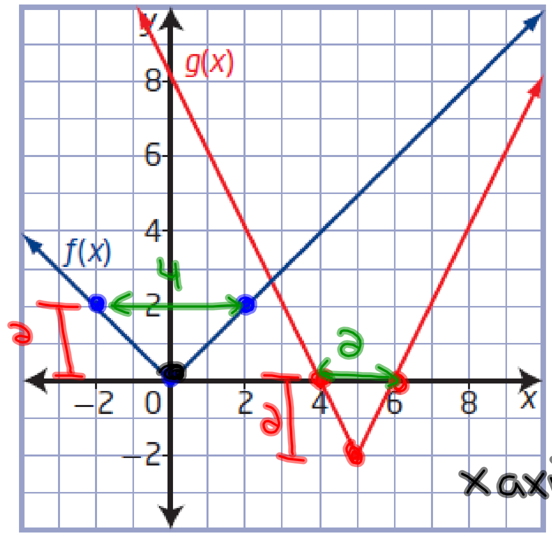
Review

11. Write the equation for the graph of $g(x)$ as a transformation of the equation for the graph of $f(x)$.



① VSF: $\frac{2}{2} = 1$ $a = 1$

② HSF: $\frac{2}{4} = \frac{1}{2}$ $b = 2$

③ Reflections: None

④ VT:
x axis $(0,0) \longrightarrow (5,-2)$
Down 2 $k = -2$

$$g(x) = f(2(x - 5)) - 2$$

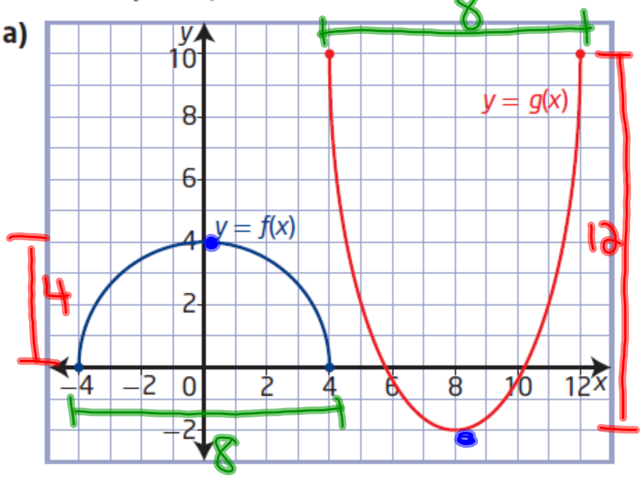
⑤ HT
y axis $(0,0) \longrightarrow (5,-2)$
Right 5 $h = 5$

$$\boxed{g(x) = 1f(2(x-5)) - 2}$$

Review

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

a)



$y = -3f(x - 8) + 10$

① VSF: $\frac{12}{4} = 3$ $a = 3$

② HSF: $\frac{8}{8} = 1$ $b = \frac{1}{1}$

* b is the reciprocal of the horizontal stretch

③ Reflections:

Vertical reflection in x-axis $(a<0)$

④ VT:

x-axis $(4,0) \rightarrow (12,10)$

Up 10 units $k=10$

⑤ HT:

y-axis $(0,4) \rightarrow (8,-2)$

8 units right $h=8$

$y = af(b(x-h))+k$

$y = -3f(1(x-8))+10$

RST

7. Describe, using an appropriate order, how to obtain the graph of each function from the graph of $y = f(x)$. Then, give the mapping for the transformation.

reflection in the y-axis, horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{3}$, and translation of 6 units right and 2 units up; $(x, y) \rightarrow \left(-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right)$

$3y - 6 = f(-2x + 12)$

$$\frac{3y}{3} = \frac{1f(-2x+12)}{3} + \frac{6}{3}$$ (Divide a + k)

$$y = \frac{1}{3}f(-2x+12) + 2$$ (Factor)

$$y = \frac{1}{3}f(-2(x-6)) + 2$$

$a = \frac{1}{3}$ $b = -2$ $h = 6$ $k = 2$

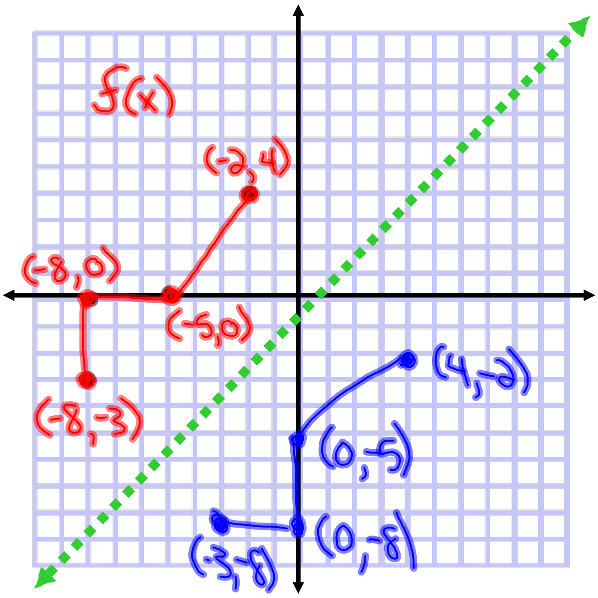
VSF $= \frac{1}{3}$ HSF $= \frac{1}{2}$ HT = Right 6 VT = Up 2

+ horizontal reflection in y-axis.

Mapping Rule:

$$(x, y) \longrightarrow \left[-\frac{1}{2}(x) + 6, \frac{1}{3}y + 2\right]$$

Inverse Relations



a) Sketch the Inverse

$(x,y) \rightarrow (y,x)$

b) Is the Inverse a function?

No because $f(x)$ does not pass the horizontal line test

a) Determine the Inverse of $f(x) = 3\sqrt{x-5} + 8$

① $y = 3\sqrt{x-5} + 8$

② $x = 3\sqrt{y-5} + 8$

③ $x - 8 = 3\sqrt{y-5}$

$\frac{1}{3}(x-8) = \sqrt{y-5}$

$\frac{1}{9}(x-8)^2 = y - 5$

$\frac{1}{9}(x-8)^2 + 5 = y$

④ $\boxed{f^{-1}(x) = \frac{1}{9}(x-8)^2 + 5}$

b) State the domain of $f(x)$ and $f^{-1}(x)$

$f(x) = 3\sqrt{x-5} + 8$

D: $x \in [5, \infty)$

R: $y \in [8, \infty)$

$f^{-1}(x) = \frac{1}{9}(x-8)^2 + 5$

D: $x \in [8, \infty)$

R: $y \in [5, \infty)$

Composite Functions

Given the three functions....

$f(x) = 1 - x$ $g(x) = \sqrt{x+1}$ $h(x) = x^2 + 5$

Evaluate each of the following:

1. $(f \circ g)(3) = f(g(3))$
2. $(g \circ h)(0) = g(h(0))$
3. $(g \circ g \circ f)(-7) = g(g(f(-7)))$

① $g(3) = \sqrt{3+1} = 2$
$f(2) = 1-2 = \boxed{-1}$

② $h(0) = (0)^2 + 5 = 5$
$g(5) = \sqrt{5+1} = \boxed{\sqrt{6}}$

③ $f(-7) = 1-(-7) = 8$
$g(8) = \sqrt{8+1} = 3$
$g(3) = \sqrt{3+1} = \boxed{2}$

Page 507: $f(x) = 3x+4$ $g(x) = x^2 - 1$

④ a) $f(g(a))$
$g(a) = a^2 - 1$

$f(a^2+1) = 3(a^2-1) + 4$
$= 3a^2 - 3 + 4$
$= 3a^2 + 1$

Combining Functions

Example

- If $f(x)=\sqrt{x}$ and $g(x)=\sqrt{4-x^2}$, find the functions $f+g$, $f-g$, fg, and f/g.

**Also examine the domain of each of these new functions

$f(x)=\sqrt{x}$ ← radicand ≥ 0

$x\ge 0$

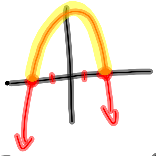
D: $\{x \mid x\ge 0, x\in R\}$

$g(x)=\sqrt{4-x^2}$ ← radicand ≥ 0

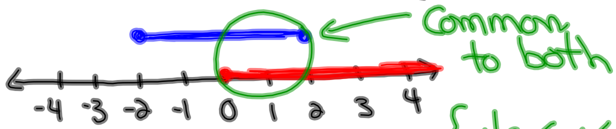
factor

$4-x^2\ge 0$ ← positive y values

$(2-x)(2+x)\ge 0$



D: $\{x \mid -2\le x\le 2, x\in R\}$



$\{x \mid 0\le x\le 2, x\in R\}$ $[0,2]$

a) $f(x)+g(x)$

$\sqrt{x}+\sqrt{4-x^2}$

$\{x \mid 0\le x\le 2\ x\in R\}$

b) $f(x)-g(x)$

$\sqrt{x}-\sqrt{4-x^2}$

$\{x \mid 0\le x\le 2\ x\in R\}$

c) $f(x)\bullet g(x)$

Ex $\sqrt{2}\bullet\sqrt{3}=\sqrt{6}$

$\sqrt{a}\bullet\sqrt{b}=\sqrt{ab}$

$\sqrt{x}\bullet\sqrt{4-x^2}$

$\sqrt{x(4-x^2)}$

$\sqrt{4x-x^3}$

$\{x \mid 0\le x\le 2\ x\in R\}$

d) $\dfrac{f(x)}{g(x)}$

$\dfrac{\sqrt{x}}{\sqrt{4-x^2}}\cdot\dfrac{(\sqrt{4-x^2})}{(\sqrt{4-x^2})}$

$\dfrac{\sqrt{4x-x^3}}{4-x^2}$ ← Non permissable values

$4-x^2\ne 0$

$4\ne x^2$

$\pm 2\ne x$

$\{x \mid 0\le x<2, x\in R\}$

Homework

Chapter Review from textbook...

Pages 56-57
#2, 3, 6, 9, 10, 11, 14, 15, 16

Practice Test
Pages 58-59
All questions

Chapter 10 Review

Pages 510-511 #1-11 (omit #5, 8, 9)

Unit Test:

- Function notation
- combinations:
- compositions:
- catalogue of essential functions
- transformations
  ↳ Reflections, Stretches, Translations
  ↳ $y = af(b(x-h)) + k$

• vertical translation shift up/down

• vertical stretch by a factor of a
• if $a<0$ reflect in x-axis

• horizontal stretch by a factor of $\frac{1}{|b|}$
• if $b<0$ reflect in y-axis

• horizontal translation shift left/right

- Mapping:
  $(x,y) \longrightarrow \left(\frac{1}{b}x+h, ay+k\right)$

⇒ Inverse Functions

- Switch "x" & "y" (Domain & Range)
- Sketch Inverses from a given graph (Reflects in line $y=x$)
- one-one function (Horizontal line)
- Switch to inverse algebraically

i.e. $f(x) = x+7$

$x = y+7$

$x-7 = y$

$f^{-1}(x) = x-7$