluate. ${}_6C_2$

A. 15
B. 18
C. 30
D. 36

___ **18.** How many ways can 3 representatives be chosen from a soccer team of 16 players?

A. 1120
B. 560
C. 3360
D. 1580

19. Suppose that 3 teachers and 6 students volunteered to be on a graduation committee. The committee must consist of 1 teachers and 2 students. How many different graduation committees does the principal have to choose from?

A. 45
B. 60
C. 90
D. 180

___ 20. Which of the following is equivalent to ${}_{17}C_{10}$?

A. $\binom{10}{7}$
B. $7!\binom{17}{7}$
C. $7!\binom{10}{7}$
D. $\binom{17}{7}$

___ 21. Which of the following is equivalent to ${}_{24}C_{10}$?

A. $\frac{24!}{14!}$
B. $\frac{24!}{14! \cdot 10!}$
C. $\frac{24!}{12! \cdot 12!}$
D. $\frac{24!}{10!}$

___ 22. Solve for n. ${}_nC_1 = 30$

A. $n = 6$
B. $n = 10$
C. $n = 30$
D. $n = 60$

___ 23. How many ways can the 6 starting positions on a hockey team (1 goalie, 2 defense, 3 forwards) be filled from a team of 2 goalies, 4 defense, and 7 forwards?

A. 420
B. 500
C. 858
D. 1716

___ 24. From a standard deck of 52 cards, how many different five-card hands are there with at least four black cards?

A. 388 700
B. 649 740
C. 1 299 480
D. 454 480

___ 25. Nine boys and twelve girls have signed up for a trip. Only six students will be selected to go on the trip. Determine the probability that there will be equal numbers of boys and girls on the trip.

A. 17.23%
B. 22.61%
C. 27.35%
D. 34.06%

Short Answer

1. Write the following expression using factorial notation. $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

2. A baseball coach is determining the batting order for the nine players she is fielding. The coach has already decided who will bat first and second. How many different batting orders are possible?

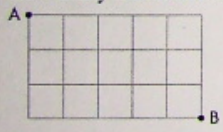
3. Solve for n, where $n \in I$.
$$\frac{(n+10)!}{(n+9)!} = 20$$

4. Solve for n, where $n \in I$.
$$\frac{(n+1)!}{2(n-1)!} = 6$$

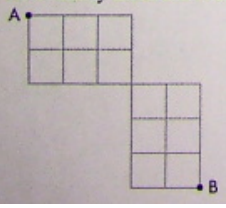
5. There are nine different marbles in a bag. Suppose you reach in and draw one at a time, and do this three times. How many ways can you draw the three marbles if you do not replace the marble each time?

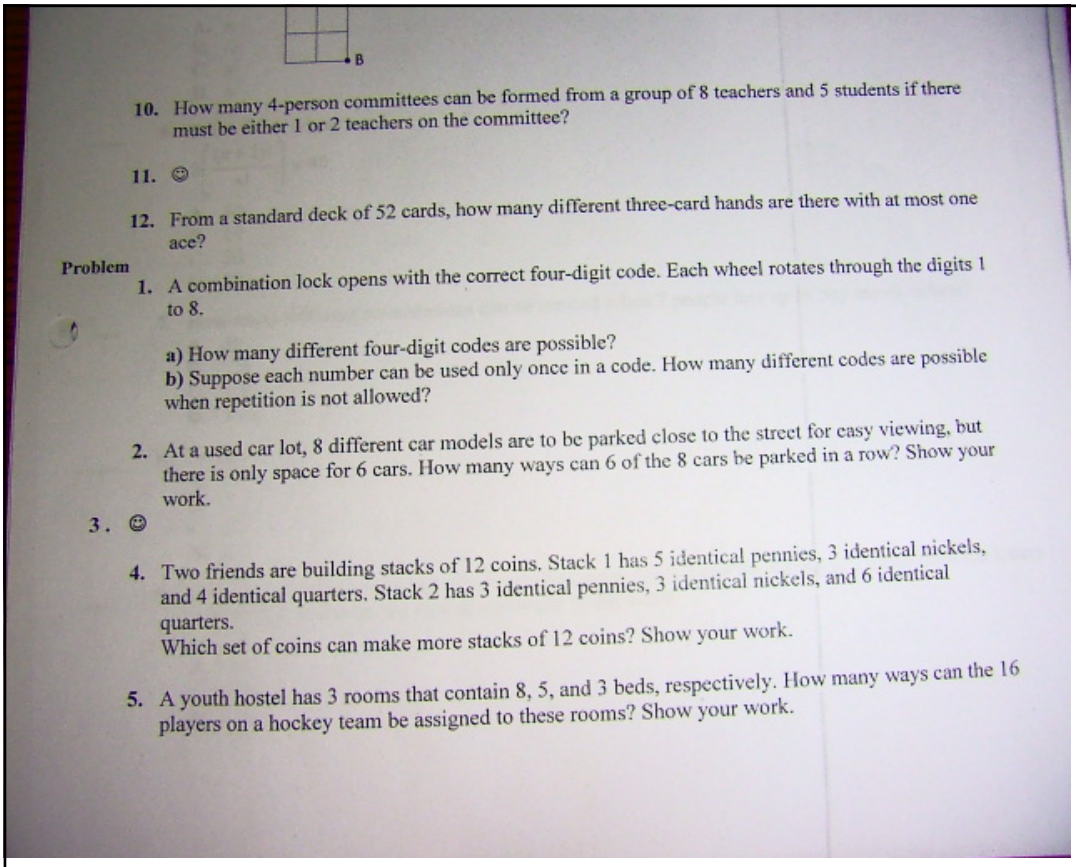
6. ☺ 7. ☺

8. How many different routes are there from A to B, if you only travel south or east?



9. How many different routes are there from A to B, if you only travel south or east?





10. How many 4-person committees can be formed from a group of 8 teachers and 5 students if there must be either 1 or 2 teachers on the committee?

11. ☺

12. From a standard deck of 52 cards, how many different three-card hands are there with at most one ace?

Problem

1. A combination lock opens with the correct four-digit code. Each wheel rotates through the digits 1 to 8.

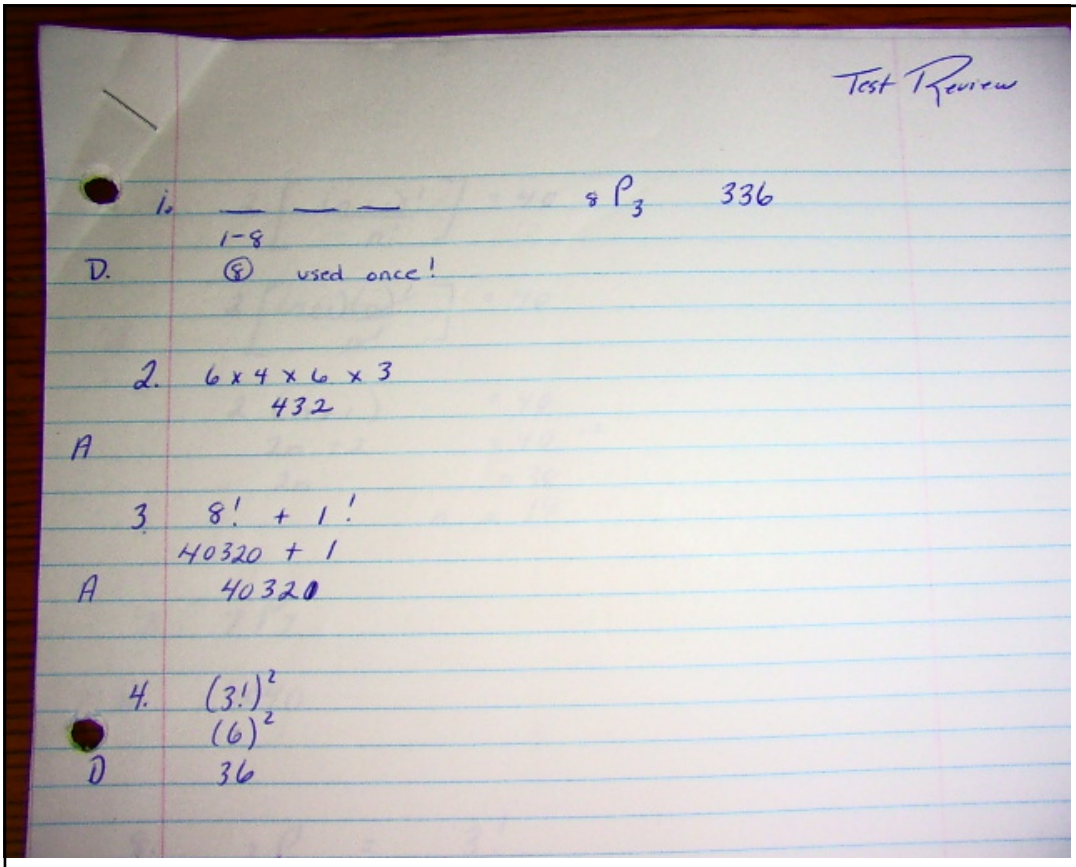
   a) How many different four-digit codes are possible?
   b) Suppose each number can be used only once in a code. How many different codes are possible when repetition is not allowed?

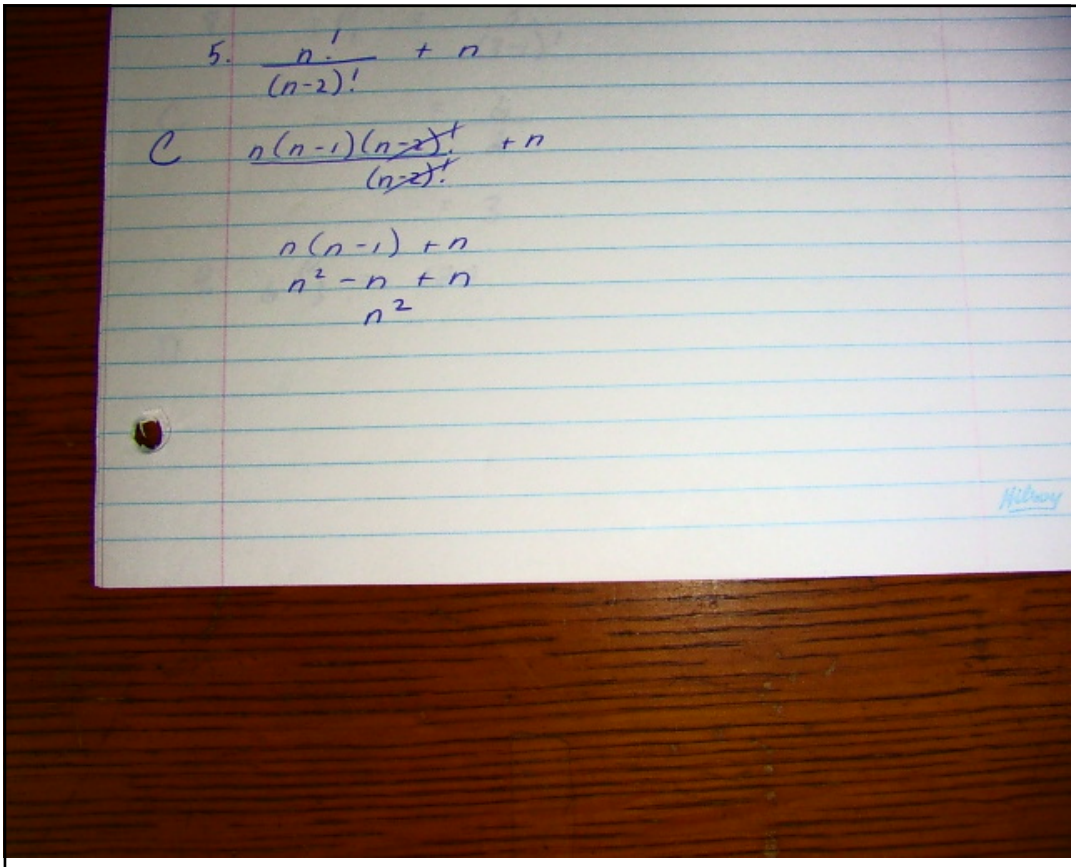
2. At a used car lot, 8 different car models are to be parked close to the street for easy viewing, but there is only space for 6 cars. How many ways can 6 of the 8 cars be parked in a row? Show your work.

3. ☺

4. Two friends are building stacks of 12 coins. Stack 1 has 5 identical pennies, 3 identical nickels, and 4 identical quarters. Stack 2 has 3 identical pennies, 3 identical nickels, and 6 identical quarters.
   Which set of coins can make more stacks of 12 coins? Show your work.

5. A youth hostel has 3 rooms that contain 8, 5, and 3 beds, respectively. How many ways can the 16 players on a hockey team be assigned to these rooms? Show your work.


Test Review
1. _ _ _ $_8P_3$ 336
1-8
D. ⑧ used once!
2. 6 x 4 x 6 x 3
432
A
3. 8! + 1!
40320 + 1
A 40321
4. $(3!)^2$
$(6)^2$
D 36



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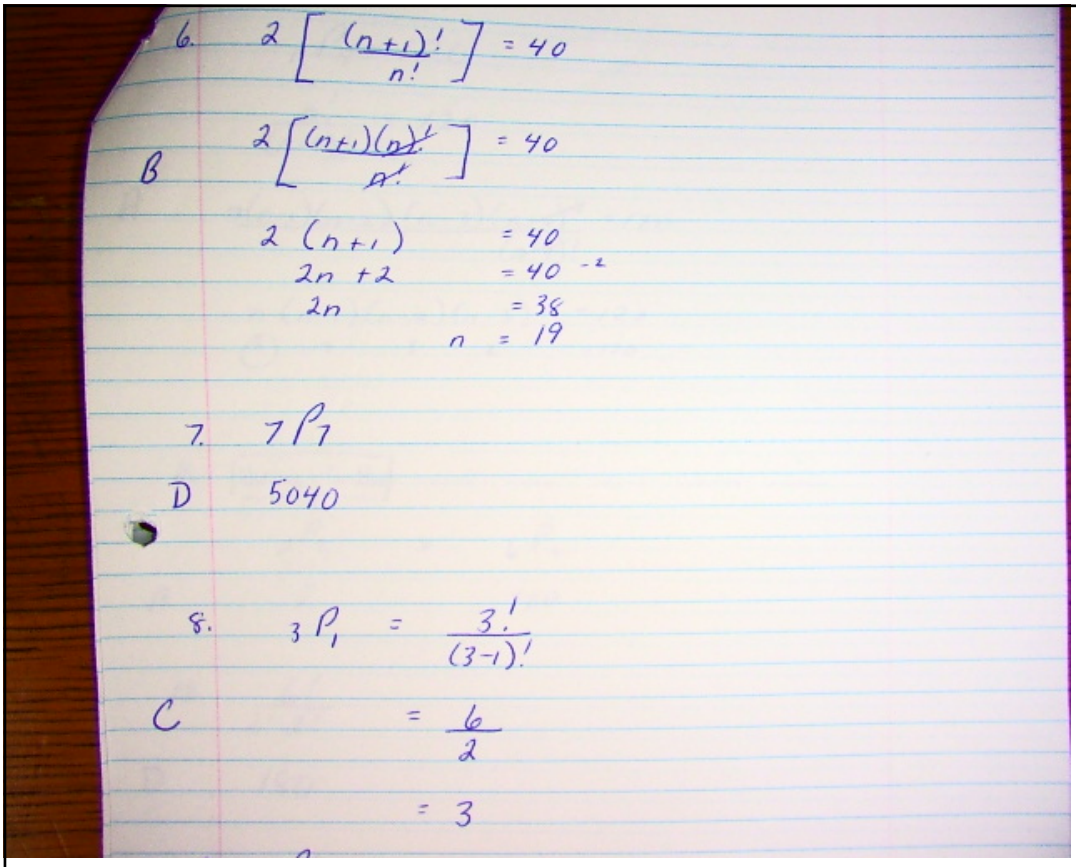


6. $2\left[\frac{(n+1)!}{n!}\right] = 40$

B $2\left[\frac{(n+1)(n)!}{n!}\right] = 40$

$2(n+1) = 40$

$2n + 2 = 40$ -2

$2n = 38$

$n = 19$

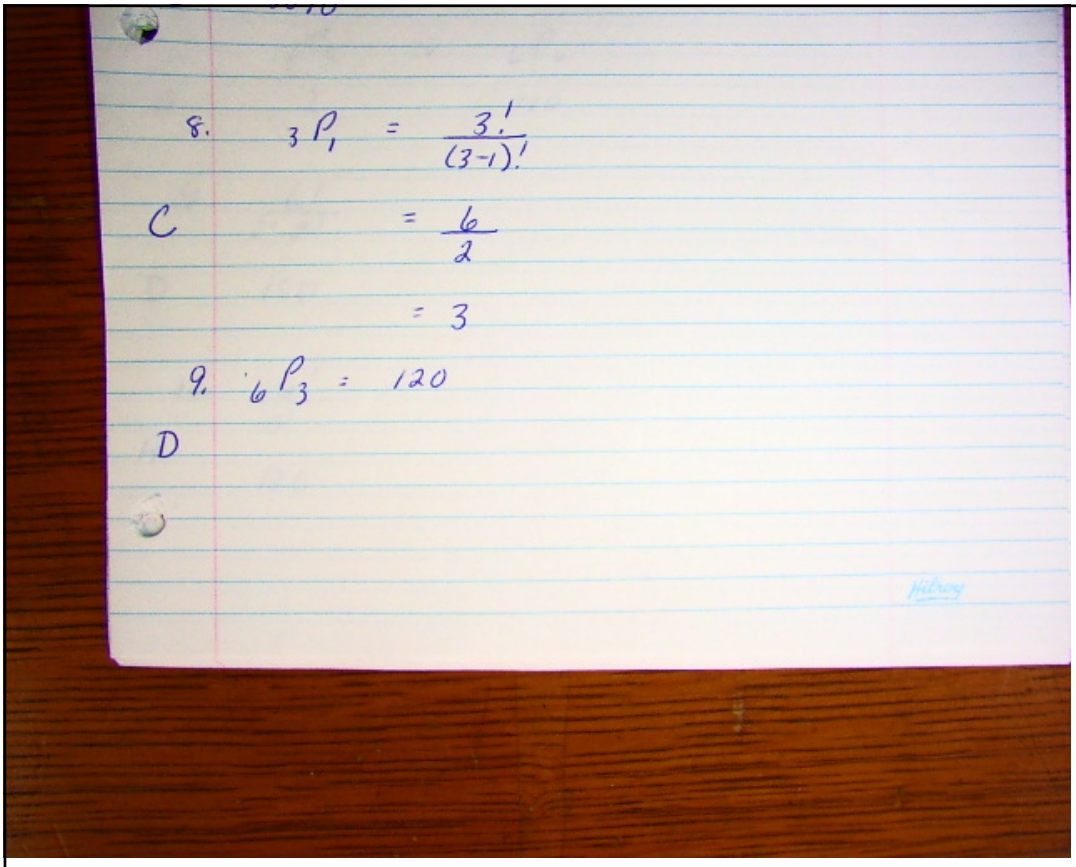
7. $_7P_7$

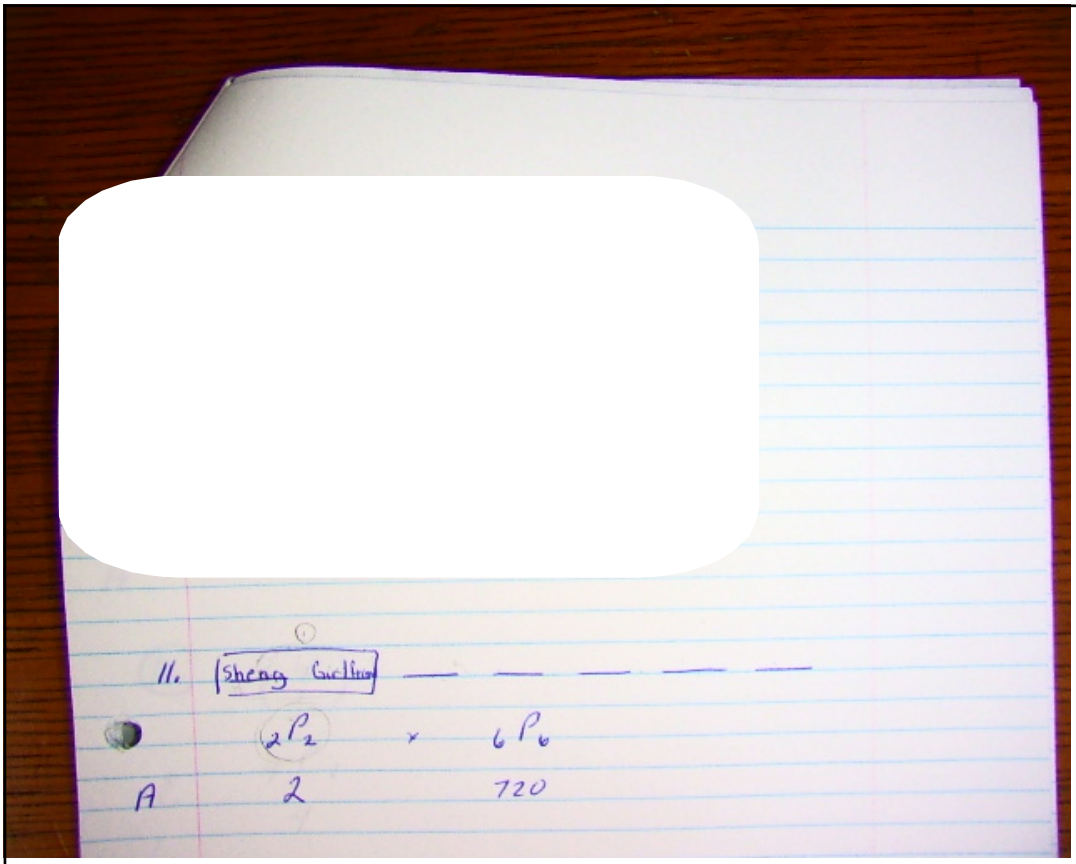
D 5040

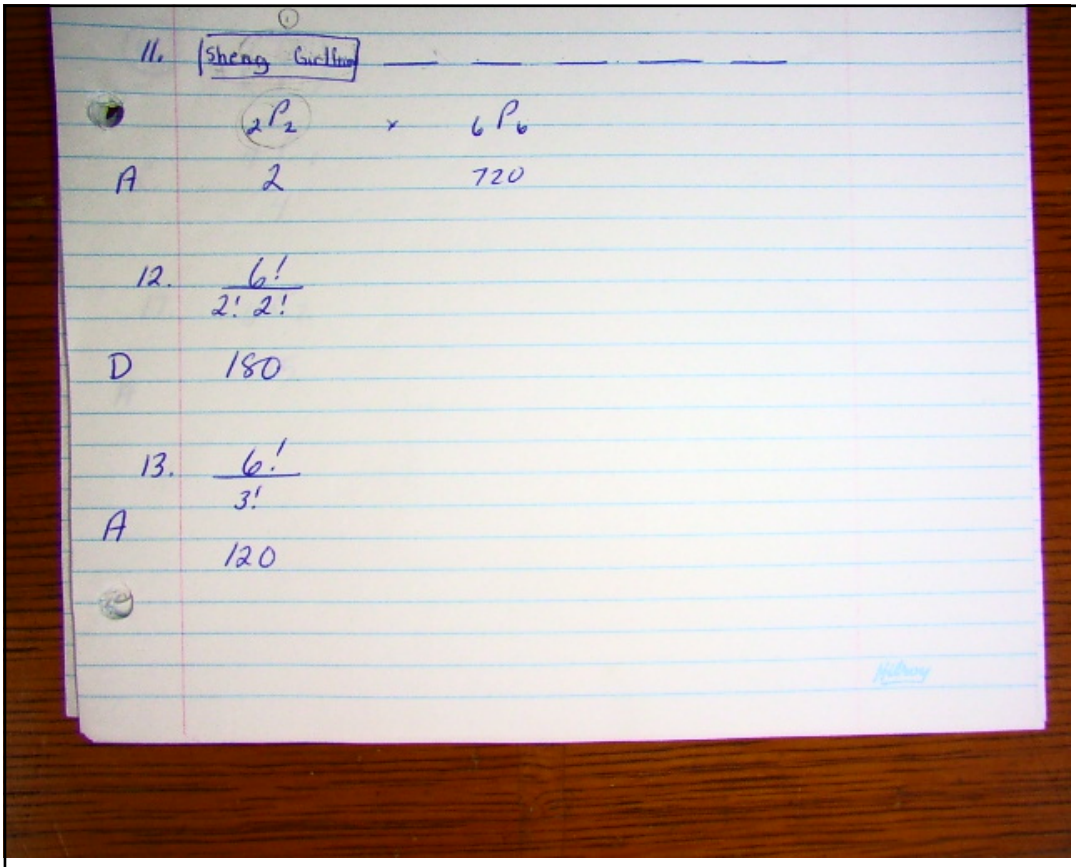
8. $_3P_1 = \frac{3!}{(3-1)!}$

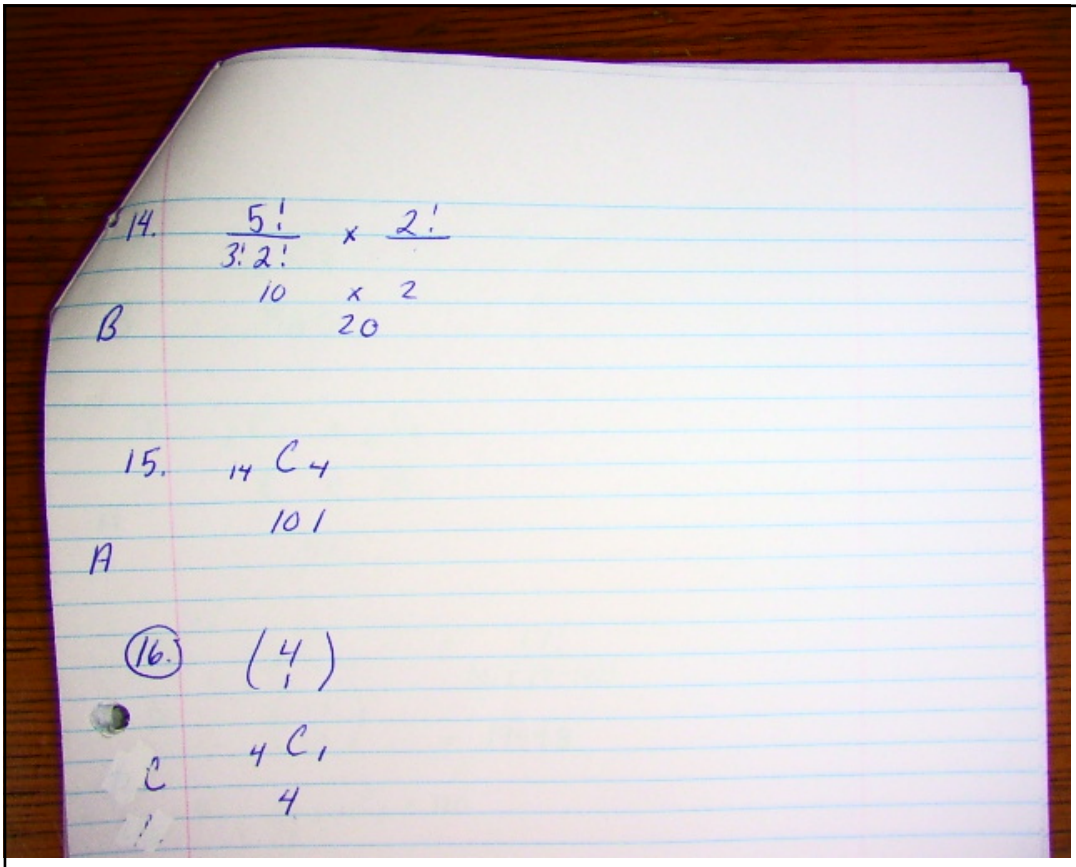
C $= \frac{6}{2}$

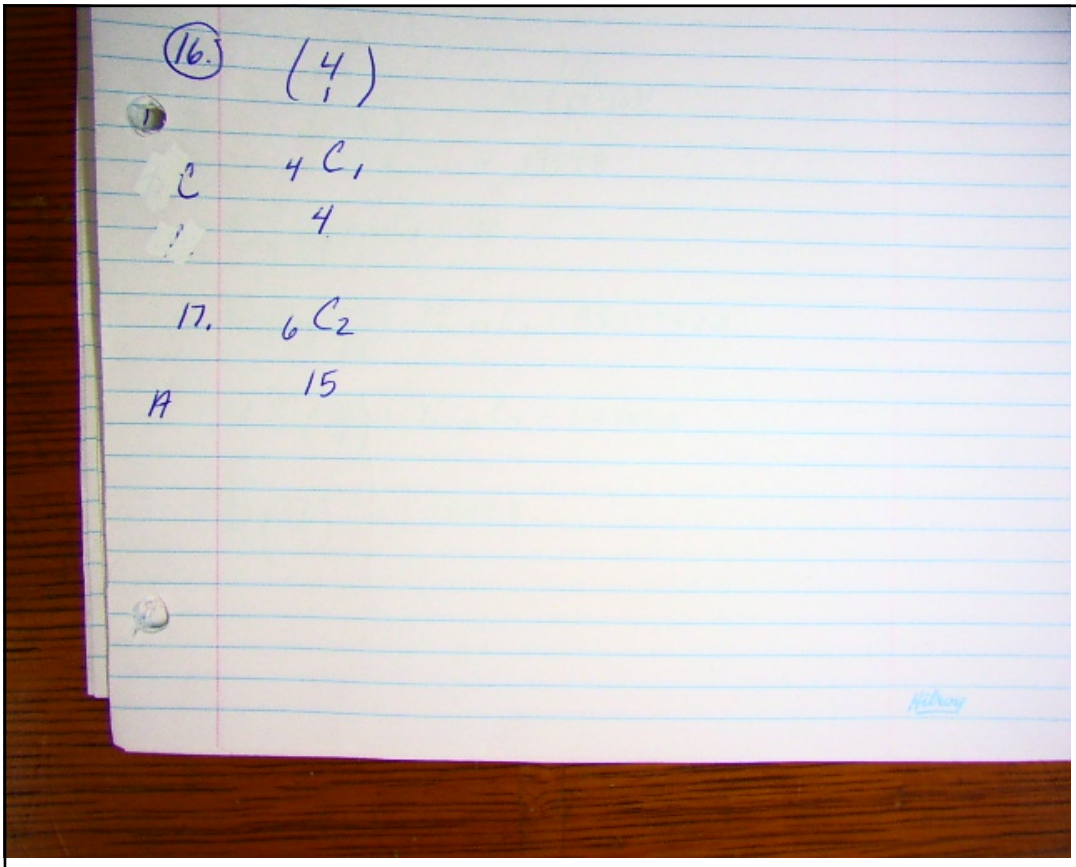
$= 3$


8. ${}_3P_1 = \frac{3!}{(3-1)!}$
C
$= \frac{6}{2}$
$= 3$
9. ${}_6P_3 = 120$
D


11. Sheag Curling
2P2 x 6P6
A 2 720


11. Sheng Guelfuo
$_2P_2$ × $_6P_6$
A 2 720
12. $\frac{6!}{2!\,2!}$
D 180
13. $\frac{6!}{3!}$
A
120


14. 5! / 3! 2! × 2!
10 × 2
B 20
15. 14 C 4
10 1
A
16. (4 1)
4 C 1
4



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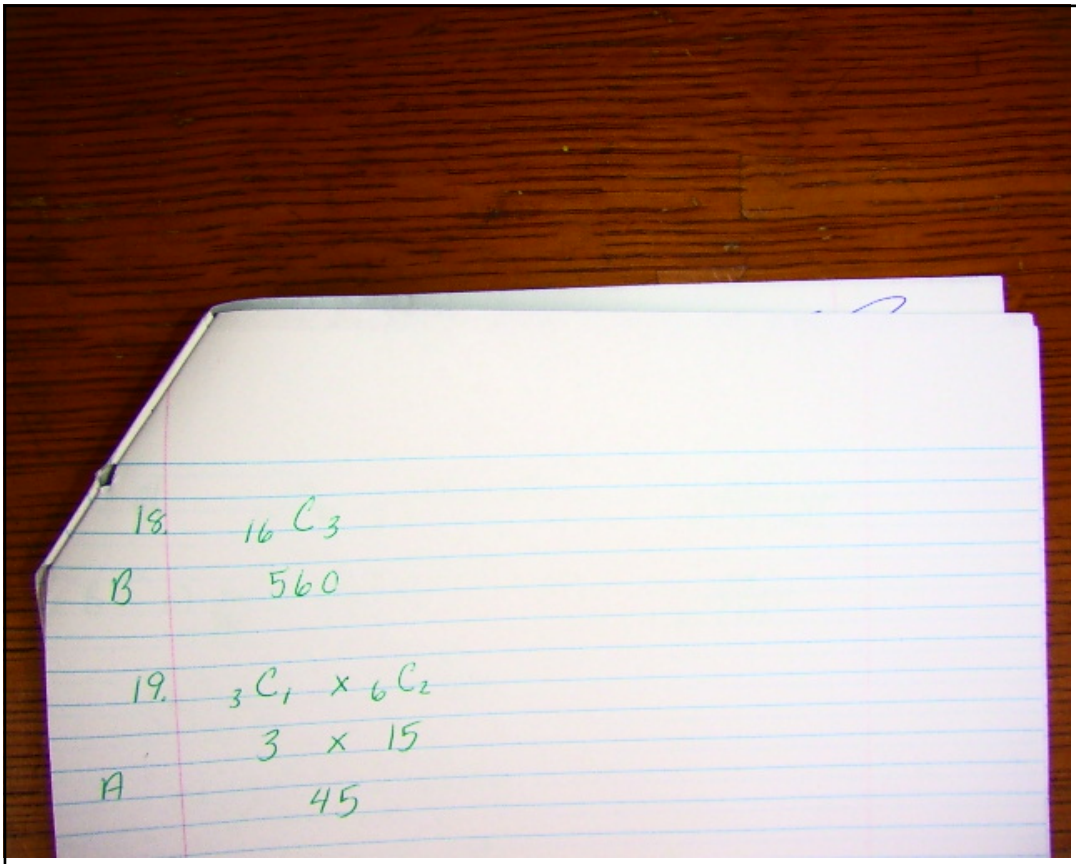

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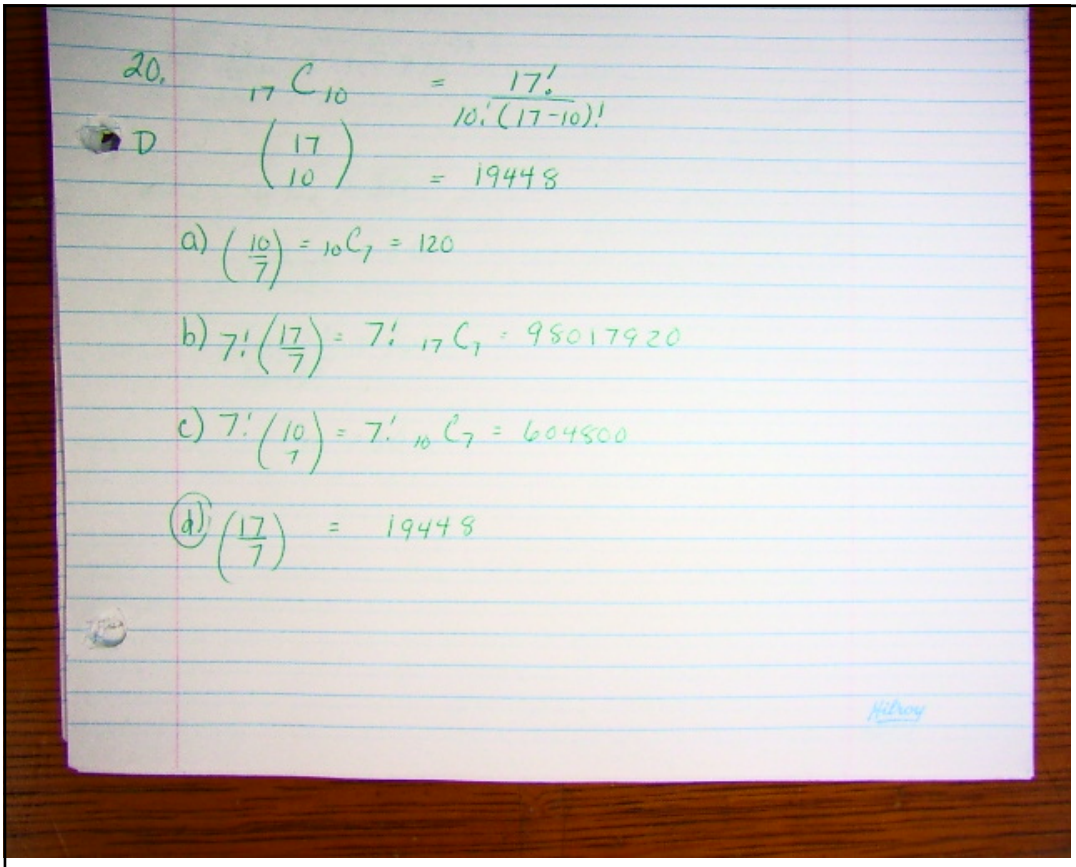
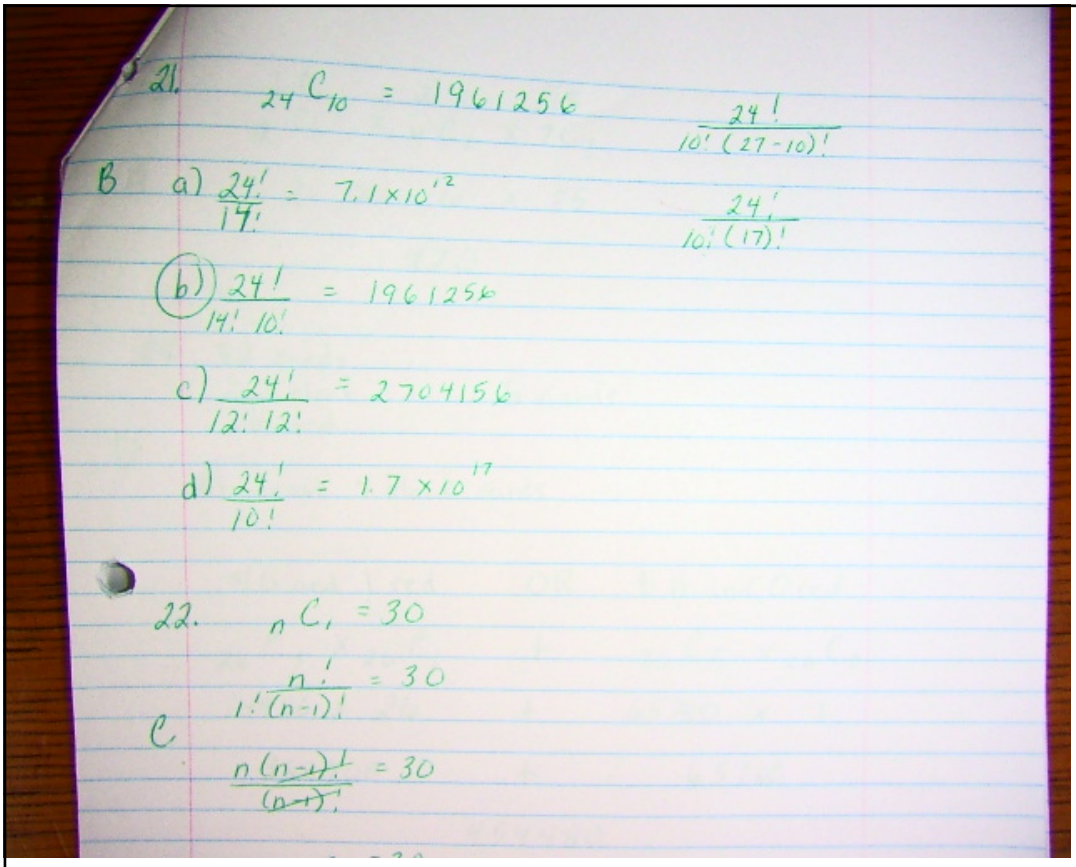
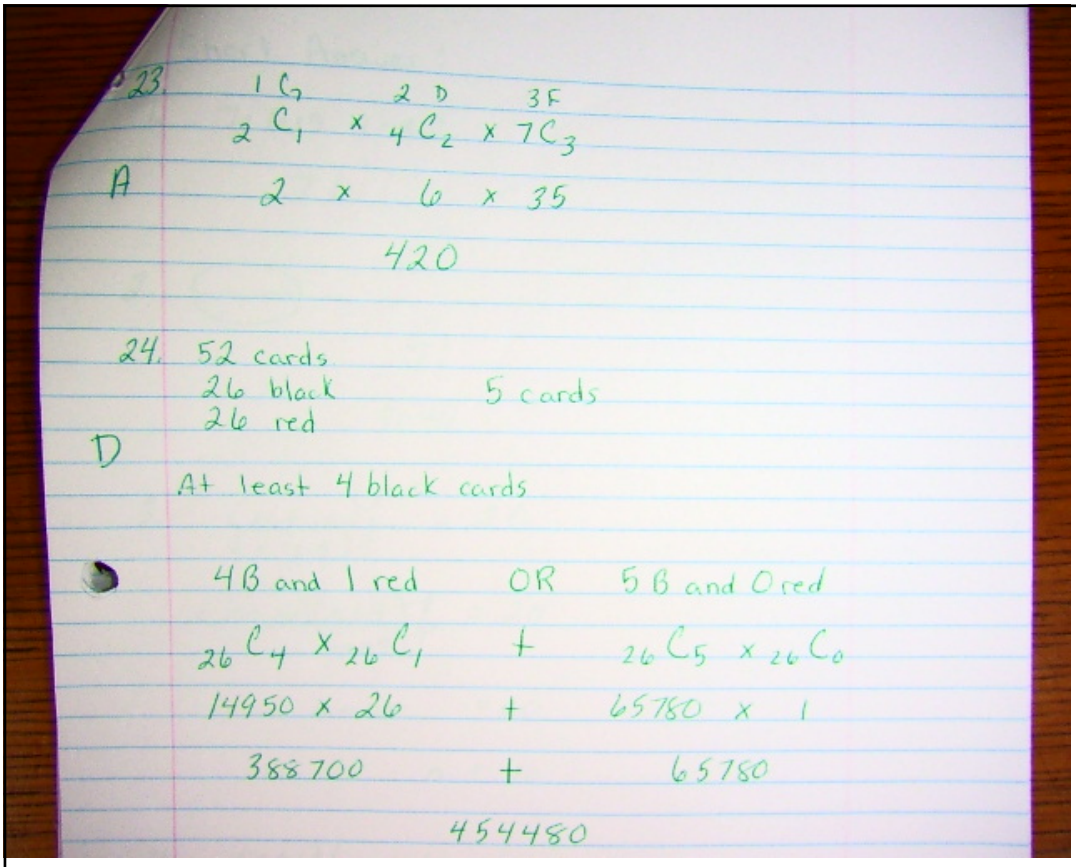

20.
D
$_{17}C_{10} = \frac{17!}{10!(17-10)!}$
$\binom{17}{10} = 19448$
a) $\binom{10}{7} = {}_{10}C_7 = 120$
b) $7!\binom{17}{7} = 7!\ {}_{17}C_7 = 98017920$
c) $7!\binom{10}{7} = 7!\ {}_{10}C_7 = 604800$
d) $\binom{17}{7} = 19448$


21. $_{24}C_{10} = 1961256$
$\frac{24!}{10!(27-10)!}$
$\frac{24!}{10!(17)!}$
B a) $\frac{24!}{17!} = 7.1 \times 10^{12}$
b) $\frac{24!}{14!\,10!} = 1961256$
c) $\frac{24!}{12!\,12!} = 2704156$
d) $\frac{24!}{10!} = 1.7 \times 10^{17}$
22. $_nC_1 = 30$
$\frac{n!}{1!(n-1)!} = 30$
C
$\frac{n(n-1)!}{(n-1)!} = 30$



23. $_2C_1 \times {}_4C_2 \times {}_7C_3$ (1 C, 2 D, 3 F)

A

$2 \times 6 \times 35$

420

24. 52 cards
26 black
26 red
5 cards

D

At least 4 black cards

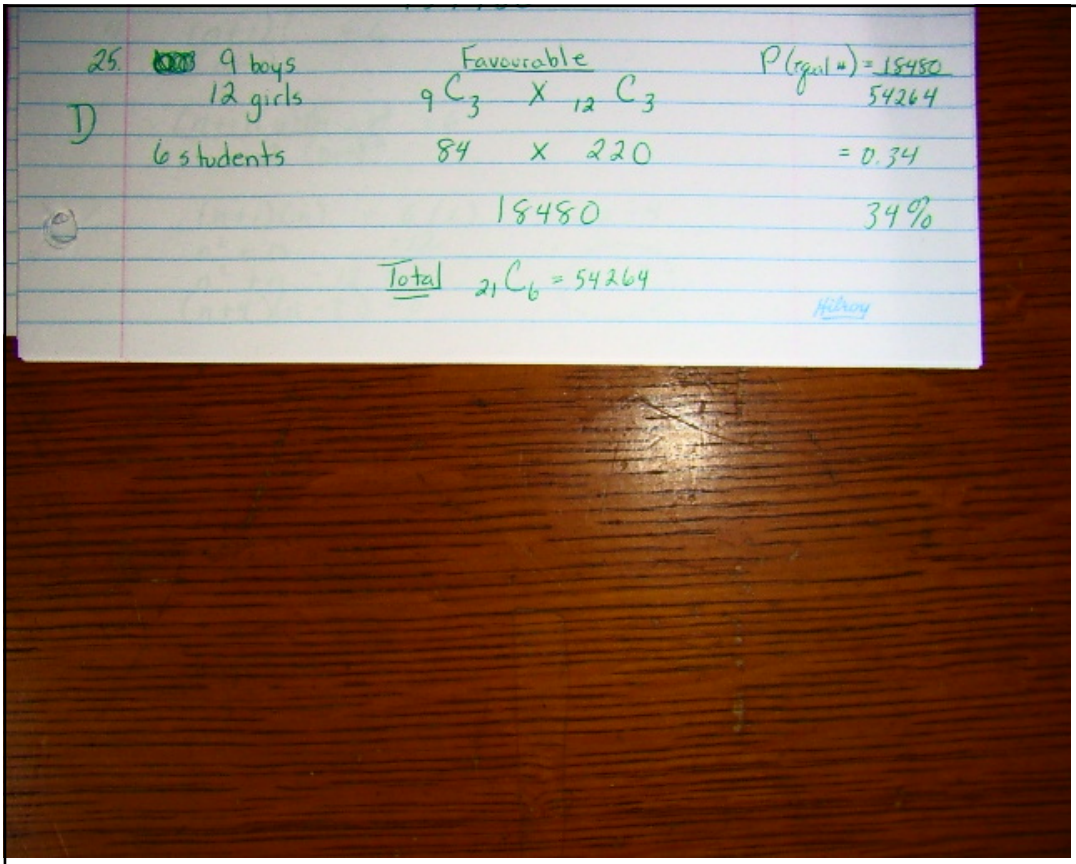
4 B and 1 red OR 5 B and 0 red

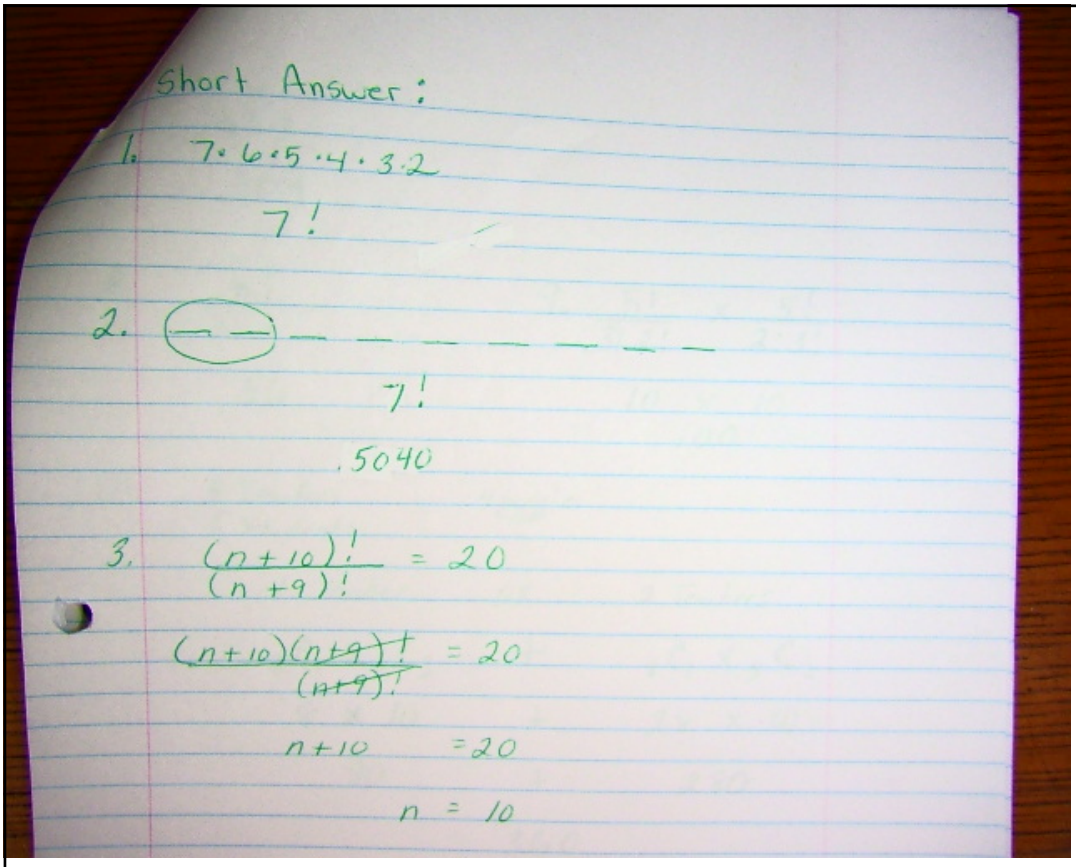
${}_{26}C_4 \times {}_{26}C_1 + {}_{26}C_5 \times {}_{26}C_0$

$14950 \times 26 + 65780 \times 1$

$388700 + 65780$

454480


25.
9 boys
12 girls
6 students
D
Favourable
$_9C_3 \times {}_{12}C_3$
84 x 220
18480
Total $_{21}C_6 = 54264$
P(equal #) = 18480/54264
= 0.34
34%
Hilary



Short Answer:

1. $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

$7!$

2. — — — — — — — — —

$7!$

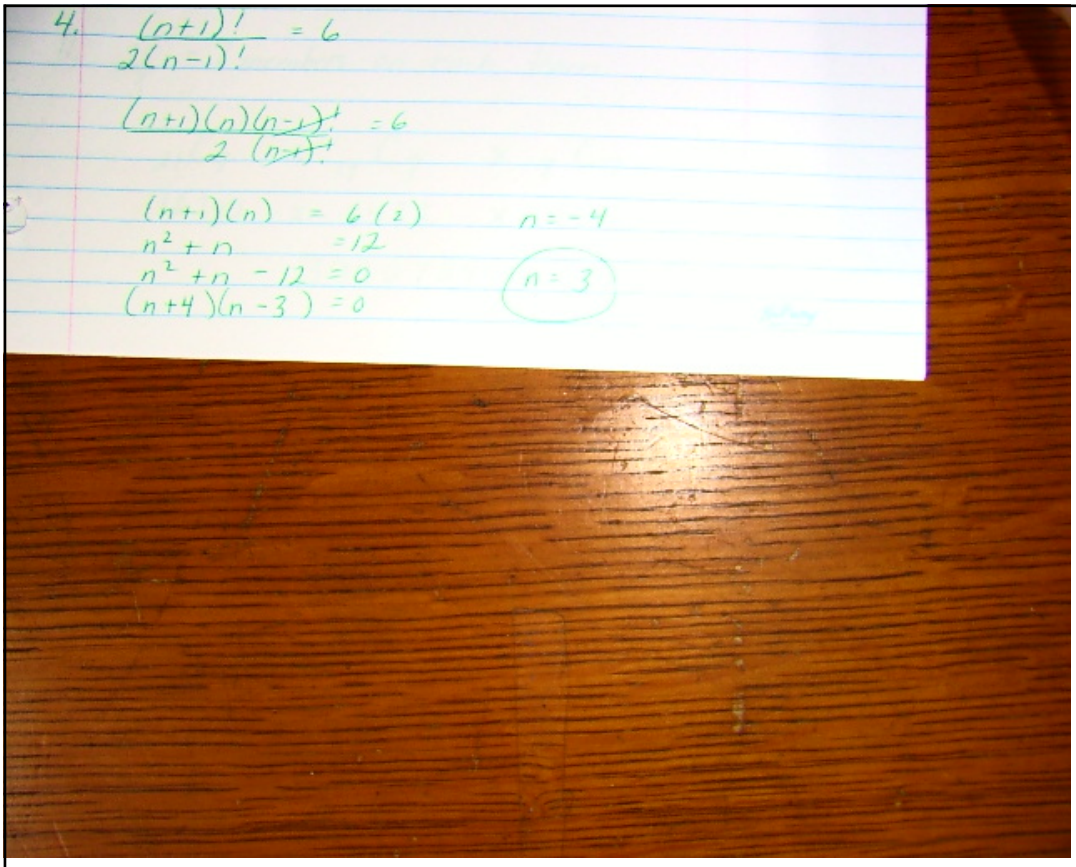
5040

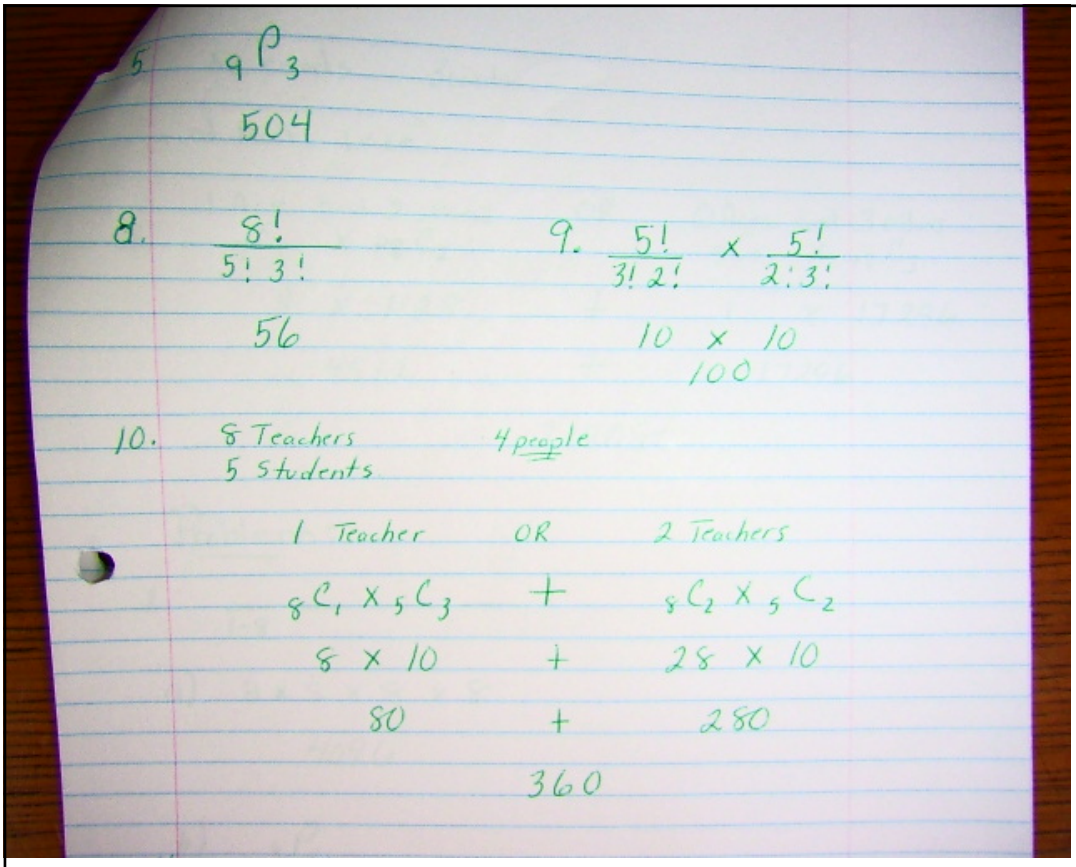
3. $\frac{(n+10)!}{(n+9)!} = 20$

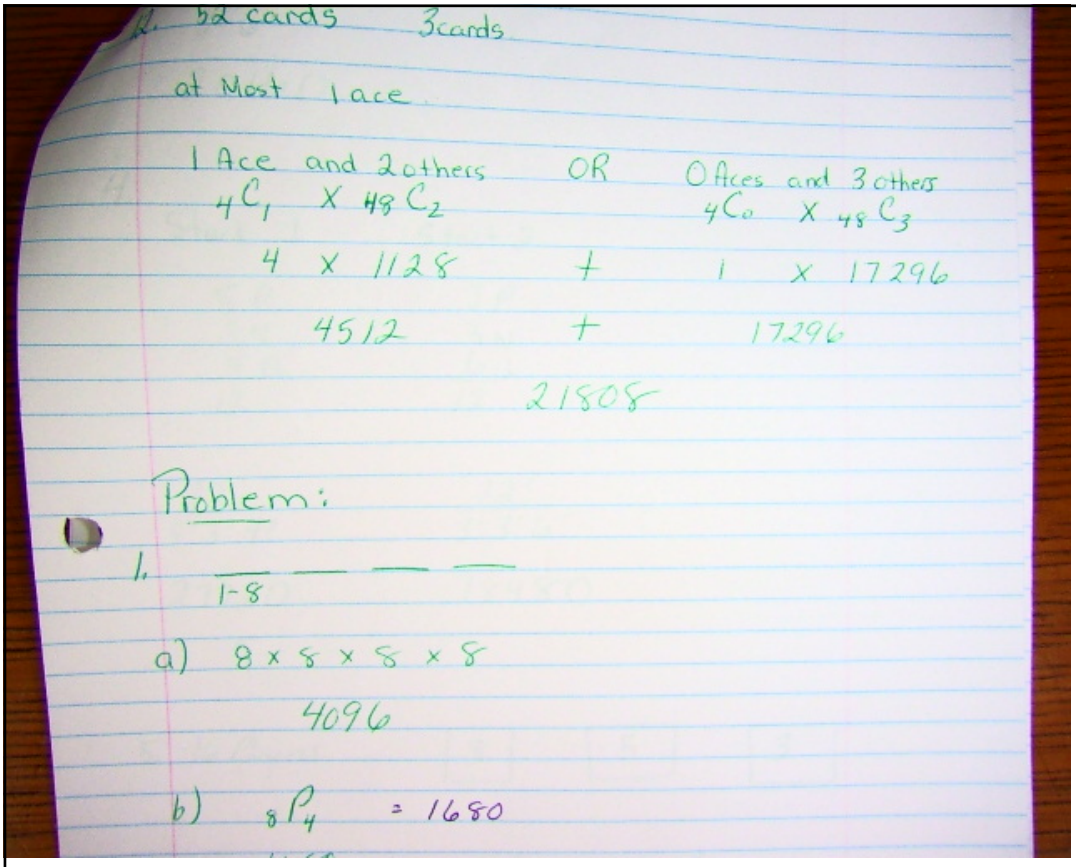
$\frac{(n+10)(n+9)!}{(n+9)!} = 20$

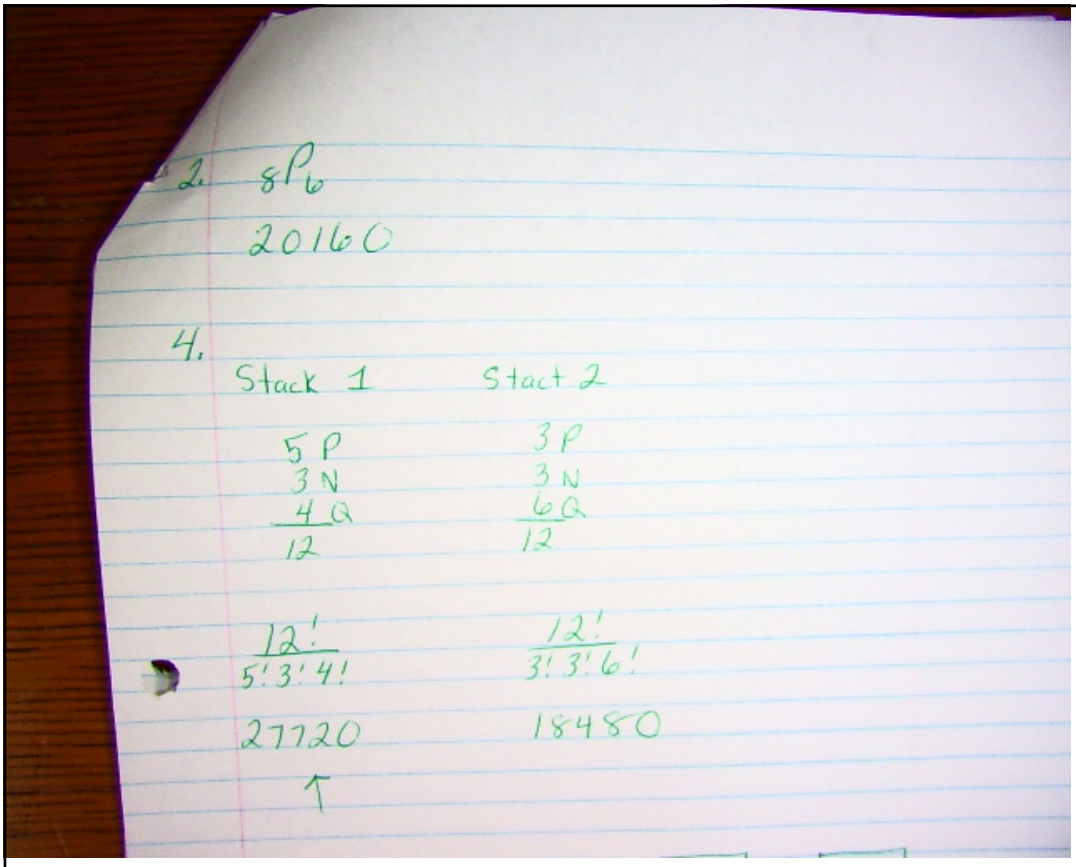
$n+10 = 20$

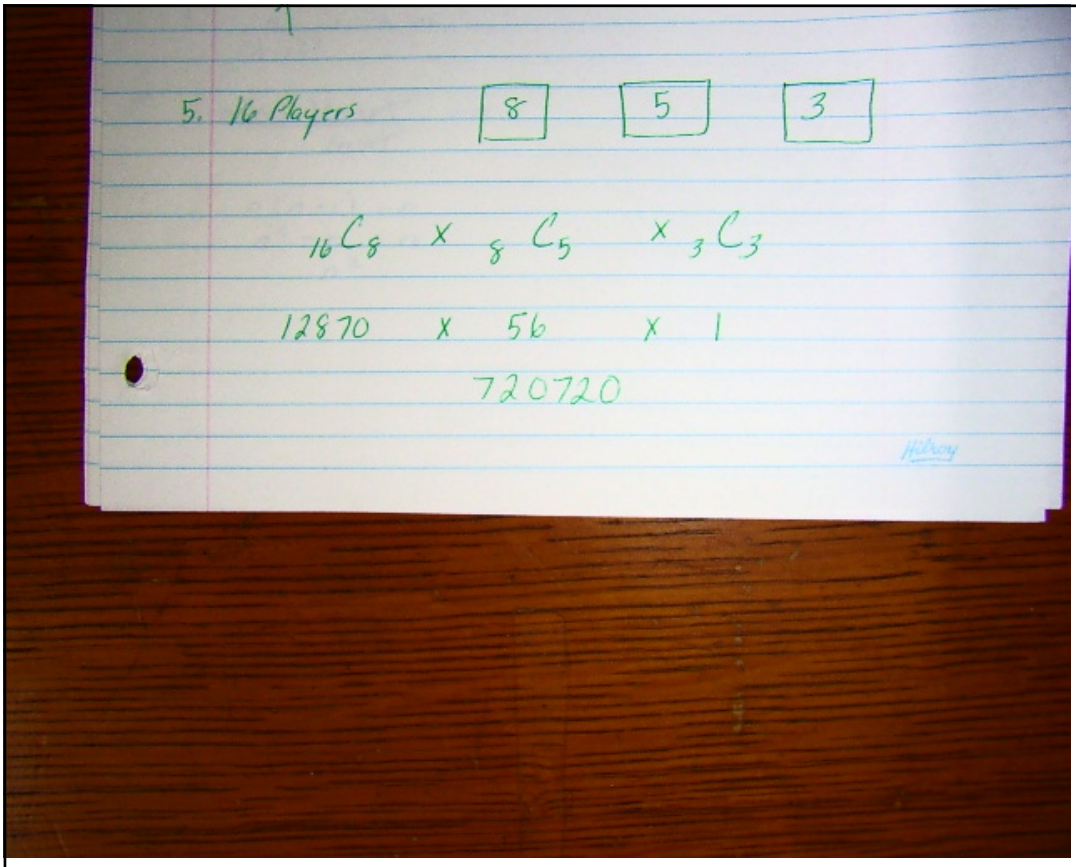
$n = 10$


4. $\frac{(n+1)!}{2(n-1)!} = 6$
$\frac{(n+1)(n)(n-1)!}{2(n-1)!} = 6$
$(n+1)(n) = 6(2)$
$n^2 + n = 12$
$n^2 + n - 12 = 0$
$(n+4)(n-3) = 0$
$n = -4$
$n = 3$


5. $_9P_3$
504
8. $\frac{8!}{5!3!}$
56
9. $\frac{5!}{3!2!} \times \frac{5!}{2!3!}$
10 x 10
100
10. 8 Teachers
5 Students
4 people
1 Teacher OR 2 Teachers
$_8C_1 \times {}_5C_3$ + $_8C_2 \times {}_5C_2$
8 x 10 + 28 x 10
80 + 280
360


52 cards 3 cards
at Most 1 ace
1 Ace and 2 others
$_4C_1 \times {}_{48}C_2$
4×1128
4512
OR
0 Aces and 3 others
$_4C_0 \times {}_{48}C_3$
1×17296
17296
+
+
21808
Problem:
1.
1-8
a) $8 \times 8 \times 8 \times 8$
4096
b) $_8P_4 = 1680$


2. $_8P_6$
20160
4.
Stack 1
5 P
3 N
4 Q
12
12!
5!3!4!
27720
Stact 2
3 P
3 N
6 Q
12
12!
3!3!6!
18480



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