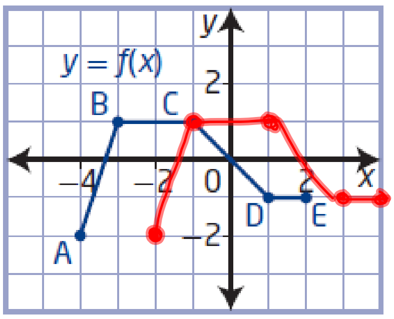
Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + 5$	$(x, y) \to (x, y + 5)$	
H	$y = f(x + 7)$	$(x, y) \to (x - 7, y)$	$h = -7$
H	$y = f(x - 3)$	$(x, y) \to (x + 3, y)$	$h = 3$
V	$y = f(x) - 6$	$(x, y) \to (x, y - 6)$	$k = -6$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x, y) \to (x - 4, y - 9)$	$h = -4$ $k = -9$
horizontal and vertical	$y = f(x - 4) - 6$	$(x, y) \to (x + 4, y - 6)$	$h = 4$ $k = -6$
H+V	$y = f(x + 2) + 3$	$(x, y) \to (x - 2, y + 3)$	$h = -2$ $k = 3$
horizontal and vertical	$y = f(x - h) + k$	$(x, y) \to (x + h, y + k)$	

Questions from Homework

2

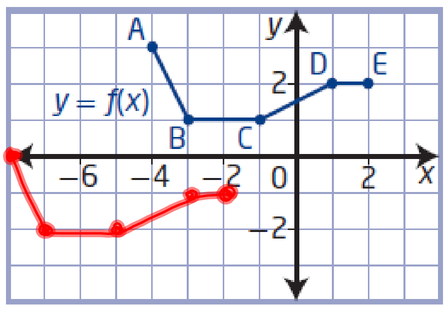


2b) $h(x) = f(x-2)$ $h=2$

$(x,y) \longrightarrow (x+2, y)$

A (-4,-2)	A' (-2,-2)
B (-3,1)	B' (-1,1)
C (-1,1)	C' (1,1)
D (1,-1)	D' (3,-1)
E (2,-1)	E' (4,-1)

4



$h=-4$ $k=-3$

a) $g(x) = f(x+4) - 3$

$(x,y) \longrightarrow (x-4, y-3)$

A(-4,3)	A' (-8,0)
B(-3,1)	B' (-7,-2)
C(-1,1)	C' (-5,-2)
D(1,2)	D' (-3,-1)
E(2,2)	E' (-2,-1)

Transformations:

New Functions From Old Functions

Translations

Stretches

Reflections

Reflections and Stretches

Focus on…

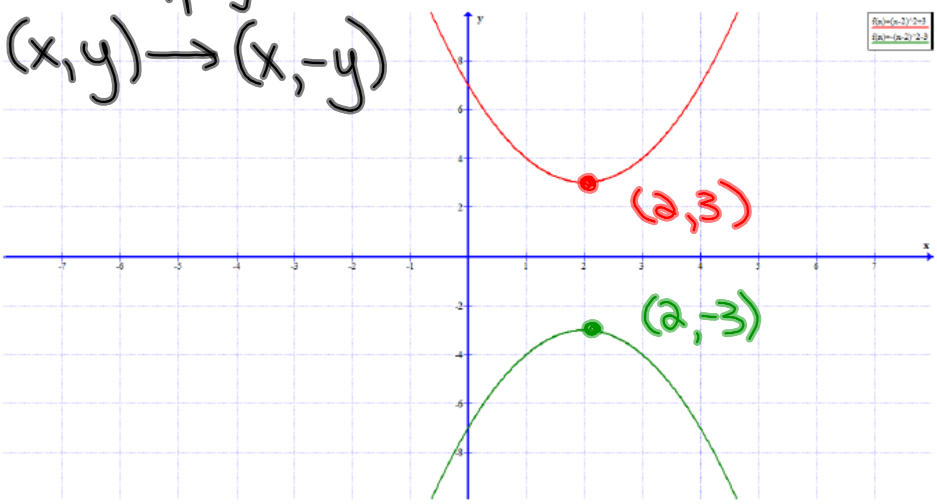
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis. (vertical reflection)

Mapping

$(x, y) \rightarrow (x, -y)$

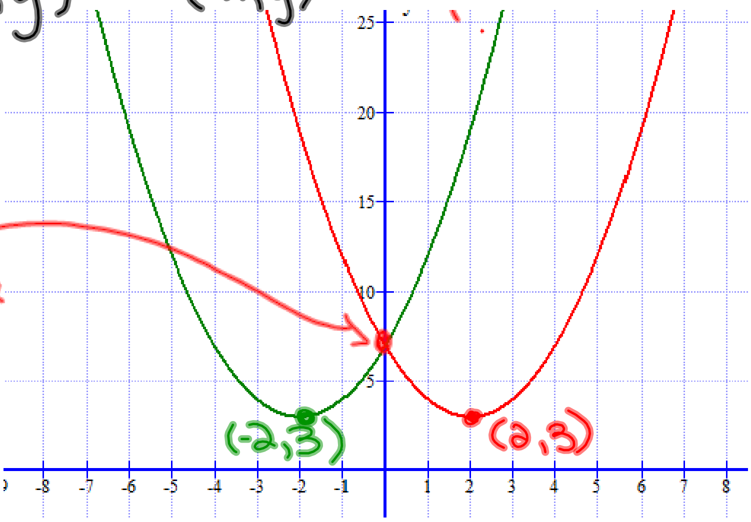


- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis. (horizontal reflection)

Mapping:

$(x, y) \rightarrow (-x, y)$

Invariant Point



invariant point

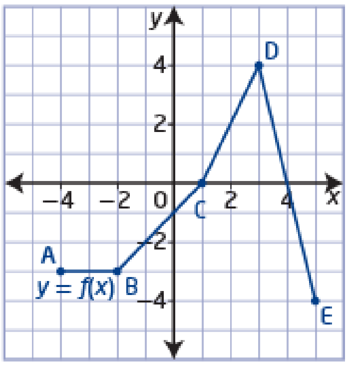
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



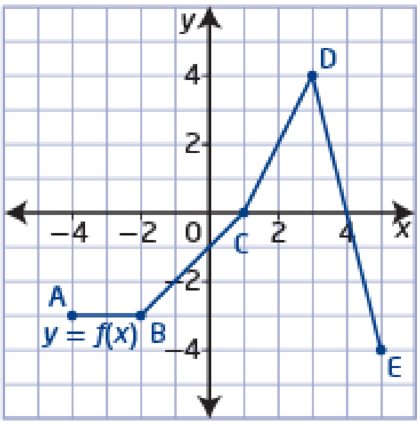
Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below

- Invariant Points

$(x,y) \longrightarrow (x,-y)$



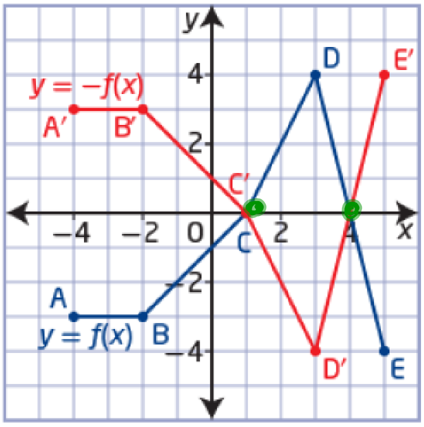
A(-4,-3) A'(-4,3)

B(-2,-3) B'(-2,3)

C(1,0) C'(1,0)

D(3,4) D'(3,-4)

E(5,-4) E'(5,4)



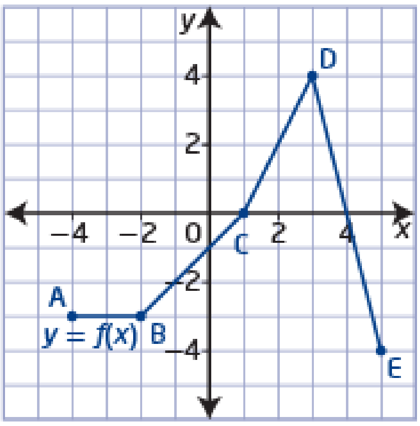
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

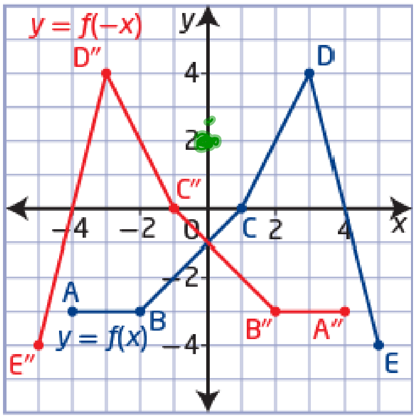
- Sketch $y = f(-x)$ on the axis below

• Invariant Point

$(x,y) \rightarrow (-x,y)$



A(-4,-3) A'(4,-3)
B(-2,-3) B'(2,-3)
C(1,0) C'(-1,0)
D(3,4) D'(-3,4)
E(5,-4) E'(-5,-4)



Homework

Page 28 #1, 3

stretch & compression

- a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches (Page 20)

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Ex: ① $y = 3f(x)$ $\quad (x,y) \rightarrow (x, 3y)$

$a = 3$

② $y = -2f(x)$ $\quad (x,y) \rightarrow (x, -2y)$

$a = -2$

③ $y = |4x|$ $\quad (x,y) \rightarrow (\frac{1}{4}x, y)$

$b = 4$

④ $y = f(\frac{2}{5}x)$ $\quad (x,y) \rightarrow (\frac{5}{2}x, y)$

$b = \frac{2}{5}$

⑤ $y + 3 = -2f(4x)$ $\quad (x,y) \rightarrow (\frac{1}{4}x, -2y - 3)$

$a = -2 \quad b = 4 \quad k = -3$

Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

Example 2

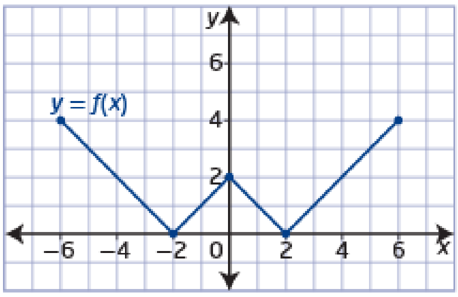
Graph $y = af(x)$

Given the graph of $y = f(x)$,

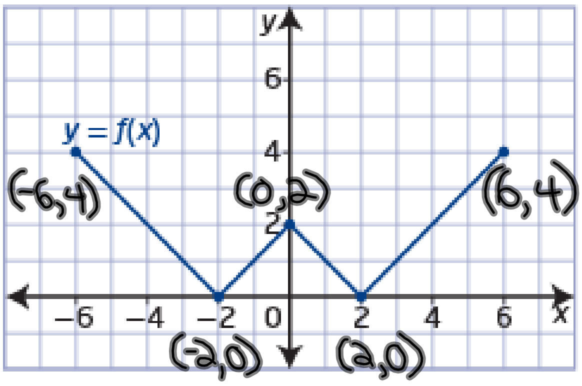
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

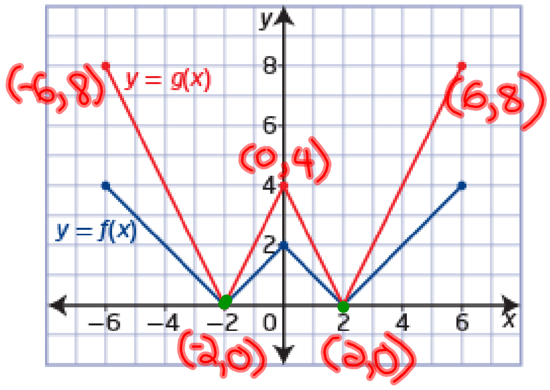
a) $g(x) = 2f(x)$

b) $g(x) = \frac{1}{2}f(x)$



a) $g(x) = 2f(x)$ $a=2$ $(x,y) \longrightarrow (x, 2y)$



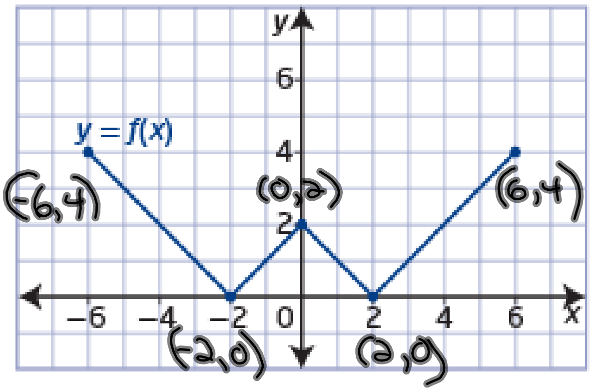


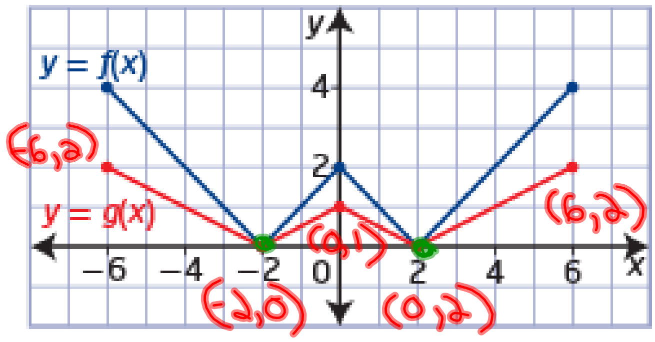
The invariant points are $(-2, 0)$ and $(2, 0)$.

For $f(x)$, the domain is
$\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$, Interval Notation
and the range is
$\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \leq y \leq 8, y \in \mathrm{R}\}$, or $[0, 8]$.

b) $g(x) = \frac{1}{2}f(x)$ $a = \frac{1}{2}$ $(x, y) \rightarrow (x, \frac{1}{2}y)$





The invariant points are (−2, 0) and (2, 0).

For $f(x)$, the domain is
$\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$,
and the range is
$\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6,$
and the range is $\{y \mid 0 \leq y \leq 2, y \in \mathbb{R}\}$, or $[0, 2]$.

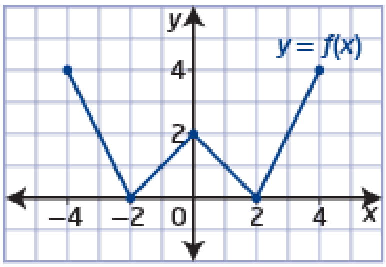
Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Example 3

Graph $y = f(bx)$

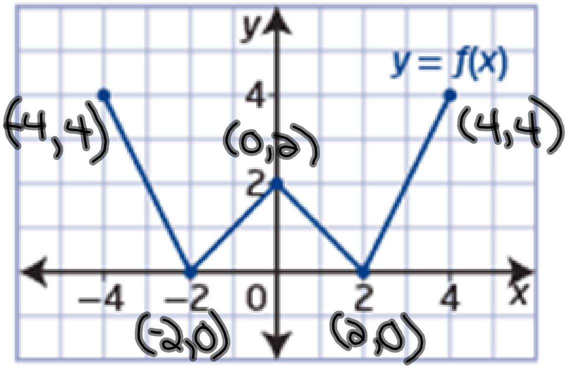
Given the graph of $y = f(x)$,
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

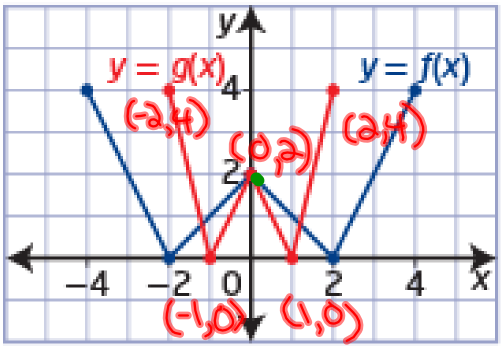


a) $g(x) = f(2x)$

b) $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$ $b=2$ $(x,y) \rightarrow (\frac{1}{2}x, y)$



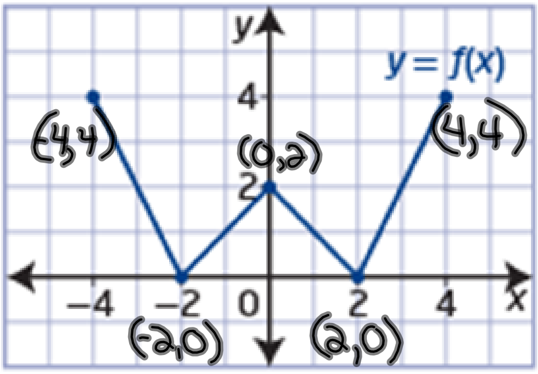


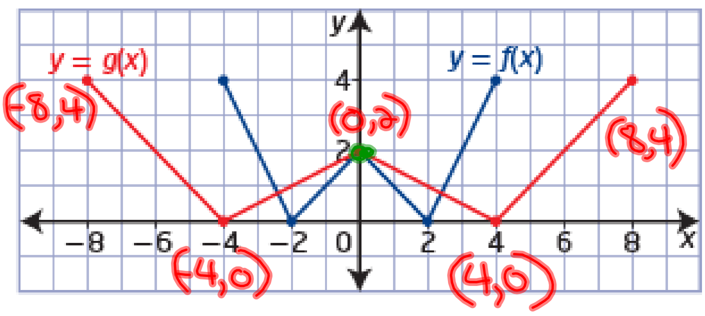
The invariant point is (0, 2).

For $f(x)$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathrm{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -2 \leq x \leq 2, x \in \mathrm{R}\}$, or $[-2, 2]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

b) $g(x) = f\left(\frac{1}{2}x\right)$ $b = \frac{1}{2}$ $(x, y) \rightarrow (2x, y)$



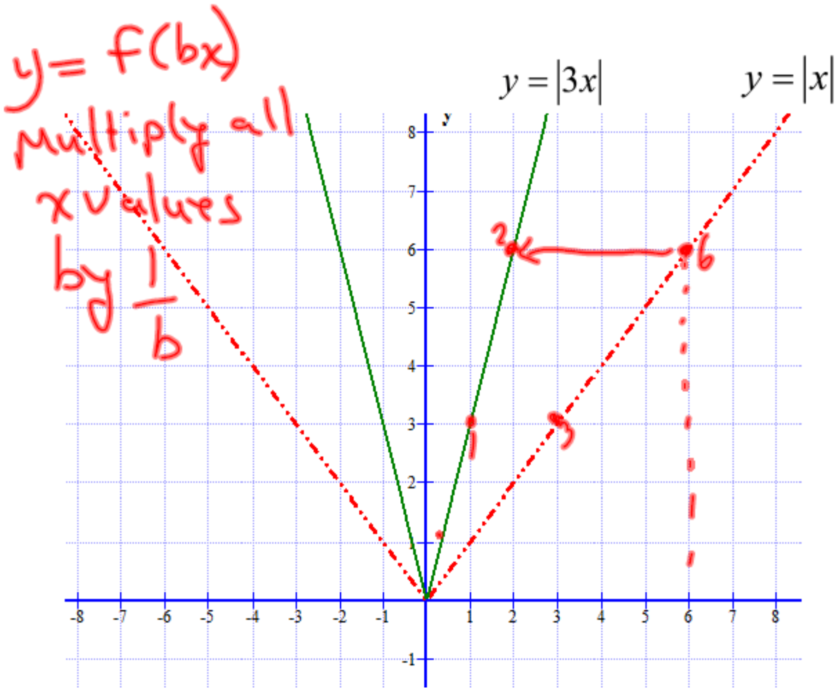


The invariant point is (0, 2).

For $f(x)$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathrm{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -8 \leq x \leq 8, x \in \mathrm{R}\}$, or $[-8, 8]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

Horizontal Stretch or Compression...



Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

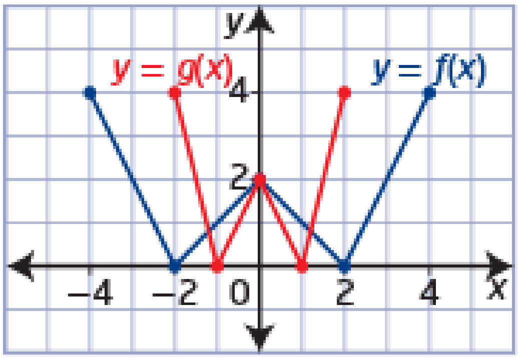
$y = -3f(-2x) + 7$ $(x,y) \rightarrow \left(-\frac{1}{2}x, -3y+7\right)$

$a=-3$ $b=-2$ $k=7$

Homework

Page 28 # 2, 5, 6, 7

Determine the Equation of a Translated Function:


y
y = g(x)
y = f(x)
4
2
−4
−2
0
2
4
x