

Warm-Up...

Given that $(-2, 5)$ is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

$$(1) y = 3f(x)$$

$$\begin{aligned} a &= 3 & (x, y) &\rightarrow (x, 3y) \\ b &= 1 \\ h &= 0 & (-2, 5) &\rightarrow (-2, 15) \\ k &= 0 \end{aligned}$$

$$(2) y = f\left(-\frac{1}{3}x\right)$$

$$\begin{aligned} a &= 1 & (x, y) &\rightarrow (-3x, y) \\ b &= \frac{1}{3} \\ h &= 0 & (-2, 5) &\rightarrow (6, 5) \\ k &= 0 \end{aligned}$$

$$(3) y = 4f\left[\frac{1}{2}(x+5)\right] - 3$$

$$\begin{aligned} a &= 4 & (x, y) &\rightarrow (2x-5, 4y-3) \\ b &= \frac{1}{2} & (-2, 5) &\rightarrow (-9, 17) \\ h &= -5 \\ k &= -3 \end{aligned}$$

$$(4) y - 5 = -2f(-2x + 6)$$

Factor
↓

$$y = -2f(-2(x-3)) + 5$$

$$\begin{aligned} a &= -2 & (x, y) &\rightarrow \left(\frac{1}{2}x + 3, 2y + 5\right) \\ b &= -2 & (-2, 5) &\rightarrow (4, -5) \\ h &= 3 \\ k &= 5 \end{aligned}$$

Summary of Transformations...

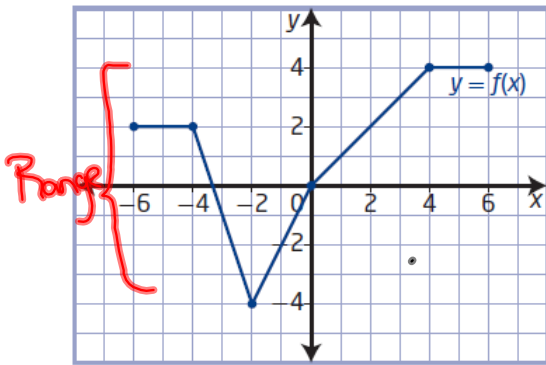
Transformations of the graphs of functions	
$f(x) + k$	shift $f(x)$ up c units
$f(x) - k$	shift $f(x)$ down c units
$f(x + h)$	shift $f(x)$ left c units
$f(x - h)$	shift $f(x)$ right c units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$a f(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$
	When $c > 1$ – vertical stretching of $f(x)$ Multiply the y values by c
$f(bx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$
	When $c > 1$ – horizontal shrinking of $f(x)$ Divide the x values by c

Handwritten notes:

- vertical translation (for $f(x) + k$ and $f(x) - k$)
- horizontal (for $f(x + h)$ and $f(x - h)$)
- horizontal reflection (for $f(-x)$)
- vertical (for $-f(x)$)
- vertical stretch (for $a f(x)$)
- horizontal stretch (for $f(bx)$)

Questions from Homework

6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a=2$



$$(x, y) \rightarrow (x, 2y)$$

$$D: \{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$$

$$\text{or } [-6, 6]$$

$$R: \{y \mid -8 \leq y \leq 8, y \in \mathbb{R}\}$$

$$\text{or } [-8, 8]$$

2. a) Copy and complete the table of values for the given functions.

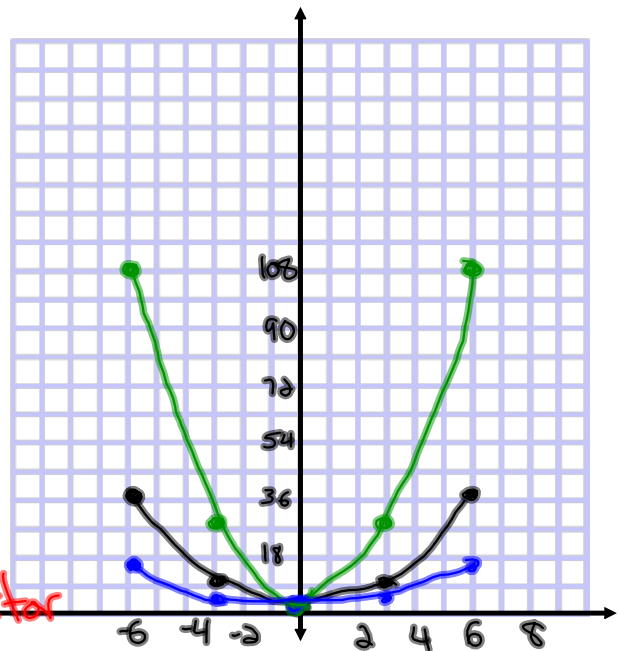
x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
-6	36	108	12
-3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

↑
stretched vertically by a factor of 3

$$(x, y) \rightarrow (x, 3y)$$

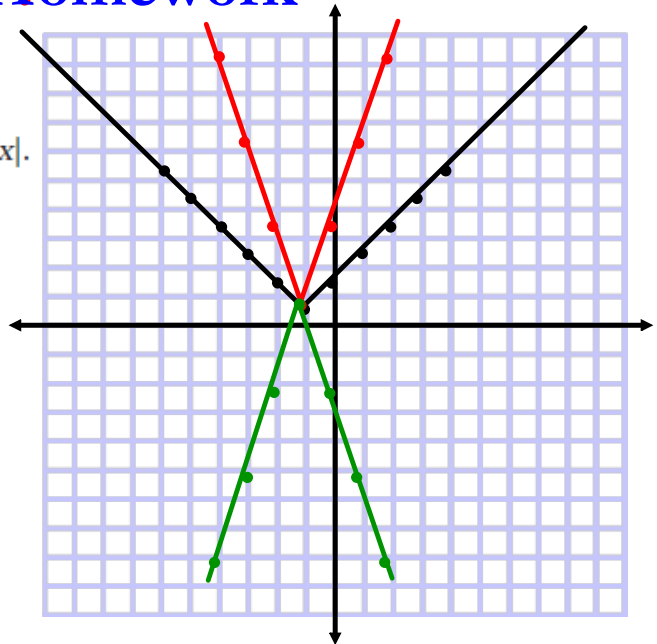
↑
compressed vertically by a factor of $\frac{1}{3}$

$$(x, y) \rightarrow (x, \frac{1}{3}y)$$



Questions from Homework

10. Thomas and Sharyn discuss the order of the transformations of the graph of $y = -3|x|$ compared to the graph of $y = |x|$. Thomas states that the reflection must be applied first. Sharyn claims that the vertical stretch should be applied first.



- a) Sketch the graph of $y = -3|x|$ by applying the reflection first.
- b) Sketch the graph of $y = -3|x|$ by applying the stretch first.
- c) Explain your conclusions. Who is correct?

Neither
 It does not matter what order the reflections and stretches are done in

Extend

14. Consider the function $f(x) = (x + 4)(x - 3)$. Without graphing, determine the zeros of the function after each transformation.

- a) $y = 4f(x)$ $a = 4$ $(x, y) \rightarrow (x, 4y)$
- b) $y = f(-x)$ horizontal reflection $(x, y) \rightarrow (-x, y)$
- c) $y = f\left(\frac{1}{2}x\right)$ $b = \frac{1}{2}$ $(x, y) \rightarrow (2x, y)$
- d) $y = f(2x)$ $b = 2$ $(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$

← quadratic
 $f(x) = x^2 + x - 12$

x intercepts

$$\begin{array}{l|l} x+4=0 & x-3=0 \\ \hline x=-4 & x=3 \\ (-4,0) & (3,0) \end{array}$$

a) $(-4, 0) + (3, 0)$

b) $(4, 0) + (-3, 0)$

c) $(-8, 0) + (6, 0)$

d) $(-2, 0) + \left(\frac{3}{2}, 0\right)$

Transformations:

$$g(x) = \underline{-3}f(\underline{4}(x-\underline{4}))-\underline{10}$$

$$a = -3$$

$$b = 4$$

$$h = 4$$

$$k = -10$$

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

- a) y axis
- b) $\frac{1}{4}$
- c) x axis
- d) 3
- e) x axis
- f) 4
- g) 10

Transformations:

$$y = f(x) \longrightarrow y = af(b(x - h)) + k$$

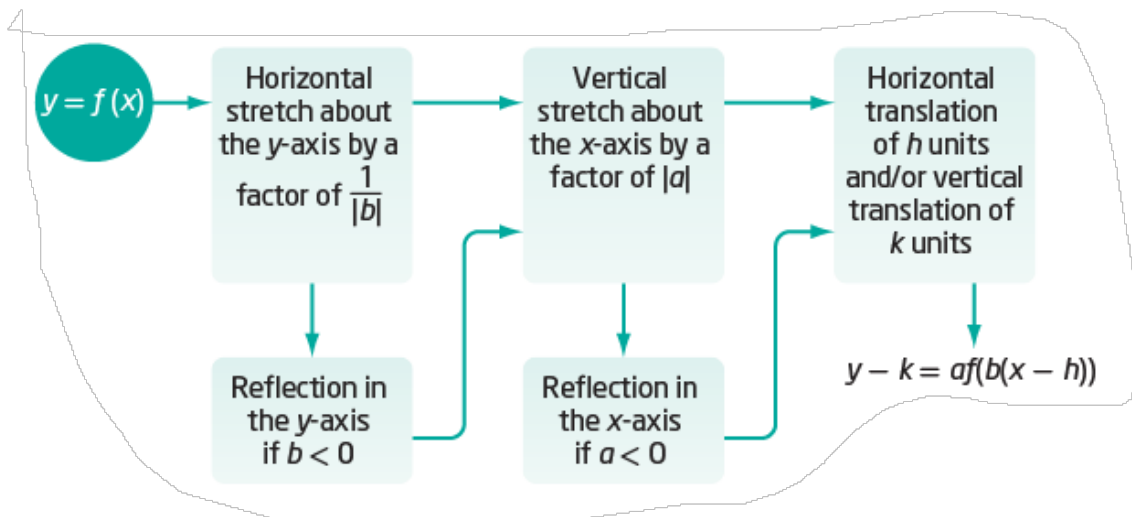
Mapping Rule: $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember...RST



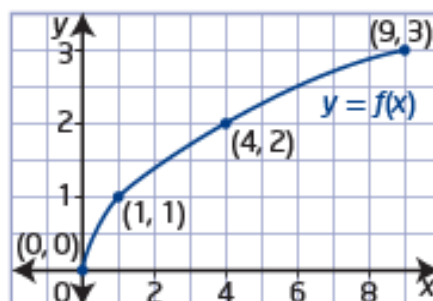
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

b) $y = f(3x + 6)$



a) $y = 3f(2x)$ $a=3$ $b=2$ $h=0$ $k=0$

The graph of $y = f(x)$ is horizontally stretched about the y -axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x -axis by a factor of 3 .

Base: $y = \sqrt{x}$

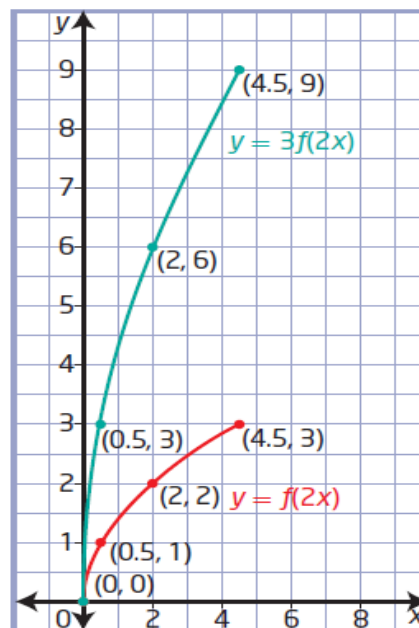
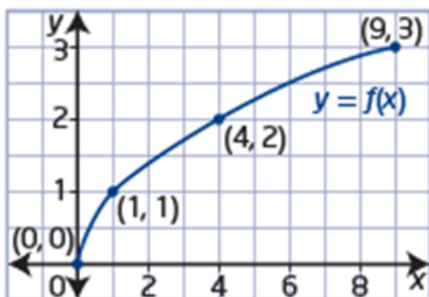
$(x, y) \rightarrow (\frac{1}{2}x, 3y)$

$(0, 0) \rightarrow (0, 0)$

$(1, 1) \rightarrow (\frac{1}{2}, 3)$

$(4, 2) \rightarrow (2, 6)$

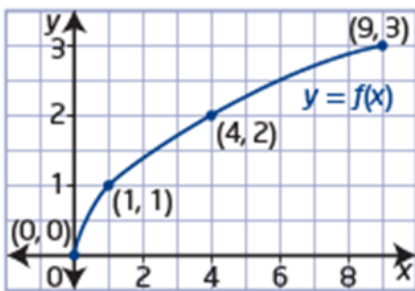
$(9, 3) \rightarrow (\frac{9}{2}, 9)$



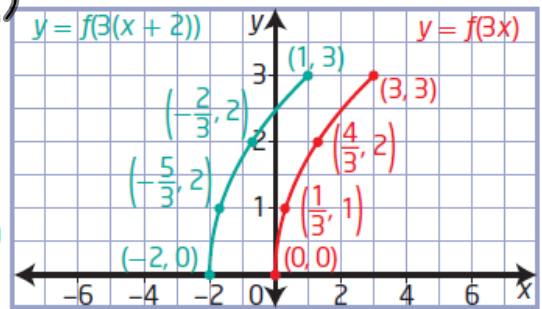
Factor

b) $y = f(3x + 6)$ $a=1$ $b=3$ $h=-2$ $k=0$
 $y = f(3(x+2))$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.



$(x,y) \rightarrow (\frac{1}{3}x-2, y)$
 $(0,0) \rightarrow (-2, 0)$
 $(1,1) \rightarrow (-\frac{5}{3}, 1)$
 $(4,2) \rightarrow (-\frac{2}{3}, 2)$
 $(9,3) \rightarrow (1, 3)$



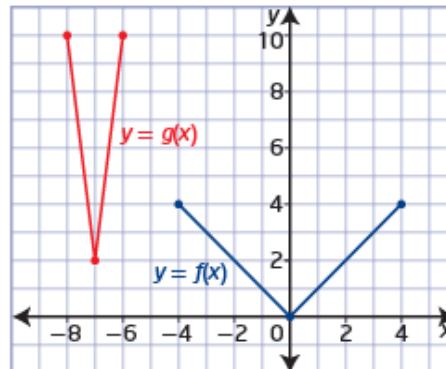
Homework

Page 38 # 3-6

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

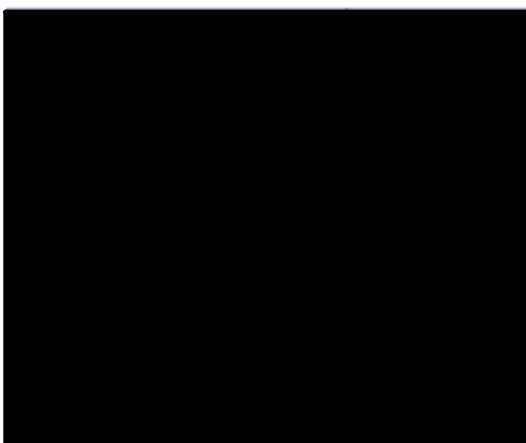
Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$$(-4, 4) \rightarrow (-8, 10)$$

$$(0, 0) \rightarrow (-7, 2)$$

$$(4, 4) \rightarrow (-6, 10)$$

The equation of the transformed function is XXXXXXXXXX



How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x -axis by a factor of 2, stretched horizontally about the y -axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x -axis by a factor of 2, stretched horizontally about the y -axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.