Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
H	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
H	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$
V	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$
horizontal and vertical	$y = f(x-4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$
H+V	$y = f(x+2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+k)$

$h = -7$

$h = 3$

$K = -6$

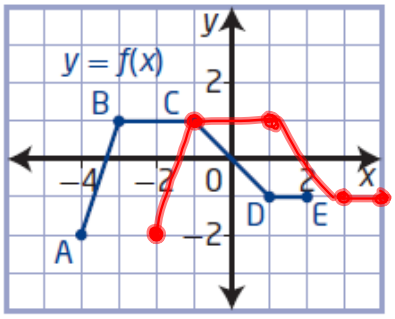
$h = -4 \quad K = -9$

$h = 4 \quad K = -6$

$h = -2 \quad K = 3$

Questions from Homework

②

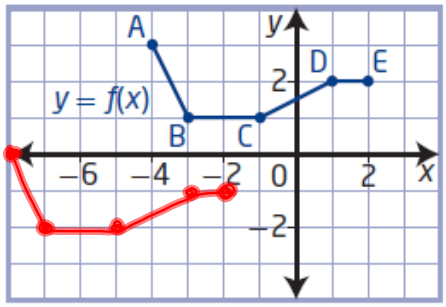


②b $h(x) = f(x-2)$ $h=2$

$(x,y) \longrightarrow (x+2, y)$

A (-4,-2)	A' (-2,-2)
B (-3,1)	B' (-1,1)
C (-1,1)	C' (1,1)
D (1,-1)	D' (3,-1)
E (2,-1)	E' (4,-1)

④



$h=-4$ $k=-3$

④a $s(x) = f(x+4) - 3$

$(x,y) \longrightarrow (x-4, y-3)$

A(-4,3)	A' (-8,0)
B(-3,1)	B' (-7,-2)
C(-1,1)	C' (-5,-2)
D(1,2)	D' (-3,-1)
E(2,2)	E' (-2,-1)

Questions from Homework

10. The graph of $f(x) = |x|$ is transformed to the graph of $g(x) = f(x - 9) + 5$.

a) Determine the equation of the function $g(x)$.

b) Compare the graph of $g(x)$ to the graph of the base function $f(x)$.

c) Determine three points on the graph of $f(x)$. Write the coordinates of the image points if you perform the horizontal translation first and then the vertical translation.

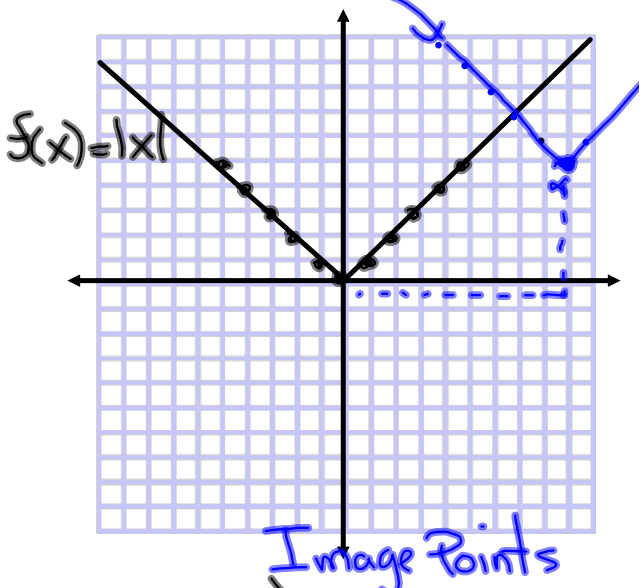
d) Using the same original points from part c), write the coordinates of the image points if you perform the vertical translation first and then the horizontal translation.

e) What do you notice about the coordinates of the image points from parts c) and d)? Is the order of the translations important?

$h = 9$ shift right 9

$k = 5$ shift up 5

a) $g(x) = |x-9| + 5$



c) Points on $f(x)$

$(-1,1)$ $(0,0)$ $(1,1)$

$(x,y) \rightarrow (x+9, y+5)$

Image Points

$(8,6)$ $(9,5)$ $(10,6)$

Transformations:

New Functions From Old Functions

Translations

Stretches

Reflections

Reflections and Stretches

Focus on...

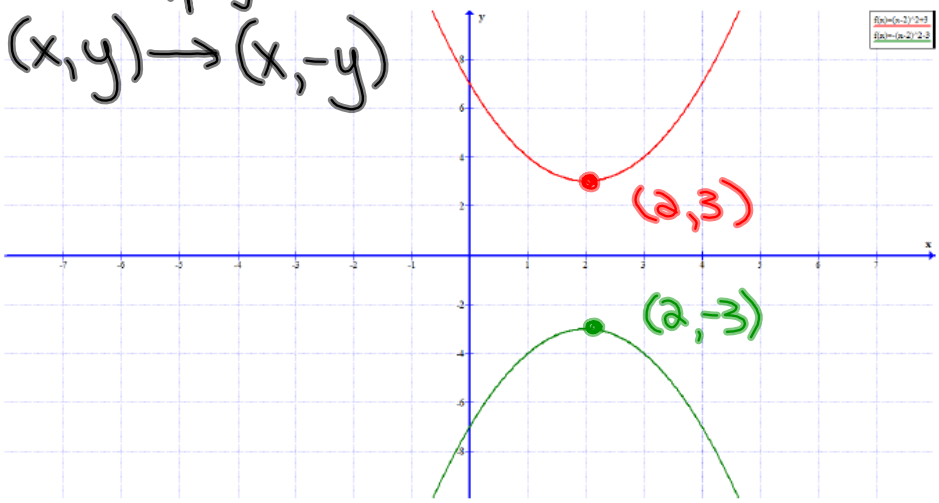
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis. (vertical reflection)

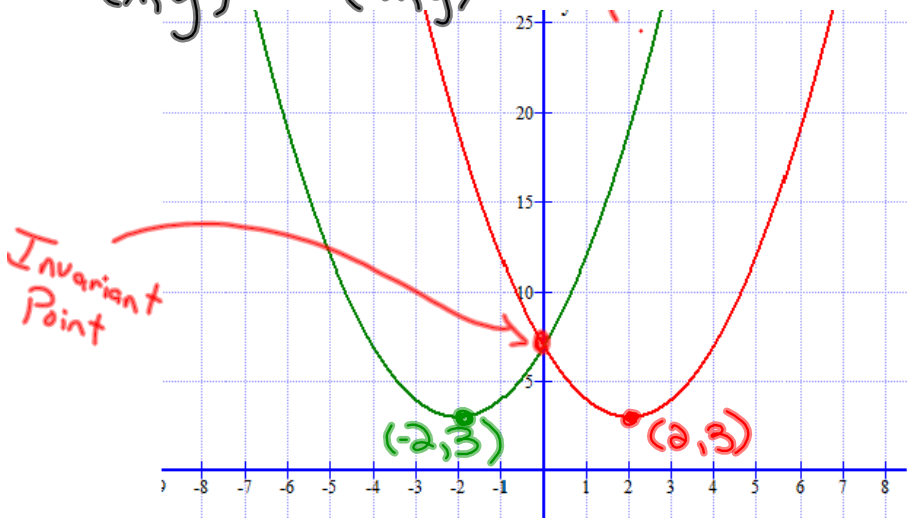
Mapping

$(x,y) \rightarrow (x,-y)$



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis. (horizontal reflection)

Mapping: $(x, y) \rightarrow (-x, y)$



invariant point

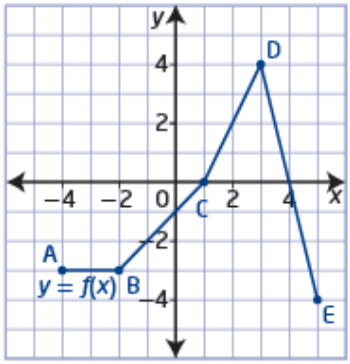
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



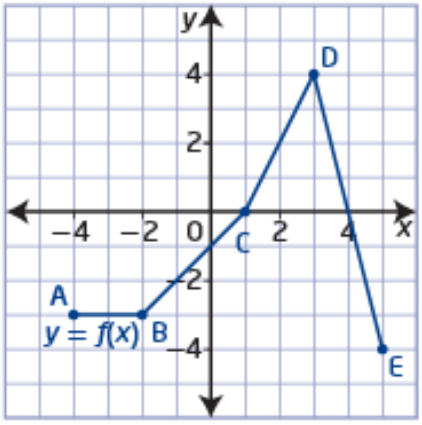
Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below

$(x,y) \rightarrow (x,-y)$

- Invariant Points



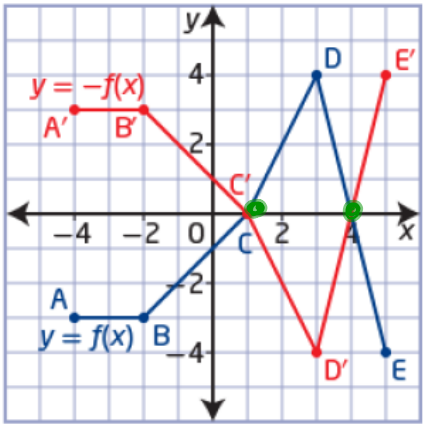
A (-4,-3) A' (-4,3)

B (-2,-3) B' (-2,3)

C (1,0) C' (1,0)

D (3,4) D' (3,-4)

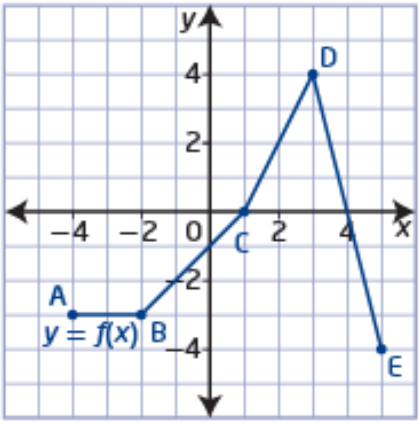
E (5,-4) E' (5,4)



Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

- Sketch $y = f(-x)$ on the axis below



$(x,y) \rightarrow (-x,y)$

A (-4,-3) A' (4,-3)

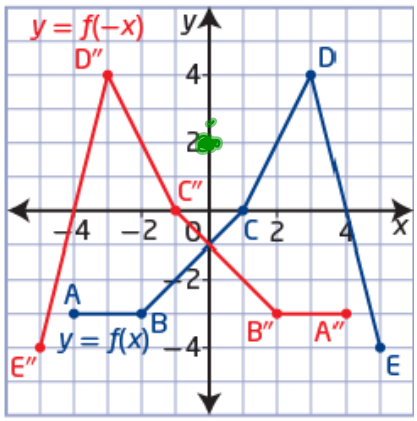
B (-2,-3) B' (2,-3)

C (1,0) C' (-1,0)

D (3,4) D' (-3,4)

E (5,-4) E' (-5,-4)

- Invariant Point



Homework

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stretch

- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor

- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection