Correct Homework Sheet

② c) $g(x) = (x-5)^3(7x^5+2x)^9(4-2x^3)^5$

$$g'(x) = 3(x-5)^2(1)(7x^5+2x)^9(4-2x^3)^5 + 9(7x^5+2x)^8(35x^4+2)(x-5)^3(4-2x^3)^5 + 5(4-2x^3)^4(-6x^2)(x-5)^3(7x^5+2x)^9$$

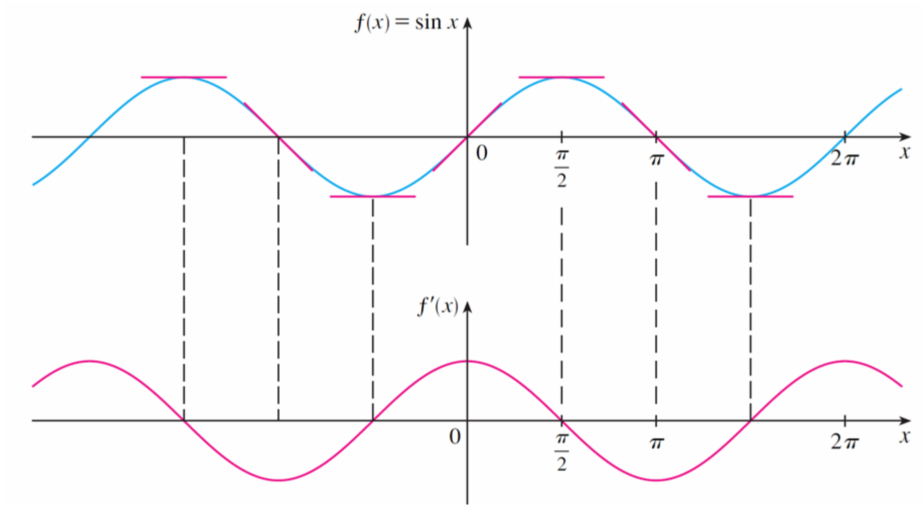
③ c) $\dfrac{\left[x^5 - x\sqrt{4-x^2}\right]^6}{12\sqrt{x}(5x^3-8)^7}$

$$\frac{\left[12\sqrt{x}(5x^3-8)^7\right]\left[6(x^5-x\sqrt{4-x^2})^5(5x^4 - x(\tfrac{1}{2})(4-x^2)^{-1/2}(-2x) - 1\sqrt{4-x^2})\right] - \left[x^5 - x\sqrt{4-x^2}\right]^6\left[12\sqrt{x}(7)(5x^3-8)^6(15x^2) + 6x^{-1/2}(5x^3-8)^7\right]}{\left[12\sqrt{x}(5x^3-8)^7\right]^2}$$

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x\cos h + \cos x\sin h - \sin x}{h}$$

$$= \lim_{h\to 0}\left[\frac{\sin x\cos h - \sin x}{h} + \frac{\cos x\sin h}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\sin x\cos h - \sin x}{h} + \frac{\cos x\sin h}{h}\right]$$

$$= \lim_{h\to 0}\left[\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)\right]$$

$$= \lim_{h\to 0}\sin x\cdot\lim_{h\to 0}\frac{\cos h - 1}{h} + \lim_{h\to 0}\cos x\cdot\lim_{h\to 0}\frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h\to 0$,

    $$\lim_{h\to 0}\sin x = \sin x \text{ and } \lim_{h\to 0}\cos x = \cos x$$

- With some work we can also show that

$$\lim_{h\to 0}\frac{\sin h}{h} = 1 \text{ and } \lim_{h\to 0}\frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0}\sin x\cdot\lim_{h\to 0}\frac{\cos h - 1}{h} + \lim_{h\to 0}\cos x\cdot\lim_{h\to 0}\frac{\sin h}{h}$$

$$= (\sin x)\cdot 0 + (\cos x)\cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Let's Practice...

Differentiate the following:

$y = \sin 3x$

$u = 3x$
$du = 3$

$y' = \cos 3x\,(3)$
$y' = 3\cos 3x$

$y = \sin(x+2)$

$u = x+2$
$du = 1$

$y' = \cos(x+2)\cdot 1$
$y' = \cos(x+2)$

$y = \sin(kx+d)$

$u = kx+d$
$du = k$

$y' = \cos(kx+d)\cdot k$
$y' = k\cos(kx+d$

Ex #2.

Differentiate:

a) $y = \sin(x^3)$

$y' = \cos x^3 \cdot 3x^2$

$y' = 3x^2 \cos x^3$

b) $y = \sin^3 x$

$y = [\sin x]^3$

$y' = 3(\sin x)^2 \cos x \cdot 1$

$y' = 3\sin^2 x \cos x$

c) $y = \sin^3(x^2 - 1)$

$y = [\sin(x^2 - 1)]^3$

$y' = 3[\sin(x^2-1)]^2 [2x \cos(x^2-1)]$

$y' = 6x \sin^2(x^2-1)\cos(x^2-1)$

Ex #3.

Differentiate: $fg' + f'g$

$y = (x^2)(\cos x)$

$y' = x^2(-\sin x \cdot 1) + 2x\cos x$

$y' = -x^2\sin x + 2x\cos x$

$y' = 2x\cos x - x^2\sin x$

$y' = x(2\cos x - x\sin x)$

Homework

Do Questions 1 and 3 from Exercise 7.2 Page 313

Worksheet on derivatives of trigonometric functions

Attachments

Derivatives Worksheet.doc