Questions from homework

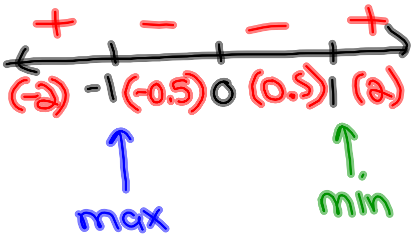
② f) $h(x)=3x^5-5x^3$

$h'(x)=15x^4-15x^2$

$h'(x)=15x^2(x^2-1)$

$h'(x)=15x^2(x+1)(x-1)$

CV: $x=-1,0,1$



local max:

$h(-1)=3(-1)^5-5(-1)^3$ $(-1,2)$

$=-3+5$

$=2$

Local min:

$h(1)=3(1)^5-5(1)^3$ $(1,-2)$

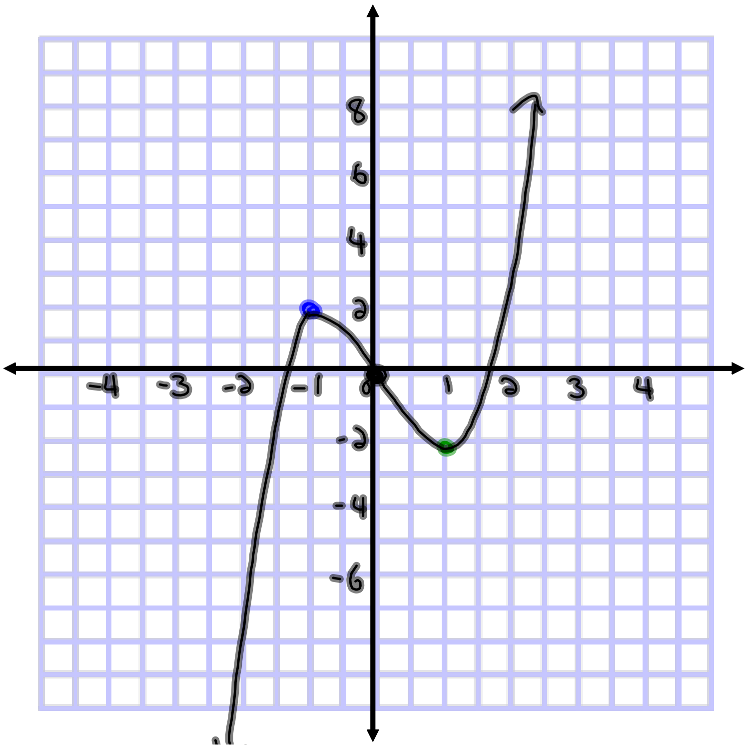
$=3-5$

$=-2$

Increasing on $(-\infty,-1)$ + $(1,\infty)$

Decreasing on $(-1,0)$ + $(0,1)$

or $(-1,1)$



Questions from homework

$$y=\frac{(x-1)^3}{x^2} = \frac{x^3-3x^2+3x-1}{x^2}$$

$\boxed{\text{Intercepts:}}$

x int (y=0)

$(x-1)^3=0$

$x-1=0$

$x=1$

(1,0)

y int (x=0)

$y=\frac{-1}{0}=$ undefined

No y intercept

$\boxed{\text{Asymptotes:}}$

V.A.

$x^2=0$

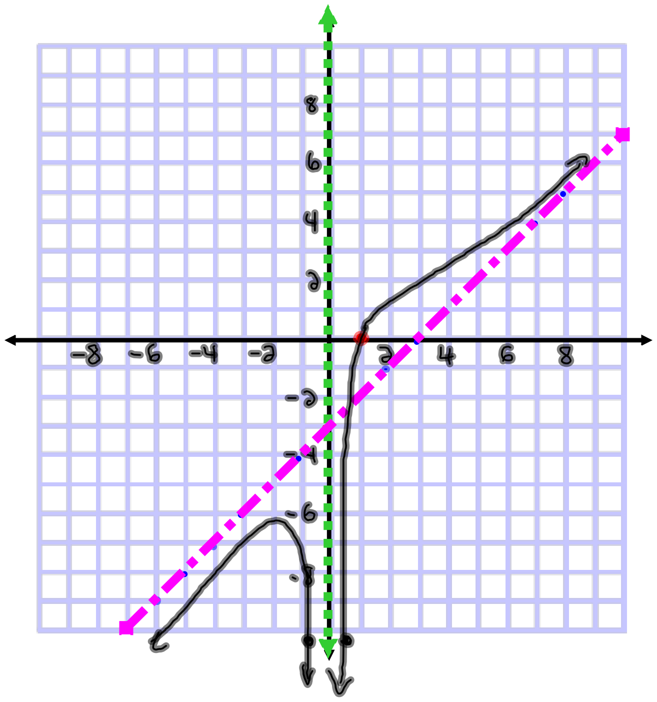
$x=0$

$$\begin{array}{r} x-3 \\ \underline{x^2}\overline{)x^3-3x^2+3x-1} \\ -x^3\quad\downarrow\qquad\quad \\ \hline -\underline{3x^2}+3x-1 \\ -3x^2\qquad\quad \\ \hline 3x-1\,R \end{array}$$

S.A.

$y=x-3$

$m=\frac{1}{1}$ $b=-3$



$$\lim_{x\to 0^-} \frac{(-)}{(+)} = -\infty$$

$x=-0.01$

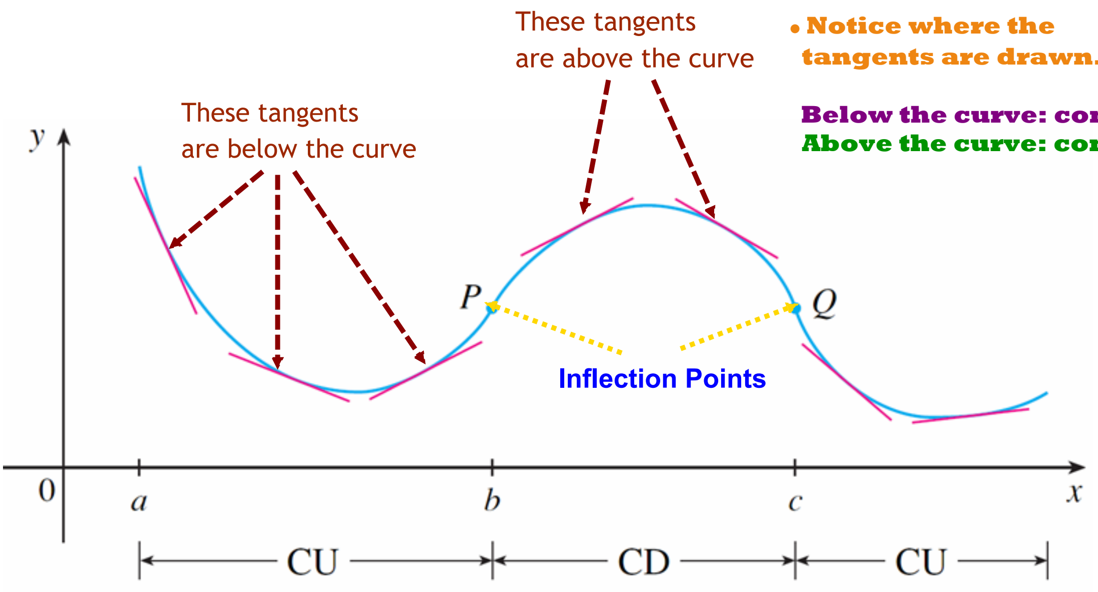
$$\lim_{x\to 0^+} \frac{(-)}{(+)} = -\infty$$

$x=0.01$

Concavity

- **Notice where the tangents are drawn...**

Below the curve: concave up
Above the curve: concave down



These tangents are below the curve

These tangents are above the curve

Inflection Points

- In general, the graph of *f* is called ***concave upward*** on an interval *I* if it lies above all its tangents.
- It is called ***concave downward*** on *I* if it lies below all of these tangents.
- A point where a curve changes its direction of concavity is called an ***inflection point***.

If $f'(x) > 0$ then $f(x)$ is increasing,
so if $f''(x) > 0$ then $f'(x)$ is increasing.

Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.
(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Thus there is a point of inflection at any point where the second derivative changes sign.

Determine where the curve $y = x^3 - 3x^2 + 4x - 5$ is concave upward and concave downward

Find the points of inflection

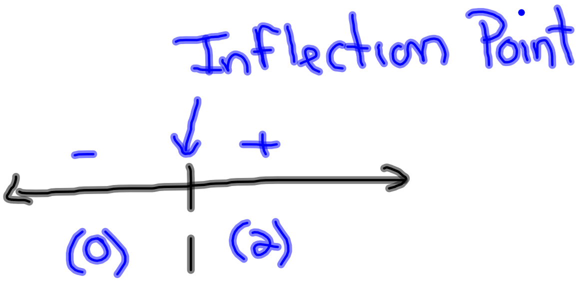
$y = x^3 - 3x^2 + 4x - 5$

$y' = 3x^2 - 6x + 4$

$y'' = 6x - 6$

$y'' = 6(x-1)$

$\boxed{\text{CV: } x = 1}$



Concave Up on $(1, \infty)$

Concave Down on $(-\infty, 1)$

Inflection Point: $(x=1)$

$y = x^3 - 3x^2 + 4x - 5$

$y = (1)^3 - 3(1)^2 + 4(1) - 5$

$y = 1 - 3 + 4 - 5$

$y = -3$

IP: $(1, -3)$

Determine where the curve $y = \frac{x}{x^2+1}$ is concave upward and concave downward

Find the points of inflection

$$y = \frac{x}{x^2+1}$$

$$y' = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \boxed{\frac{-x^2+1}{(x^2+1)^2}}$$

$$y'' = \frac{(x^2+1)^2(-2x) - (-x^2+1)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

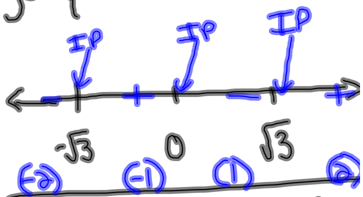
$$y'' = \frac{-2x(x^2+1)^2 + 4x(x^2-1)(x^2+1)}{(x^2+1)^4}$$

$$y'' = \frac{2x\cancel{(x^2+1)}\left[\overset{-x^2-1}{-(x^2+1)}\overset{+2x^2-2}{+2(x^2-1)}\right]}{(x^2+1)^{\cancel{4}\,3}}$$

$$y'' = \frac{2x(x^2-3)}{(x^2+1)^3} \longleftarrow \text{Always positive}$$

CV: $2x = 0$
$x = 0$

$x^2 - 3 = 0$
$x^2 = 3$
$x = \pm\sqrt{3}$



CU on $(-\sqrt{3}, 0)$ and $(\sqrt{3},$

CD on $(-\infty, -\sqrt{3})$ and $(0$

Inflection Points: $y = \frac{x}{x^2+1}$

$f(-\sqrt{3}) = -\frac{\sqrt{3}}{4}$ $\left(-\sqrt{3}, -\frac{\sqrt{3}}{4}\right)$

$F(0) = \frac{0}{1} = 0$ $(0,0)$

homework

Second Derivative Test for Local Extrema

The Second Derivative Test Suppose f'' is continuous near c.

(a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.

(b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

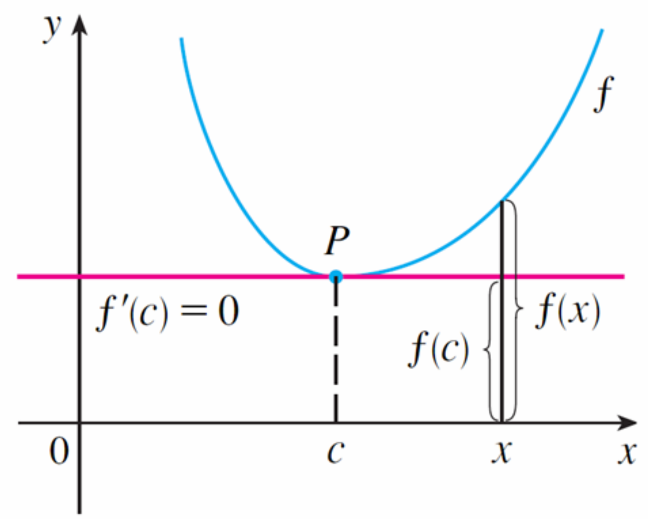


FIGURE 6

$f''(c) > 0$, f is concave upward

Example:

Examine the function $f(x) = x^4 - 4x^3$ with respect to…

- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values

Solution

Example:

Using the function: $f(x) = \frac{x^2}{x-7}$

Determine each of the following...

- Intercepts
- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values