

Higher Derivatives

Since the derivative of a function f is itself a function f' , we can take its derivative $(f')'$. The result is a function called the **second derivative** of f and is denoted by f''

In Leibniz notation we write:

$$f''(x) = \frac{d^2 y}{dx^2}$$

Find $y'' = f''(x) = \frac{d^2 y}{dx^2}$

Examples

$$y = x^6$$

$$y' = 6x^5$$

$$y'' = 30x^4$$

$$f(x) = 5x^2 + \sqrt{x} = 5x^2 + x^{1/2}$$

$$f'(x) = 10x + \frac{1}{2}x^{-1/2}$$

$$f''(x) = 10 - \frac{1}{4}x^{-3/2} = \boxed{10 - \frac{1}{4\sqrt{x^3}}}$$

$$f(x) = (2 - x^2)^{10}$$

$$f'(x) = 10(2 - x^2)^9(-2x) = (-20x)(2 - x^2)^9$$

$$f''(x) = -20x(9)(2 - x^2)^8(-2x) - 20(2 - x^2)^9$$

$$= 360x^2(2 - x^2)^8 - 20(2 - x^2)^9$$

$$= 20(2 - x^2)^8 [18x^2 - (2 - x^2)]$$

$$= \boxed{20(2 - x^2)^8(19x^2 - 2)}$$

Since the first derivative of a function can be interpreted either as the slope of a tangent line or as a rate of change, the second derivative can be interpreted as *the rate of change of the slope of the tangent line*.

This idea will be pursued later where the second derivative gives valuable information about the shape of a graph, or when dealing with acceleration.

Higher derivatives can also be defined. The **third derivative** is the derivative of the second derivative.

$$y''' = f'''(x) = \frac{d^3 y}{dx^3}$$

$y^{(4)} \rightarrow 4^{\text{th}}$ Derivative

$y^{(5)} \rightarrow 5^{\text{th}}$ "

Find the first five derivatives of

$$y = x^4 + 2x^3 - 5x^2 + 3x - 6$$

$$y' = 4x^3 + 6x^2 - 10x + 3$$

$$y'' = 12x^2 + 12x - 10$$

$$y''' = 24x + 12$$

$$y^{(4)} = 24$$

$$y^{(5)} = 0$$

Homework

Exercise 2.8

Omit 6 and 7

$$\textcircled{1} \text{ c) } f(t) = 2t - \frac{1}{t+1} = 2t - (t+1)^{-1}$$

$$f'(x) = 2 + 1(t+1)^{-2} (1) = 2 + (t+1)^{-2}$$

$$f''(x) = -2(t+1)^{-3} (1)$$

$$= \boxed{\frac{-2}{(t+1)^3}}$$

$$\textcircled{1} \text{ g) } f(x) = \sqrt{x^2+1} = (x^2+1)^{1/2}$$

$$f'(x) = \frac{1}{2} (x^2+1)^{-1/2} (2x)$$

$$= x(x^2+1)^{-1/2}$$

$$f''(x) = x \left(\frac{-1}{2} \right) (x^2+1)^{-3/2} (2x) + 1(x^2+1)^{-1/2}$$

$$= -x^2(x^2+1)^{-3/2} + (x^2+1)^{-1/2}$$

$$= (x^2+1)^{-3/2} \left[-x^2 + (x^2+1) \right]$$

$$= (x^2+1)^{-3/2} (1)$$

$$= \frac{1}{\sqrt{(x^2+1)^3}}$$