<span style="color:red">Questions from Homework</span>

m) $y = (1 + \cos^2 x)^6 = (1 + (\cos x)^2)^6$

$y' = 6(1 + (\cos x)^2)^5 \cdot 2(\cos x)(-\sin x)$

$y' = 6(1 + \cos^2 x)^5 (-2 \sin x \cos x)$

$y' = -6(1 + \cos^2 x)^5 (2 \sin x \cos x)$  <span style="color:blue">Double Angle Identity</span>

$y' = -6(1 + \cos^2 x)^5 (\sin 2x)$

$\boxed{y' = -6 \sin 2x (1 + \cos^2 x)^5}$

b  $y = \dfrac{\sin x}{1 + \cos x}$

$y' = \dfrac{(1 + \cos x)(\cos x)(1) - \sin x (-\sin x)(1)}{(1 + \cos x)^2}$

$y' = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$  <span style="color:blue">Pythagorean Identity</span>

$y' = \dfrac{1 + \cos x}{(1 + \cos x)^2} = \boxed{\dfrac{1}{1 + \cos x}}$

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<span style="color:red">Questions from Homework</span>

g) $y = \sin^{-2}(x^3) = \left(\sin(x^3)\right)^{-2}$

$y' = -2\left(\sin(x^3)\right)^{-3} \cdot \cos(x^3) \cdot 3x^2$

$y' = \dfrac{-6x^2 \cos(x^3)}{\left(\sin(x^3)\right)^3} = \boxed{\dfrac{-6x^2 \cos(x^3)}{\sin^3(x^3)}}$

p) $y = \cos^3(\sin x) = \left[\cos(\sin x)\right]^3$

$y' = 3\left[\cos(\sin x)\right]^2 \cdot \left(-\sin(\sin x)\right) \cdot (\cos x)$

$y' = -3\left[\cos(\sin x)\right]^2 \sin(\sin x) \cos x$

$\boxed{y' = -3\cos^2(\sin x) \sin(\sin x) \cos x}$

u) $y = \cos^2\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right) = \left[\cos\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)\right]^2$

$y' = 2\cos\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)\left(-\sin\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)\left(\dfrac{-1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\right)\right)$

$y' = \boxed{2\cos\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)\sin\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)}\left(\dfrac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\right)$

<span style="color:blue">Double Angle Identity</span>

$y' = \sin 2\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)\left(\dfrac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\right)$

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<span style="color:red">## Questions from Homework</span>

③a) $y = 2\sin x$  @ $\left(\frac{\pi}{6}, 1\right)$

① Differentiate:       ② Find $y'\left(\frac{\pi}{6}\right)$

$y' = (0)\sin x + 2(\cos x \cdot 1)$       $y'\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right)$

$y' = 2\cos x$       $y'\left(\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2}\right)$

③ Find equation using $y - y_1 = m(x - x_1)$

$y - 1 = \sqrt{3}\left(x - \frac{\pi}{6}\right)$
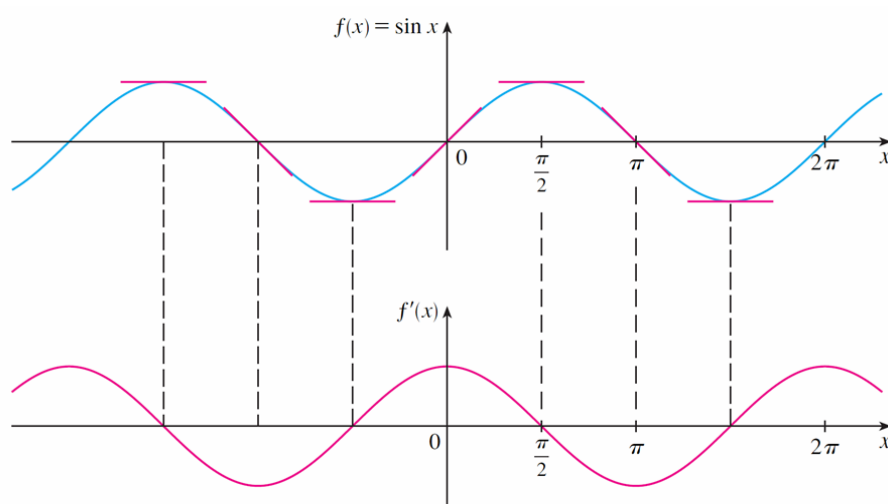
$y - 1 = x\sqrt{3} - \frac{\pi\sqrt{3}}{6}$

$6y - 6 = 6x\sqrt{3} - \pi\sqrt{3}$

$0 = 6x\sqrt{3} - 6y - \pi\sqrt{3} + 6$

**Derivatives of Trigonometric Functions**

# The Sine Function

■ We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.

■ On the next slide we graph $f(x)$ = sin $x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.

    ■ Note that $x$ is measured in <u>radians</u>.

■ The derivative graph resembles the graph of the cosine!

### Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h \to 0} \left[ \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[ \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h \to 0} \left[ \sin x \left( \frac{\cos h - 1}{h} \right) + \cos x \left( \frac{\sin h}{h} \right) \right]$$

$$= \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since $x$ is constant while $h \to 0$,

$$\lim_{h \to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h \to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h \to 0} \sin x \cdot \lim_{h \to 0} \frac{\cos h - 1}{h} + \lim_{h \to 0} \cos x \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

# Rules to differentiate trigonometric functions:

**Given that "u" represents some differentiable function...**

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

## Let's Practice...

Differentiate the following:

$$f(x) = \frac{1}{1+\tan x}$$

$$f'(x) = \frac{(1+\tan x)(0) - 1(\sec^2 x)(1)}{(1+\tan x)^2}$$

$$= \frac{-\sec^2 x}{(1+\tan x)^2}$$

$$f(x) = \frac{1}{1+\tan x} = (1+\tan x)^{-1}$$

$$f'(x) = -(1+\tan x)^{-2} \cdot (\sec^2 x)(1)$$

$$= \frac{-\sec^2 x}{(1+\tan x)^2}$$

## Ex #2.

Differentiate:

$$f(x) = 2\csc^3(3x^2) \; = 2\left(\csc(3x^2)\right)^3$$

$$f'(x) = 6\left(\csc(3x^2)\right)^2 \cdot -\csc(3x^2)\cot(3x^2) \cdot 6x$$

$$= -36x\csc^2(3x^2)\csc(3x^2)\cot(3x^2)$$

$$\boxed{= -36x\csc^3(3x^2)\cot(3x^2)}$$

# Homework

Worksheet on derivatives of trigonometric functions

# Attachments

Derivatives Worksheet.doc