Ex. 2.6

⑥ n) $y=\sqrt{x+\sqrt{x+\sqrt{x}}} = \left(x+\left(x+x^{1/2}\right)^{1/2}\right)^{1/2}$

$y' = \frac{1}{2}\left(x+\left(x+x^{1/2}\right)^{1/2}\right)^{-1/2}\left[1+\frac{1}{2}\left(x+x^{1/2}\right)^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)\right]$

$y' = \left[\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\right]\left[1+\left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(1+\frac{1}{2\sqrt{x}}\right)\right]$

$y' = \left[\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\right]\left[1+\left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right)\right]$

$y' = \left[\frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\right]\left[1+\frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}\right]$

Correct Homework Sheet

⑥ d) $G(x) = (x^4 - x + 1)^2 (x^2 - 2)^3$

$$G'(x) = (x^4-x+1)^2(3)(x^2-2)^2(2x) + 2(x^4-x+1)(4x^3-1)(x^2-2)^3$$

$$= 6x(x^4-x+1)^2(x^2-2)^2 + 2(x^4-x+1)(x^2-2)^3(4x^3-1)$$

$$3x^5 - 3x^2 + 3x + 4x^5 - x^2 - 8x^3 + 2$$

$$= 2(x^4-x+1)(x^2-2)^2\left[3x(x^4-x+1) + (x^2-2)(4x^3-1)\right]$$

$$= 2(x^4-x+1)(x^2-2)^2(7x^5 - 8x^3 - 4x^2 + 3x + 2)$$

⑧ $y - y_1 = m(x - x_1)$

Point: $(2, \frac{1}{2})$ (x_1, y_1)

(i) $y = \frac{1}{\sqrt{20-x^4}} = \frac{1}{(20-x^4)^{1/2}} = (20-x^4)^{-1/2}$

$m = \text{Slope} = y'(2)$

$$y' = -\frac{1}{2}(20-x^4)^{-3/2}(-4x^3)$$

$$y' = \frac{2x^3}{(20-x^4)^{3/2}}$$

$$y' = \frac{2x^3}{\sqrt{(20-x^4)^3}}$$

(ii) $y'(2) = \frac{2(2)^3}{\sqrt{(20-(2)^4)^3}}$

$$= \frac{16}{\sqrt{64}}$$

$$\boxed{m = 2}$$

(iii) $y - y_1 = m(x - x_1)$

$y - \frac{1}{2} = 2(x - 2)$

$y - \frac{1}{2} = 2x - 4 \quad \rightarrow \quad 2x - y - \frac{7}{2} = 0$

$2y - 1 = 4x - 8$

$\boxed{0 = 4x - 2y - 7}$

⑨ $g(2) = 4$

$g'(2) = 3$

$f'(4) = 5$

$F'(2) = ?$

$F(x) = f(g(x))$

$F'(x) = f'(g(x))g'(x)$

$F'(2) = f'(g(2))g'(2)$

$= [f'(4)][g'(2)]$

$= (5)(3)$

$= 15$

Correct Homework Sheet

② b) $$y = \sqrt[3]{\frac{1-x^6}{2+(5x-1)^4}} = \left[\frac{1-x^6}{2+(5x-1)^4}\right]^{1/3}$$

$$y' = \frac{1}{3}\left[\frac{1-x^6}{2+(5x-1)^4}\right]^{-2/3}\left[\frac{(2+(5x-1)^4)(-6x^5)-(1-x^6)(4)(5x-1)^3(5)}{[2+(5x-1)^4]^2}\right]$$

Correct Homework Sheet

③ b) $$f(x) = \frac{8x^3(12x^2-5x)^8}{2-3(1-32x^{10})^{1/5}}$$

$$\frac{\left[2-3(1-32x^{10})^{1/5}\right]\left[(8x^3)(8)(12x^2-5x)^7(24x-5)+(24x^2)(12x^2-5x)^8\right]-\left[8x^3(12x^2-5x)^8\right]\left[\left(-\frac{3}{5}\right)(1-32x^{10})^{-4/5}(-320x^9)\right]}{\left[2-3(1-32x^{10})^{1/5}\right]^2}$$

③ c) $$f(x) = \frac{\left[x^5 - x(4-x^2)^{1/2}\right]^6}{12x^{1/2}(5x^3-8)^7}$$

$$f'(x) = \frac{\left[12x^{1/2}(5x^3-8)^7\right]\left[6\left[x^5-x(4-x^2)^{1/2}\right]^5\left[5x^4-\left[x\left(\frac{1}{2}\right)(4-x^2)^{-1/2}(-2x)+(4-x^2)^{1/2}\right]\right]\right] - \left[x^5-x(4-x^2)^{1/2}\right]^6\left[(12x^{1/2})(7)(5x^3-8)^6(15x^2)+(6x^{-1/2})(5x^3-8)^7\right]}{\left[12x^{1/2}(5x^3-8)^7\right]^2}$$

To be handed in today
Differentiate the following (do not simplify)

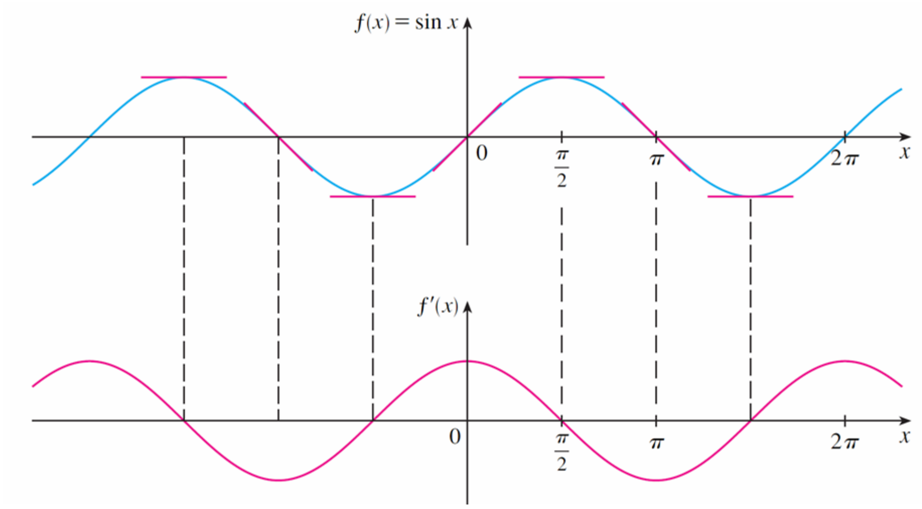
$$f(x)=\sqrt[7]{\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}}$$

$$=\left[\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}\right]^{1/7}$$

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Let's Practice...

Differentiate the following:

$y = \sin 3x$ $y = \sin(x + 2)$ $y = \sin(kx + d)$

Ex #2.

Differentiate:

a) $y = \sin(x^3)$

b) $y = \sin^3 x$

c) $y = \sin^3(x^2 - 1)$

Ex #3.

Differentiate:

$$y = x^2 \cos x$$

Homework

Worksheet on derivatives of trigonometric functions

Attachments

Derivatives Worksheet.doc