Questions from Homework

Related Rates

In a related rates problem, we are given the rate of change of one quantity and we are to find the rate of change of a related quantity. To do this, we find an equation that relates the two quantities and use the *Chain Rule* to differentiate both sides of the equation *with respect to time.*

Related Rates

1. Draw a diagram

2. List what is given in differentiation notation $\frac{da}{dt}$, $\frac{dv}{dt}$, etc.

3. List what is to be found in differentiation notation.

4. Find an appropriate equation that relates the variables in steps 2 and 3.

5. Differentiate with respect to time.

6. Substitute the values given and solve for the unknown.

Areas and Volumes

The length of a square is 4m and is increasing at a rate of 1.25m/min. How fast is the *area* of the square increasing?

Hint!
write down what is given
find an equation that relates the two quantities

$l = 4m$

$A = l^2$

$\frac{dl}{dt} = 1.25m/min$

$\frac{dA}{dt} = 2l\frac{dl}{dt}$

$\frac{dA}{dt} = ?$

$\frac{dA}{dt} = 2(4)(1.25)$

$\frac{dA}{dt} = 10m^2/min$

The area of the square is increasing at a rate $10m^2/min$.

Suppose you tossed a stone into a lake. A circular ripple starts and moves outward with its radius increasing at a rate of 5cm/sec. How fast is the *area* of the circle increasing after 3 seconds? ***(Hint: what would the radius be at 3 seconds?)***

$\frac{dr}{dt} = 5cm/sec$

$\frac{dA}{dt} = ?$

$t = 3s$

* $r = 15cm$
(5x3)

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi (15)(5)$

$\frac{dA}{dt} = 150\pi \ cm^2/sec$

The area of the circle is increasing at a rate of $150\pi \ cm^2/sec$.

Volumes/Surface Areas of Spheres

A *spherical* snowball is melting in such a way that its volume is *decreasing* at a rate of 1 cm^3/min. At what rate is the radius of the snowball decreasing if the original radius is 5 cm?

Hint!
write down what is given
find an equation that relates the two quantities

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = -1\text{ cm}^3/\text{min}$$

$$r = 5\text{cm}$$

$$\frac{dr}{dt} = ?$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-1 = 4\pi(5)^2\frac{dr}{dt}$$

$$-1 = 100\pi\frac{dr}{dt}$$

$$\frac{-1}{100\pi} = \frac{dr}{dt}$$

$$-0.00318\text{ cm/min} = \frac{dr}{dt}$$

The radius is decreasing at a rate of 0.00318 cm/min.

A beach ball is being inflated so that its surface area is *increasing* at a rate of 100 cm^2/sec. Find the rate at which the radius is increasing if the original radius is 2 cm?

$$A = 4\pi r^2$$

$\frac{dA}{dt} = 100\text{cm}^2/\text{sec}$

$r = 2\text{cm}$

$\frac{dr}{dt} = ?$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$100 = 8\pi(2)\frac{dr}{dt}$$

$$100 = 16\pi \frac{dr}{dt}$$

$$\frac{100}{16\pi} = \frac{dr}{dt}$$

$$1.989 \text{ cm/sec} = \frac{dr}{dt}$$

The radius is increasing at a rate of 1.989 cm/sec.

Homework