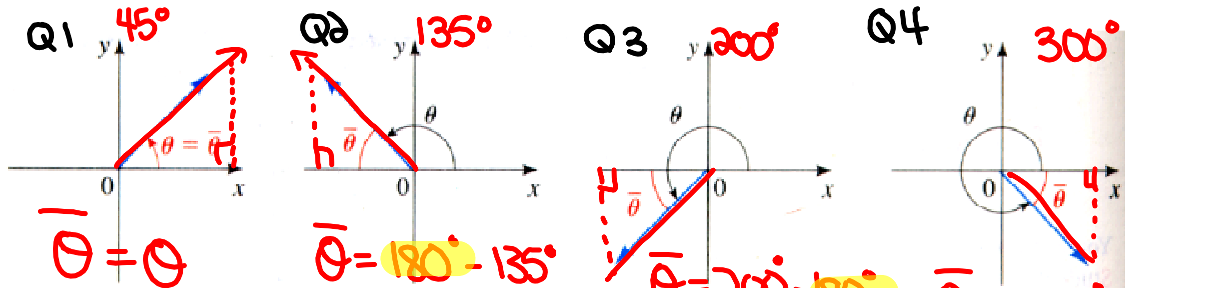
Reference Triangles:

Definition 17 *The reference angle $\overline{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

0 and $\frac{\pi}{2}$ rads

The picture below illustrates this concept.



$\overline{\theta} = \theta$
$= 45°$

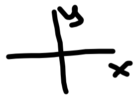
$\overline{\theta} = 180° - 135°$
$= 45°$

$\overline{\theta} = 200° - 180°$
$= 20°$

$\overline{\theta} = 360° - 300°$
$= 60°$

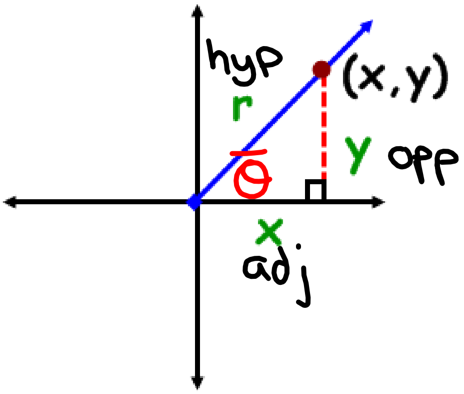
What is the significance of reference angles?

Angles on the Cartesian Plane



- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



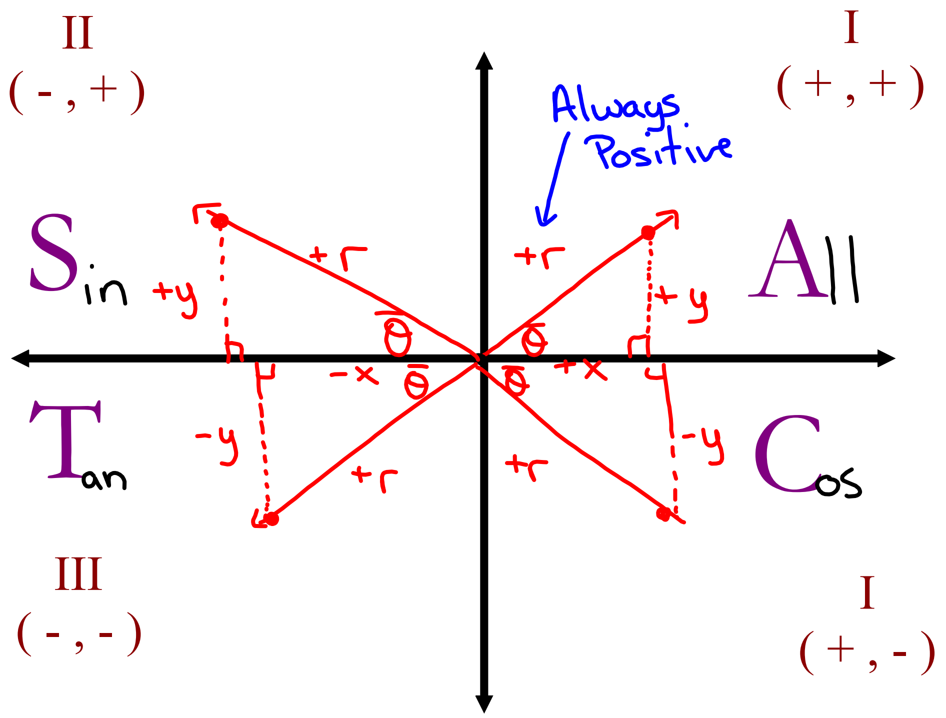
Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$
"Primary"	"Reciprocal"

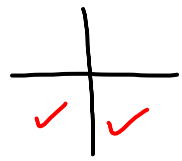
TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are POSITIVE in...



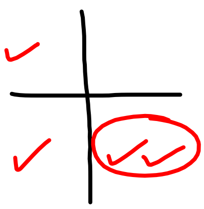
Where is θ if...

$\csc\theta < 0$
$(\sin\theta)$



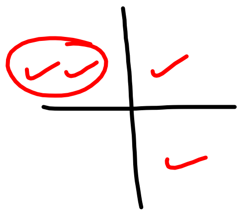
Quad 3 or Quad 4

$\sin\theta < 0$ & $\tan\theta < 0$



Quad 4

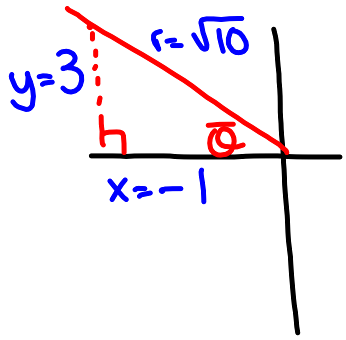
$\csc\theta > 0$ & $\cot\theta < 0$
$(\sin\theta)$ $(\tan\theta)$



Quad 2

If $\sec\theta = -\sqrt{10}$ *and* $\sin\theta > 0$, determine the value of $\csc\theta$.

$\sec\theta = -\frac{\sqrt{10}}{1}$ $\quad \frac{r}{x}$



$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

Choose $y = 3$

$r = \sqrt{10}$

$x = -1$

$\csc\theta = \frac{hyp}{opp} = \frac{r}{y} = \frac{\sqrt{10}}{3}$

Example

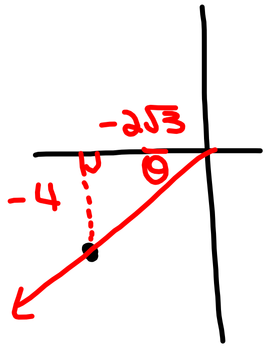
Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $\left(-2\sqrt{3}, -4\right)$

x, y

Given:

$x = -2\sqrt{3}$

$y = -4$



$\tan\bar{\theta} = \dfrac{y}{x}$

$\tan\bar{\theta} = \dfrac{-4}{-2\sqrt{3}}$

$\tan\bar{\theta} = \dfrac{2}{\sqrt{3}}$

```
tan⁻¹(2/√(3))
      49.10660535
```

$\bar{\theta} = \tan^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

$\bar{\theta} = 49.1$

$\theta = 49.1^\circ + 180^\circ$

$\theta = 229.1^\circ$

To Convert to Radians

$229.1\left(\dfrac{\pi}{180}\right) = \boxed{3.99 \text{ rads}}$

In Radians

```
tan⁻¹(2/√(3))
      .8570719479
```

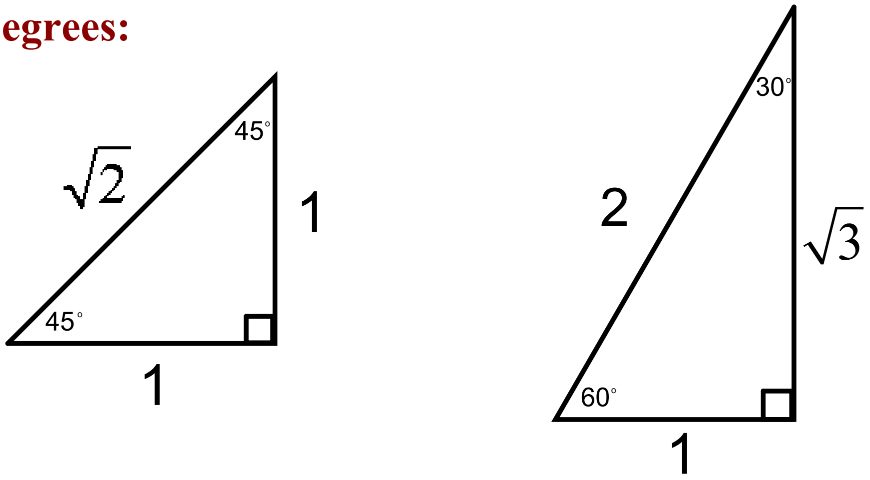
$\theta = 0.857 + \pi$

$\theta = 0.857 + 3.14$

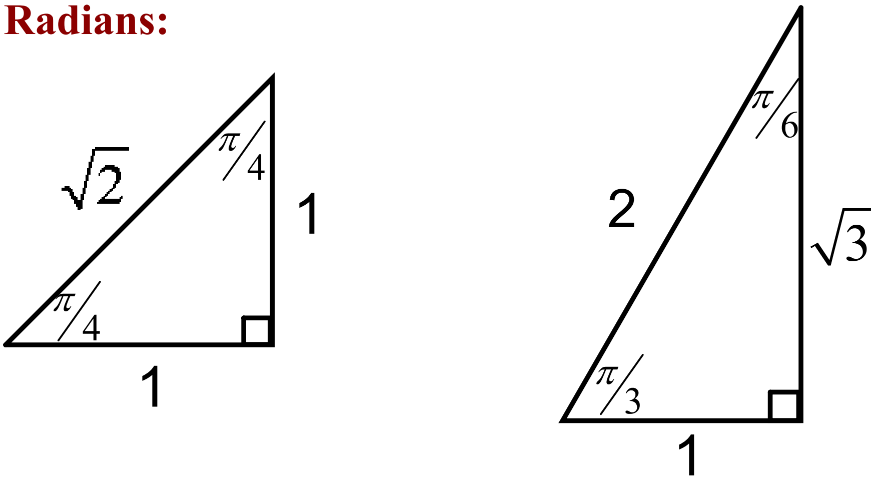
$\theta = 3.99 \text{ rads}$

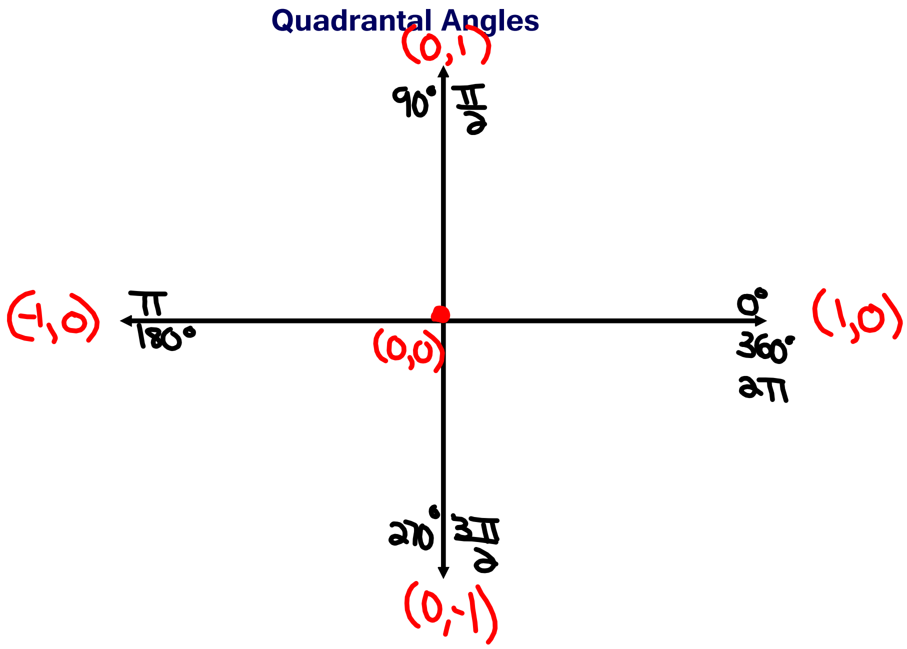
Special Angles

In Degrees:



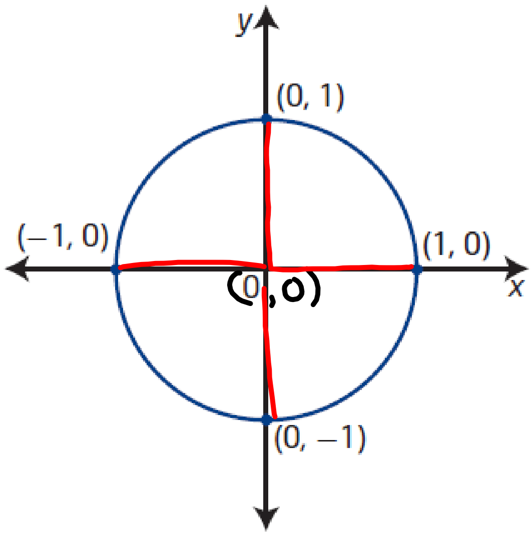
In Radians:





Unit Circle

(used for multiples of 90° or $\frac{\pi}{2}$ rads)



unit circle

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$

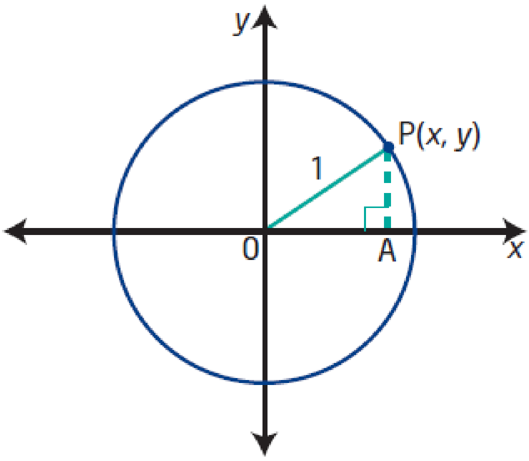
$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$

$\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{1}{y}$

$\sec\theta = \frac{1}{x}$

$\cot\theta = \frac{x}{y}$



Center: (0,0)

Radius: r=1

$x^2+y^2=r^2$

The equation of the unit circle is $x^2 + y^2 = 1$.

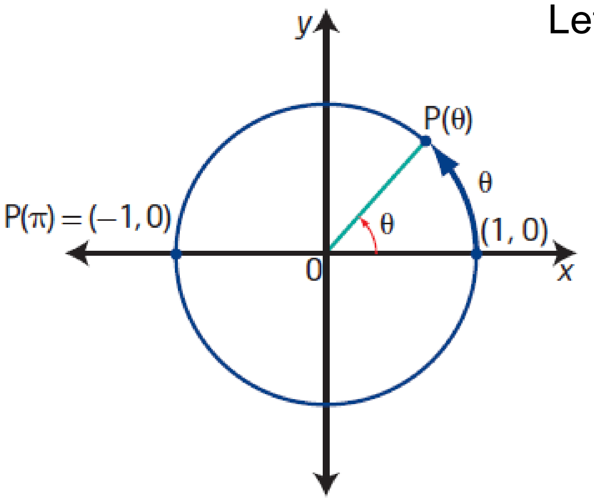
Determine the equation of a circle with centre at the origin and radius 6.

$x^2+y^2=r^2$

$x^2+y^2=(6)^2$

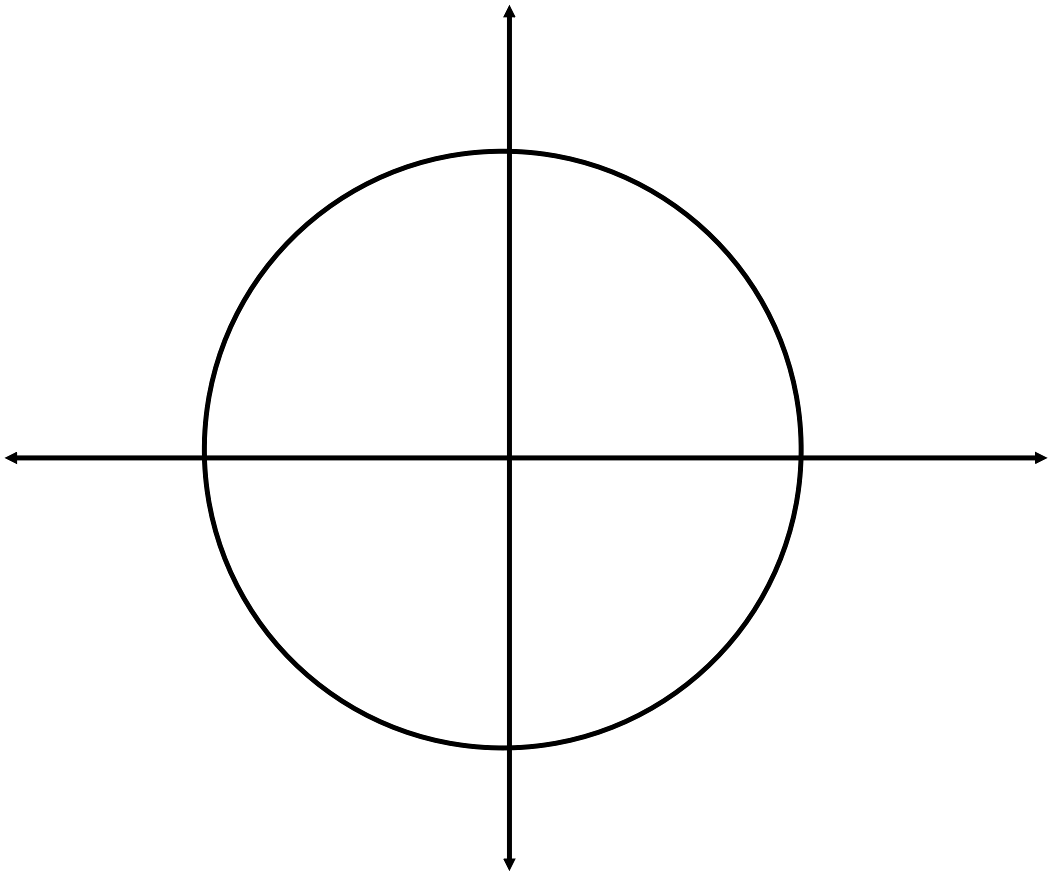
$\boxed{x^2+y^2=36}$

Special Angles on the Unit Circle:

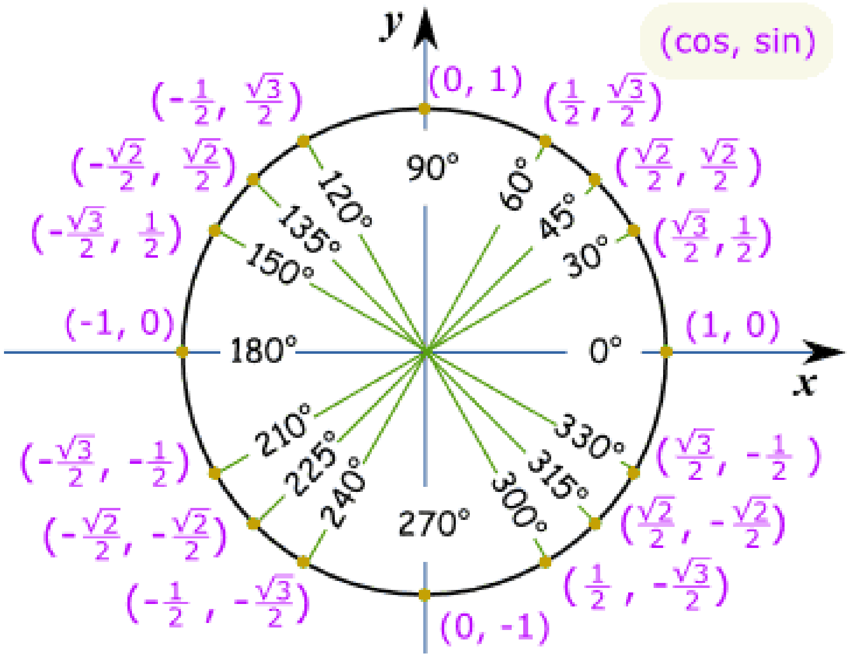


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

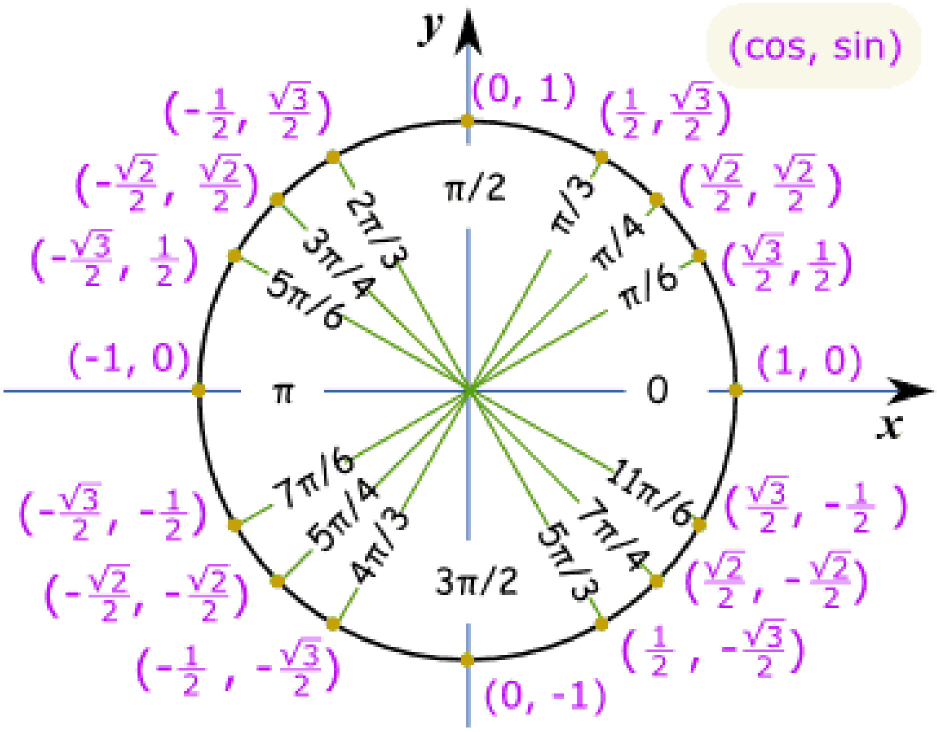


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



Problems Involving the Unit Circle:

Determine Coordinates for Points of the Unit Circle

Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case.

- the y-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III

$$x^2 + y^2 = 1$$

$$x^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$$

$$x^2 + \frac{1}{2} = 1$$

$$x^2 = 1 - \frac{1}{2}$$

$$x^2 = \frac{2}{2} - \frac{1}{2}$$

$$x^2 = \frac{1}{2}$$

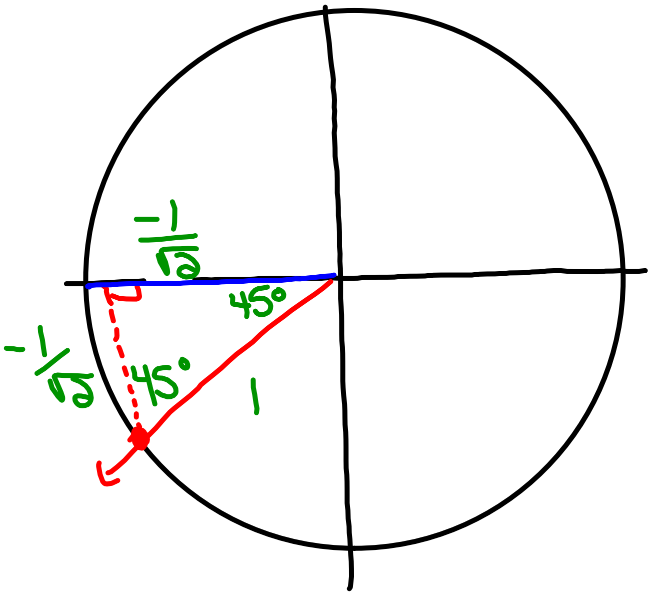
$$x = \pm\frac{\sqrt{1}}{\sqrt{2}}$$

$$x = \pm\frac{1}{\sqrt{2}}$$

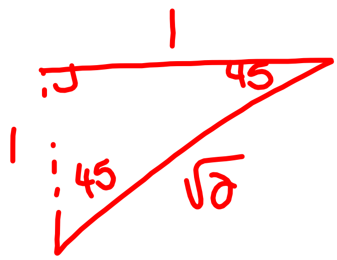
Since the point is in quadrant III $x = -\frac{1}{\sqrt{2}}$

Problems Involving the Unit Circle:

If $P(225°)$ is the point at which the terminal arm of an angle θ in standard position intersects the unit circle, determine the exact coordinates of... $P(225°)$



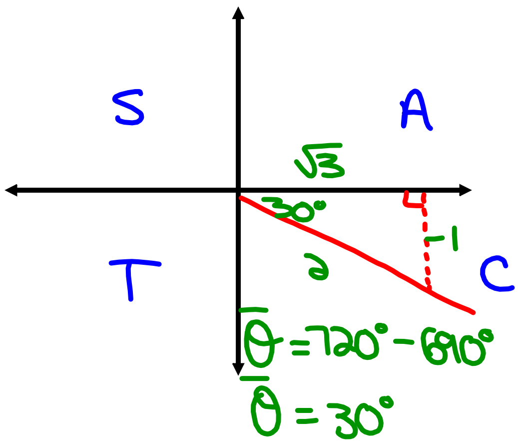
Scale this diagram to fit unit circle



$$P(225°) \rightarrow \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \text{ or } \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

Solving Trig Expressions by Sketching Angles

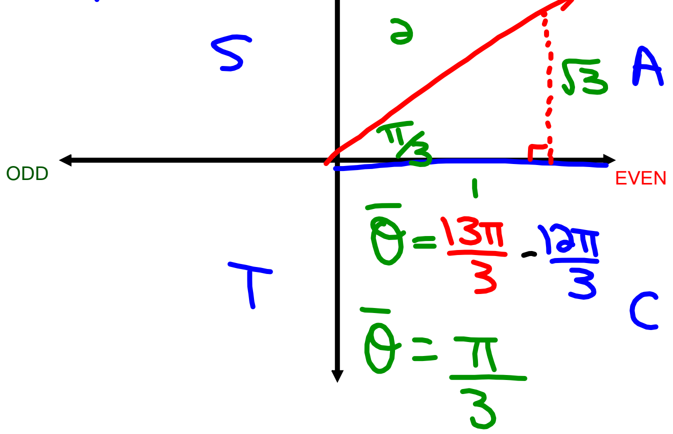
Ex. Evaluate the $\sin 690° = -\frac{1}{2}$



Ex. $\cos\frac{13\pi}{3} = +\frac{1}{2}$

$\frac{12\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$

4π



Homework

Evaluate each Trig Expression (provide a sketch of each angle)

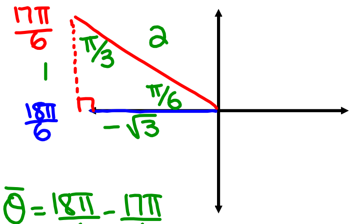
1. $\tan\dfrac{17\pi}{6}$ 2. $\sin\dfrac{15\pi}{4}$ 3. $\cos\left(-\dfrac{21\pi}{4}\right)$

$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$ $-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ $-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

Ex. $\tan\frac{17\pi}{6} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\frac{16\pi}{6}, \frac{17\pi}{6}, \frac{18\pi}{6}$

3π



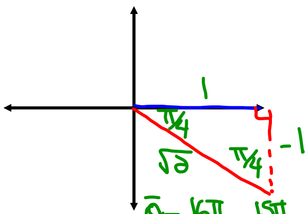
$\bar{\theta} = \frac{18\pi}{6} - \frac{17\pi}{6}$

$\bar{\theta} = \frac{\pi}{6}$

Ex. $\sin\frac{15\pi}{4} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}$

4π



$\bar{\theta} = \frac{16\pi}{4} - \frac{15\pi}{4}$

$\bar{\theta} = \frac{\pi}{4}$

Ex. $\cos\left(-\frac{21\pi}{4}\right) = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

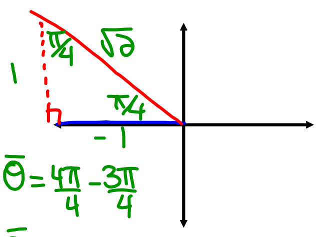
$\frac{-21\pi}{4} + \frac{6\pi}{1}$

$\frac{-21\pi}{4} + \frac{24\pi}{4} = \frac{3\pi}{4}$

$\cos\frac{3\pi}{4}$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$

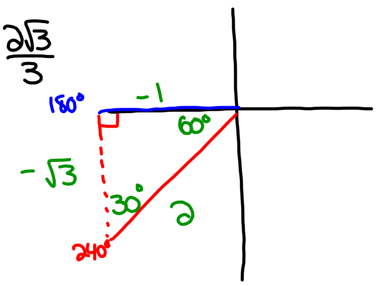
1π



$\bar{\theta} = \frac{4\pi}{4} - \frac{3\pi}{4}$

$\bar{\theta} = \frac{\pi}{4}$

$\csc 240° = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$



$\bar{\theta} = 240° - 180°$

$\bar{\theta} = 60°$

Evaluate without the use of a calculator:

$$\sin\frac{9\pi}{2}-\cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$$

Evaluate without the use of a calculator:

$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right)+\csc\left(\frac{11\pi}{2}\right)+\sin^2\left(\frac{27\pi}{4}\right)$$

Homework: Worksheet - Sketching Angles in Radians.doc

Solutions...

1. $-\frac{5}{3}$

2. $\frac{-\sqrt{6}}{3}$

3. $-2-\sqrt{3}$

4. $\frac{-5}{3}$

5. $\frac{4+3\sqrt{3}}{6}$

6. $\frac{-10}{3}$

7. 0

8. $\frac{3+3\sqrt{3}}{-2}$

Attachments

Worksheet - Sketching Angles in Radians.doc