

Arithmetic
(common difference "d")

$$t_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric
(Common Ratio "r")

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

*

$$S_n = \frac{a}{1 - r}$$

$$-1 < r < 1$$

$$\textcircled{1} \quad \textcircled{1} - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$$

$$a = 1$$

$$r = -\frac{2}{3}$$

$$S_n = \frac{a}{1-r}$$

$$= \frac{1}{1 - (-\frac{2}{3})}$$

$$= \frac{1}{\frac{5}{3}}$$

$$= 1 \times \frac{3}{5}$$

$$= \boxed{\frac{3}{5}}$$

$$\textcircled{c) \quad \textcircled{\frac{1}{4}} - \frac{5}{16} + \frac{25}{64} - \frac{125}{256} + \dots}$$

$$a = \frac{1}{4}$$

Divergent (r is too small)

$$r = -\frac{5}{4}$$

$$\textcircled{b) \quad \sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n = \textcircled{-\frac{2}{5}} + \frac{4}{25} - \frac{8}{125} + \frac{16}{625} - \dots$$

$$a = -\frac{2}{5}$$

$$r = \frac{4}{25} = -\frac{2}{5}$$

$$= \frac{4}{25} \times -\frac{2}{5}$$

$$= -\frac{2}{5}$$

$$S_n = \frac{-\frac{2}{5}}{1 - (-\frac{2}{5})}$$

$$= \frac{-\frac{2}{5}}{\frac{7}{5}}$$

$$= -\frac{2}{5} \times \frac{5}{7}$$

$$= \boxed{-\frac{2}{7}}$$

Series & Sequence Review

② 12000, —, —, —, —, 91125

$$a = 12000$$

$$t_n = ar^{n-1}$$

$$t_6 = 91125$$

$$\frac{91125}{12000} = \frac{12000r^{6-1}}{12000}$$

$$n = 6$$

$$r = ?$$

$$7.59375 = r^5$$

$$1.5 = r$$

* Annual rate of Increase:

$$1.5 - 1 = 0.5 \times 100 = 50\%$$

④ $t_1 = 3$

$$t_2 = (t_{2-1})^2$$

$$t_3 = 81$$

$$t_n = (t_{n-1})^2$$

$$= (t_1)^2$$

$$t_4 = 6561$$

$$= (3)^2$$

$$t_5 = 43046721$$

$$= 9$$

④ $t_1 = 3$ $t_n = \underline{\underline{(t_{n-1})^2}}$ "recursive rules"
previous term

$$\begin{aligned} t_2 &= (t_{2-1})^2 \\ &= (t_1)^2 \\ &= (3)^2 \\ &= 9 \end{aligned}$$

$$\begin{aligned} t_3 &= (t_2)^2 \\ &= (9)^2 \\ &= 81 \end{aligned}$$

$$\textcircled{5} \text{ c) } 4 + 8 + 12 + \dots + 400$$

$$a = 4$$

$$d = 4$$

$$t_n = 400$$

$$S_n = ?$$

Solve for "n"

$$t_n = a + (n-1)d$$

$$400 = 4 + (n-1)(4)$$

$$400 = 4 + 4n - 4$$

$$400 = 4n$$

$$100 = n$$

$$S_n = \frac{n}{2} (a + t_n)$$

$$S_{100} = \frac{100}{2} (4 + 400)$$

$$S_{100} = 50(404)$$

$$S_{100} = 20200$$

$$\textcircled{5} \text{ a) } \sum_{n=1}^5 2n+1$$

$$= 3 + 5 + 7 + 9 + 11$$

$$= 35$$

$$\begin{array}{l|l}
 \textcircled{6} & t_5 = 16 \\
 & t_5 = a + (5-1)d \\
 & t_5 = a + 4d \\
 \hline
 & a + 4d = 16 \\
 & t_8 = 25 \\
 & t_8 = a + (8-1)d \\
 & t_8 = a + 7d \\
 \hline
 & a + 7d = 25 \\
 & t_n = a + (n-1)d
 \end{array}$$

$$\begin{array}{l}
 \textcircled{7} \quad \begin{array}{l} a + 7d = 25 \\ a + 4d = 16 \\ \hline 3d = 9 \\ \boxed{d = 3} \end{array} \\
 \begin{array}{l} a + 4d = 16 \\ a + 4(3) = 16 \\ a + 12 = 16 \\ \boxed{a = 4} \end{array} \\
 \begin{array}{l} t_n = a + (n-1)d \\ t_n = 4 + (n-1)3 \\ t_n = 4 + 3n - 3 \\ \boxed{t_n = 3n + 1} \end{array}
 \end{array}$$

$$\textcircled{7} \quad t_5 = 48 \quad | \quad t_8 = 384 \quad t_n = ar^{n-1}$$

$$t_5 = ar^{5-1} \quad | \quad t_8 = ar^{8-1}$$

$$t_5 = ar^4 \quad | \quad t_8 = ar^7$$

$$\boxed{ar^4 = 48} \quad | \quad \boxed{ar^7 = 384}$$

$$\frac{ar^7 = 384}{ar^4 = 48}$$

$$r^3 = 8$$

$$\boxed{r = 2}$$

$$ar^4 = 48$$

$$a(2)^4 = 48$$

$$16a = 48$$

$$\boxed{a = 3}$$

$$t_n = ar^{n-1}$$

$$\boxed{t_n = (3)(2)^{n-1}}$$

$$\textcircled{8} \quad \textcircled{\frac{5}{8}}, \frac{5}{2}, 10, \dots, \underline{\underline{640}}.$$

$$a = \frac{5}{8}$$

$$r = 4$$

$$t_n = 640$$

$$t_n = ar^{n-1}$$
$$640 = \frac{5}{8}(4)^{n-1}$$

$$\star \frac{\log 1024}{\log 4} = 5$$

$$1024 = 4^{n-1}$$

$$4^{\textcircled{5}} = 4^{n-1}$$

$$5 = n - 1$$

$$\boxed{6 = n}$$

S+S Review 2

$$\textcircled{1} \quad t_{12} = 15$$

$$t_n = a + (n-1)d$$

$$t_{12} = a + (12-1)d$$

$$t_{12} = a + 11d$$

$$\boxed{a + 11d = 15}$$

$$S_{15} = 105$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{15} = \frac{15}{2}(2a + (15-1)d)$$

$$S_{15} = 7.5(2a + 14d)$$

$$S_{15} = 15a + 105d$$

$$\boxed{15a + 105d = 105}$$

$$a + 11d = 15$$

$$\textcircled{2} \quad a + 7d = 7$$

$$\frac{4d = 8}{d = 2}$$

$$\boxed{d = 2}$$

$$a + 7d = 7$$

$$a + 7(2) = 7$$

$$a + 14 = 7$$

$$\boxed{a = -7}$$

$$-7, -5, -3$$

