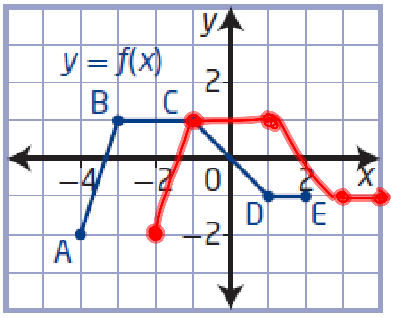
Warm-Up

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$	
H	$y = f(x + \underline{7})$	$(x, y) \rightarrow (x - 7, y)$	$h = -7$
H	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$	$h = 3$
V	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$	$k = -6$
horizontal and vertical	$y + 9 = f(x + 4)$	$(x,y) \rightarrow (x-4, y-9)$	$h = -4$ $k = -9$
horizontal and vertical	$y = f(x-4) - 6$	$(x, y) \rightarrow (x + 4, y - 6)$	$h = 4$ $k = -6$
H+V	$y = f(x+2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$	$h = -2$ $k = 3$
horizontal and vertical	$y = f(x - h) + k$	$(x,y) \rightarrow (x+h, y+k)$	

Questions from Homework

2



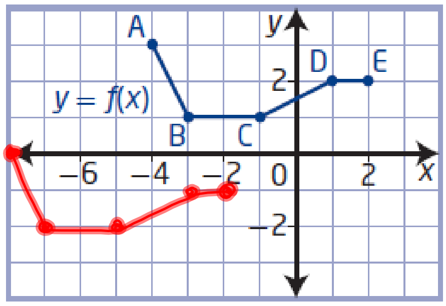
2b) $h(x) = f(x-2)$ $h=2$

$(x,y) \longrightarrow (x+2, y)$

A $(-4,-2)$	A' $(-2,-2)$
B $(-3,1)$	B' $(-1,1)$
C $(-1,1)$	C' $(1,1)$
D $(1,-1)$	D' $(3,-1)$
E $(2,-1)$	E' $(4,-1)$

$h=-4$ $k=-3$

4



a) $g(x) = f(x+4) - 3$

$(x,y) \longrightarrow (x-4, y-3)$

A$(-4,3)$	A' $(-8,0)$
B$(-3,1)$	B' $(-7,-2)$
C$(-1,1)$	C' $(-5,-2)$
D$(1,2)$	D' $(-3,-1)$
E$(2,2)$	E' $(-2,-1)$

Transformations:

New Functions From Old Functions

Translations

Stretches

Reflections

Reflections and Stretches

Focus on...

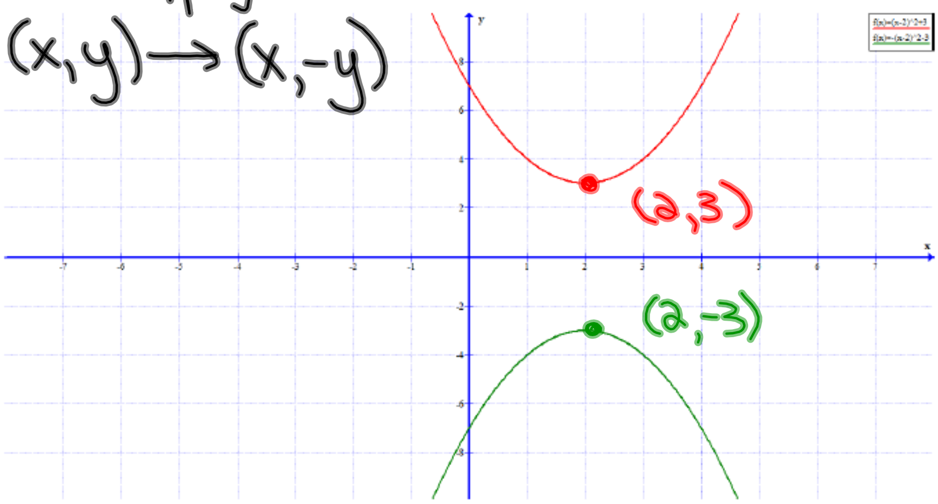
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis. (vertical reflection)

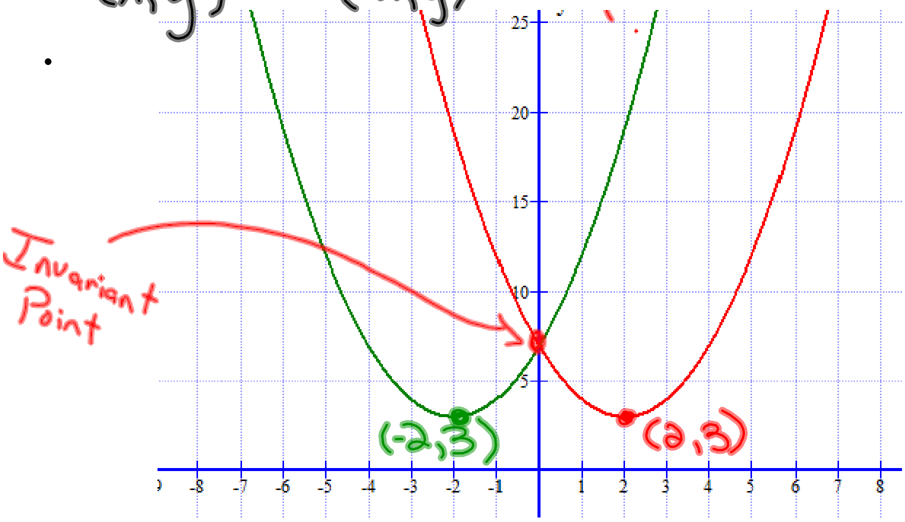
Mapping

$(x, y) \rightarrow (x, -y)$



- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis. (horizontal reflection)

Mapping:
$(x,y) \longrightarrow (-x,y)$



invariant point

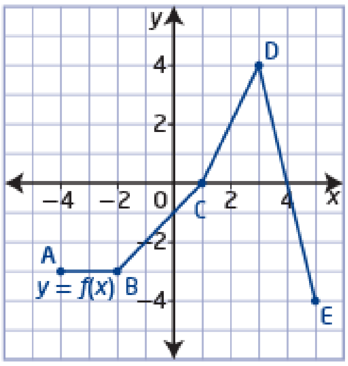
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1

Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$

a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.

b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



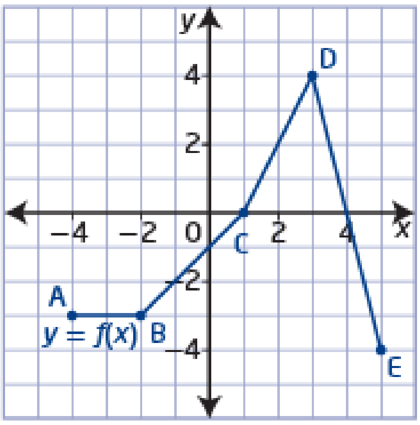
Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

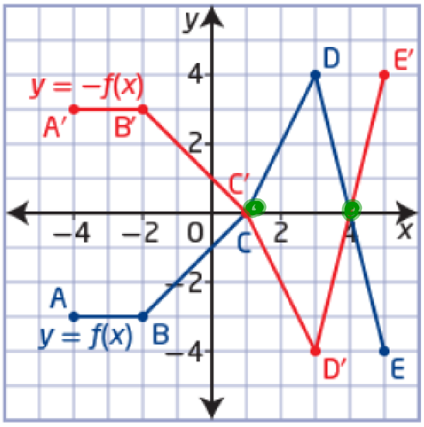
- Sketch $y = -f(x)$ on the axis below

• Invariant Points

$(x,y) \longrightarrow (x,-y)$



$A(-4,-3)$	$A'(-4,3)$
$B(-2,-3)$	$B'(-2,3)$
$C(1,0)$	$C'(1,0)$
$D(3,4)$	$D'(3,-4)$
$E(5,-4)$	$E'(5,4)$



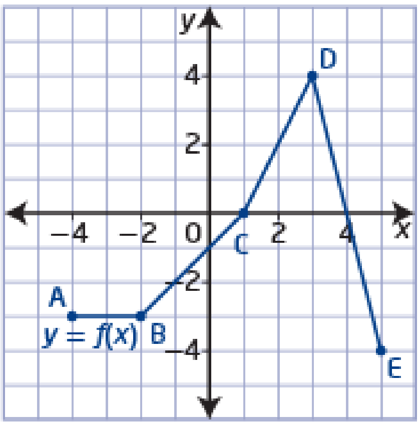
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

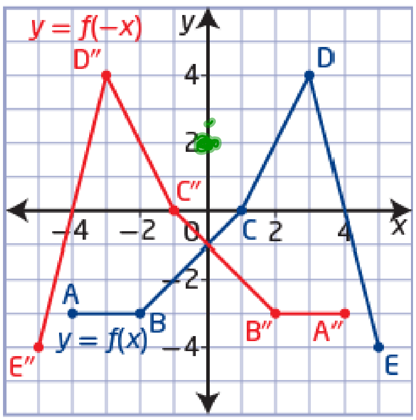
- Sketch $y = f(-x)$ on the axis below

- Invariant Point

$(x,y) \rightarrow (-x,y)$



$A(-4,-3)$	$A'(4,-3)$
$B(-2,-3)$	$B'(2,-3)$
$C(1,0)$	$C'(-1,0)$
$D(3,4)$	$D'(-3,4)$
$E(5,-4)$	$E'(-5,-4)$



Homework Page 28 #1, 3, 4

1. a) Copy and complete the table of values for the given functions.

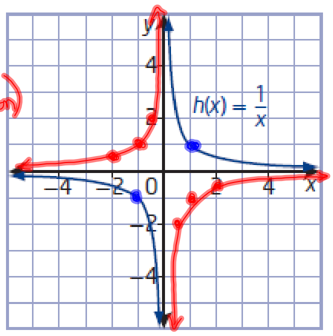
x	$f(x) = 2x + 1$	$g(x) = -f(x)$	$h(x) = f(-x)$
−4	-7	7	9
−2	-3	3	5
0	1	-1	1
2	5	-5	-3
4	9	-9	-7

$(x,y) \longrightarrow (x,-y)$

3. Consider each graph of a function.

- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes. (vertical reflection)
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.

c)



$k(x) = -h(x)$

$k(x) = -\left(\frac{1}{x}\right) = -\frac{1}{x}$

$h(x) = \frac{1}{x}$

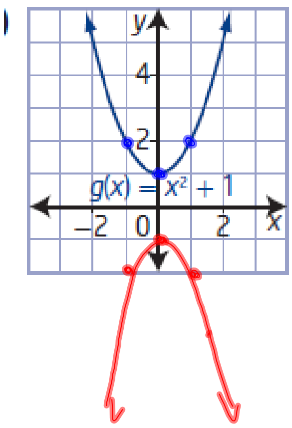
Domain: $\{x \mid x \neq 0, x \in R\}$

Range: $\{y \mid y \neq 0, y \in R\}$

$k(x) = -\frac{1}{x}$

Domain: $\{x \mid x \neq 0, x \in R\}$

Range: $\{y \mid y \neq 0, y \in R\}$



$g(x) = x^2 + 1$

x	y
-1	2
0	1
1	2

$h(x) = -g(x)$

$h(x) = -(x^2 + 1)$

$h(x) = -x^2 - 1$

x	y
-1	-2
0	-1
1	-2

$g(x) = x^2 + 1$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \geq 1, y \in R\}$

$h(x) = -x^2 - 1$

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \leq -1, y \in R\}$

stretch + compression

- a transformation in which the distance of each *x*-coordinate or *y*-coordinate from the line of reflection is multiplied by some scale factor

- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

ex. $\frac{1}{2}, \frac{2}{5}, 0.\overline{3}$

Vertical and Horizontal Stretches (see page 20)

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis. $(x, y) \rightarrow (x, ay)$
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$$(x, y) \rightarrow \left(\frac{1}{b}x, y\right)$$

Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

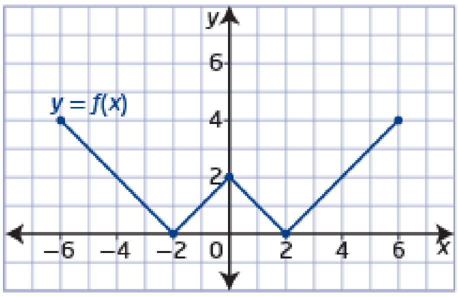
Example 2

Graph $y = af(x)$

Given the graph of $y = f(x)$,
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

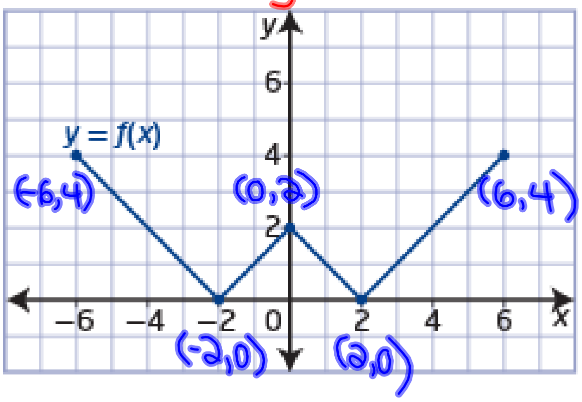
a) $g(x) = 2f(x)$

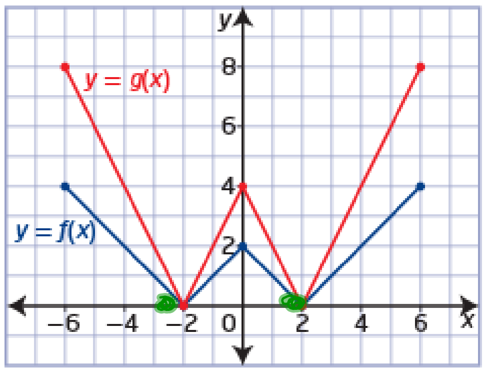
b) $g(x) = \frac{1}{2}f(x)$



a) $g(x) = 2f(x)$ $a = 2$

• vertically stretched by a factor of 2





The invariant points are (−2, 0) and (2, 0).

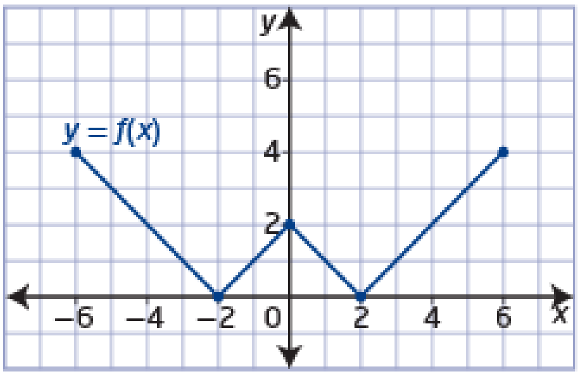
For $f(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$. ← Interval Notation

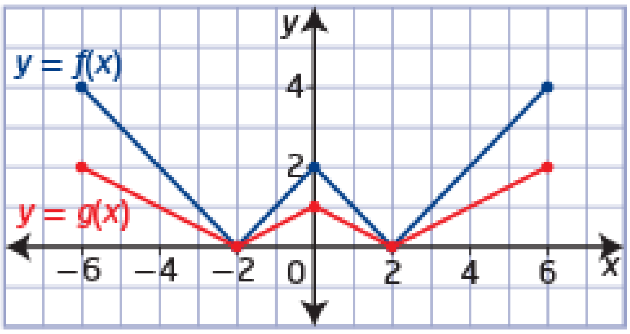
For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \leq y \leq 8, y \in \mathrm{R}\}$, or $[0, 8]$.

$(x, y) \rightarrow (x, 2y)$

-6,4	-6,8
-2,0	-2,0
0,2	0,4
2,0	2,0
6,4	6,8

b) $g(x) = \frac{1}{2}f(x)$





The invariant points are $(-2, 0)$ and $(2, 0)$.

For $f(x)$, the domain is
$\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$,
and the range is
$\{y \mid 0 \leq y \leq 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathrm{R}\}$, or $[-6, 6]$,
and the range is $\{y \mid 0 \leq y \leq 2, y \in \mathrm{R}\}$, or $[0, 2]$.

Horizontal Stretch or Compression... (reciprocals)

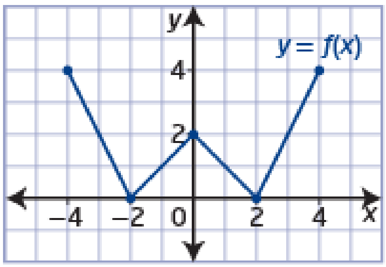
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Example 3

Graph $y = f(bx)$

Given the graph of $y = f(x)$,

- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

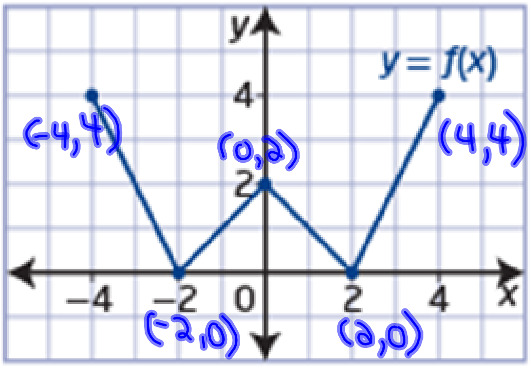


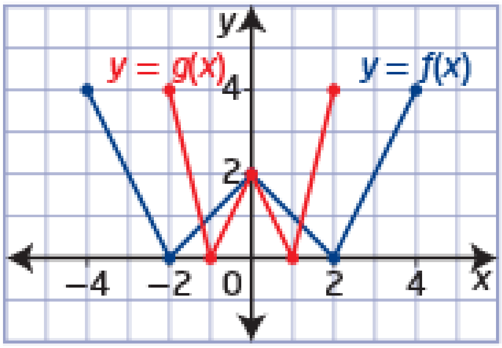
a) $g(x) = f(2x)$

b) $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$ $b=2$

- horizontally compressed by a factor of $\frac{1}{2}$





$(x, y) \longrightarrow \left(\frac{1}{2}x, y\right)$

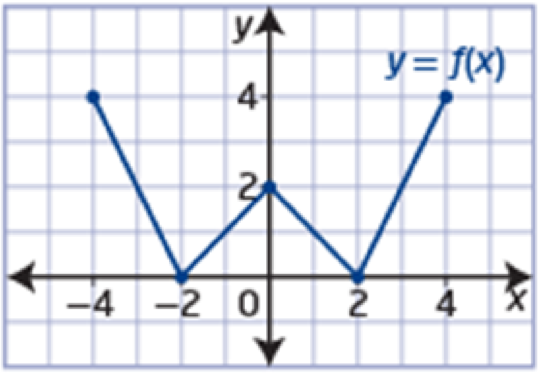
$(-4, 4)$	$(-2, 4)$
$(-2, 0)$	$(-1, 0)$
$(0, 2)$	$(0, 2)$
$(2, 0)$	$(1, 0)$
$(4, 4)$	$(2, 4)$

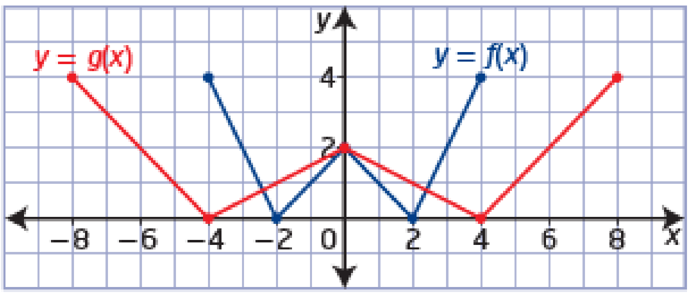
The invariant point is

For $f(x)$, the domain is $\{x \mid -4 \le x \le 4, x \in \mathbb{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -2 \le x \le 2, x \in \mathbb{R}\}$, or $[-2, 2]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathbb{R}\}$, or $[0, 4]$.

b) $g(x) = f\left(\frac{1}{2}x\right)$



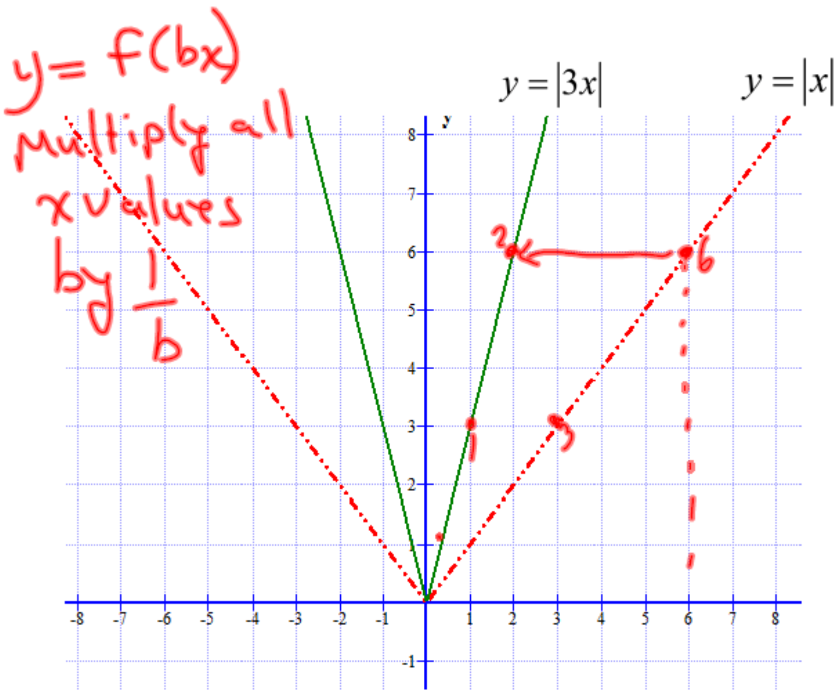


The invariant point is (0, 2).

For $f(x)$, the domain is $\{x \mid -4 \le x \le 4, x \in \mathrm{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathrm{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -8 \le x \le 8, x \in \mathrm{R}\}$, or $[-8, 8]$, and the range is $\{y \mid 0 \le y \le 4, y \in \mathrm{R}\}$, or $[0, 4]$.

Horizontal Stretch or Compression...



Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$$y = -3f(-2x) + 7$$

Homework

Page 28 # 2, 5-10, 14

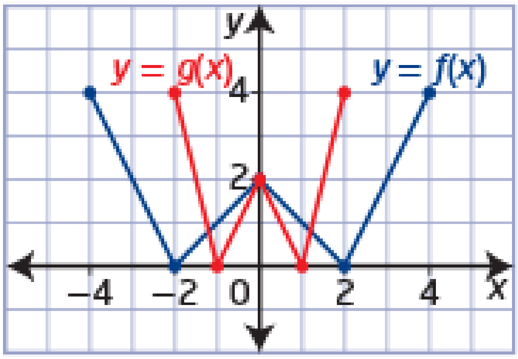
$$y = \underline{-3}f(\underline{5}(x-\underline{2})) + \underline{1}$$

$a = -3$ $b = 5$ $h = 2$ $k = 1$

- vertical reflection in x-axis
- vertical stretch by a factor of 3
- horizontally compressed by a factor of $\frac{1}{5}$
- translated 2 units right
- " 1 unit up.

$$(x, y) \longrightarrow \left(\frac{1}{5}x + 2, -3y + 1\right)$$

Determine the Equation of a Translated Function:


y
y = g(x)
y = f(x)
4
2
−4
−2
0
2
4
x