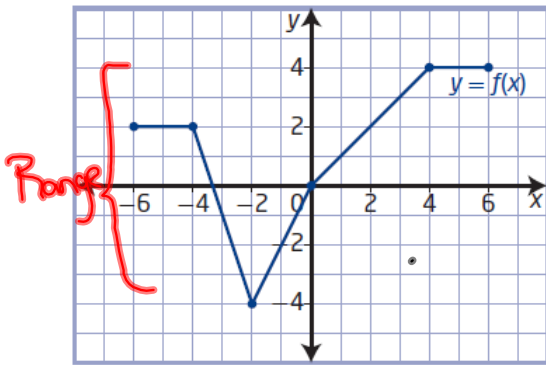


## Questions from Homework

6. The graph of the function  $y = f(x)$  is vertically stretched about the x-axis by a factor of 2.  $a=2$



$$(x, y) \rightarrow (x, 2y)$$

$$D: \{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$$

$$\text{or } [-6, 6]$$

$$R: \{y \mid -8 \leq y \leq 8, y \in \mathbb{R}\}$$

$$\text{or } [-8, 8]$$

2. a) Copy and complete the table of values for the given functions.

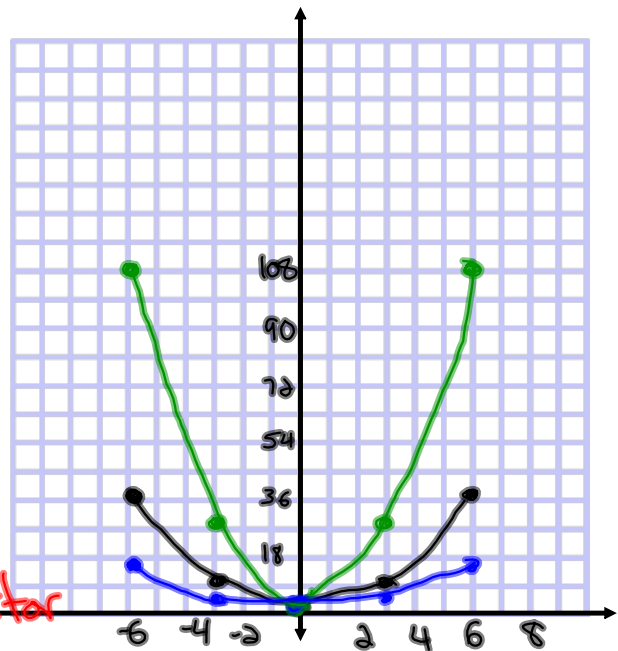
$x$	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
-6	36	108	12
-3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

↑  
stretched vertically by a factor of 3

$$(x, y) \rightarrow (x, 3y)$$

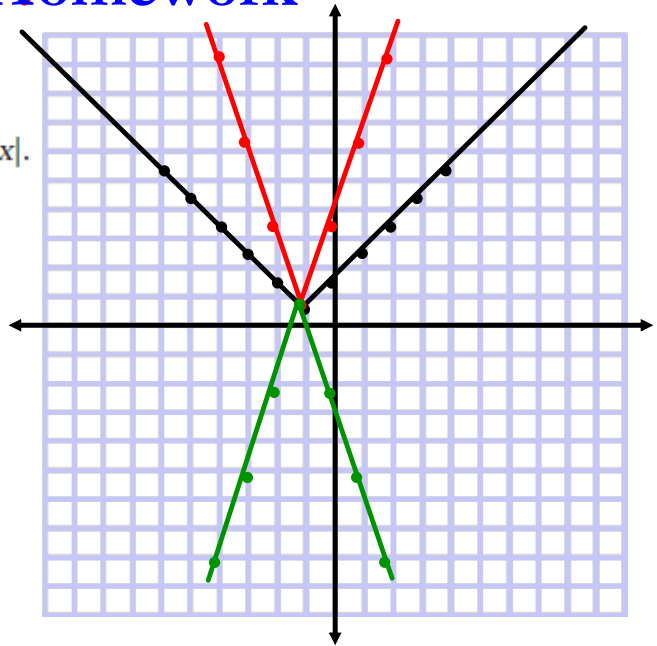
↑  
compressed vertically by a factor of  $\frac{1}{3}$

$$(x, y) \rightarrow (x, \frac{1}{3}y)$$



## Questions from Homework

10. Thomas and Sharyn discuss the order of the transformations of the graph of  $y = -3|x|$  compared to the graph of  $y = |x|$ . Thomas states that the reflection must be applied first. Sharyn claims that the vertical stretch should be applied first.



- a) Sketch the graph of  $y = -3|x|$  by applying the reflection first.
- b) Sketch the graph of  $y = -3|x|$  by applying the stretch first.
- c) Explain your conclusions. Who is correct?

Neither . . . .  
 It does not matter what order the reflections and stretches are done in

### Extend

14. Consider the function  $f(x) = (x + 4)(x - 3)$ . Without graphing, determine the zeros of the function after each transformation.

- a)  $y = 4f(x)$   $a = 4$   $(x, y) \rightarrow (x, 4y)$
- b)  $y = f(-x)$  horizontal reflection  $(x, y) \rightarrow (-x, y)$
- c)  $y = f\left(\frac{1}{2}x\right)$   $b = \frac{1}{2}$   $(x, y) \rightarrow (2x, y)$
- d)  $y = f(2x)$   $b = 2$   $(x, y) \rightarrow \left(\frac{1}{2}x, y\right)$

← quadratic  
 $f(x) = x^2 + x - 12$

x intercepts

$$\begin{array}{l|l} x+4=0 & x-3=0 \\ x=-4 & x=3 \\ (-4,0) & (3,0) \end{array}$$

a)  $(-4, 0) + (3, 0)$

b)  $(4, 0) + (-3, 0)$

c)  $(-8, 0) + (6, 0)$

d)  $(-2, 0) + \left(\frac{3}{2}, 0\right)$

## Summary of Transformations...

Transformations of the graphs of functions	
$f(x) + c$	shift $f(x)$ up $c$ units
$f(x) - c$	shift $f(x)$ down $c$ units
$f(x + h)$	shift $f(x)$ left $h$ units
$f(x - h)$	shift $f(x)$ right $h$ units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$a f(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$
	When $c > 1$ – vertical stretching of $f(x)$ Multiply the y values by $c$
$f(bx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$
	When $c > 1$ – horizontal shrinking of $f(x)$ Divide the x values by $c$

Handwritten notes:

- Vertical translation (for  $f(x) + c$  and  $f(x) - c$ )
- Horizontal translation (for  $f(x + h)$  and  $f(x - h)$ )
- Horizontal reflection (for  $f(-x)$ )
- Vertical reflection (for  $-f(x)$ )
- Vertical stretch (for  $a f(x)$  when  $c > 1$ )
- Horizontal stretch (for  $f(bx)$  when  $c < 1$ )

# Transformations:

$$y = f(x) \longrightarrow y = af(b(x - h)) + k$$

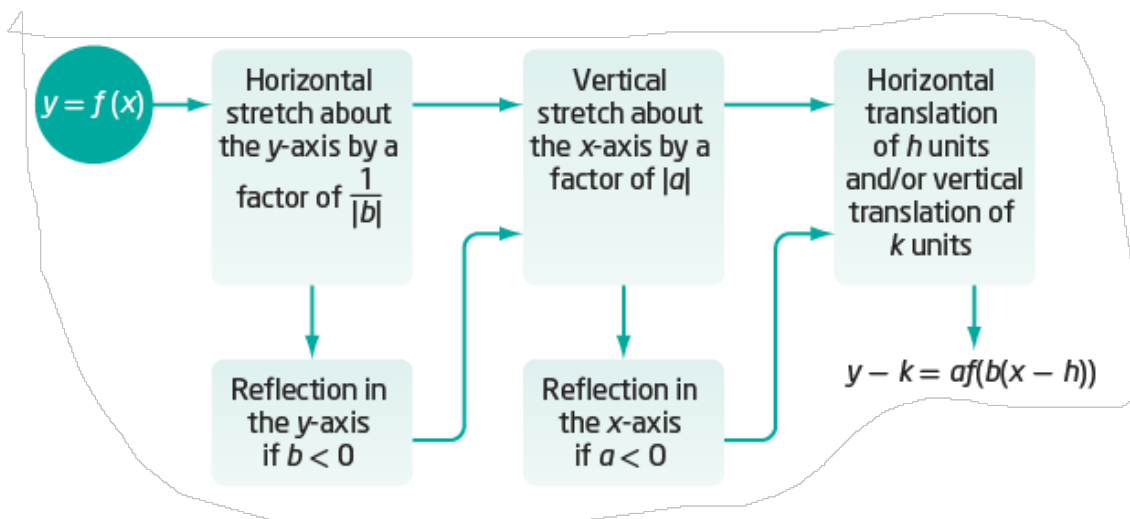
**Mapping Rule:**  $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

**Important note for sketching...**

**Transformations should be applied in following order:**

1. Reflections
2. Stretches
3. Translations

**Remember...RST**



## Warm-Up...

Given that  $(-2, 5)$  is a point on the graph of  $y = f(x)$ , determine the coordinates of this point once the following transformations are applied...

$$a=3 \quad b=1 \quad h=0 \quad k=0$$

$$(1) y = 3f(x)$$

$$(x, y) \rightarrow \left(\frac{1}{1}x + 0, 3y + 0\right)$$

$$(x, y) \rightarrow (x, 3y)$$

$$(-2, 5) \rightarrow (-2, 15)$$

$$a=1 \quad b=-\frac{1}{3} \quad h=0 \quad k=0$$

$$(2) y = f\left(-\frac{1}{3}x\right)$$

$$(x, y) \rightarrow (-3x, y)$$

$$(-2, 5) \rightarrow (6, 5)$$

$$a=4 \quad b=\frac{1}{2} \quad h=-5 \quad k=-3$$

$$(3) y = 4f\left[\frac{1}{2}(x+5)\right] - 3$$

$$(x, y) \rightarrow [2x - 5, 4y - 3]$$

$$\cdot (-2, 5) \rightarrow (-9, 17)$$

$$a=-2 \quad b=-2 \quad h=3 \quad k=5$$

$$(4) y - 5 = -2f(-2x + 6)$$

$$y = -2f(-2(x-3)) + 5$$

$$(x, y) \rightarrow \left(-\frac{1}{2}x + 3, -2y + 5\right)$$

$$(-2, 5) \rightarrow (4, -5)$$

## Transformations:

$$g(x) = -3f(4(x-4)) - 10$$

2. The function  $y = f(x)$  is transformed to the function  $g(x) = -3f(4x - 16) - 10$ . Copy and complete the following statements by filling in the blanks.

The function  $f(x)$  is transformed to the function  $g(x)$  by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

$$\begin{aligned} a &= 3 \\ b &= 4 \\ h &= 4 \\ k &= -10 \end{aligned}$$

- a) y axis
- b)  $\frac{1}{4}$
- c) x axis
- d) 3
- e) x axis
- f) 4
- g) 10

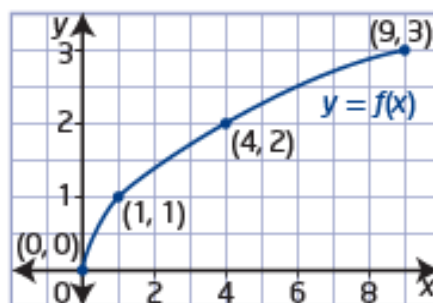
## Example 1

### Graph a Transformed Function

Describe the combination of transformations that must be applied to the function  $y = f(x)$  to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a)  $y = 3f(2x)$

b)  $y = f(3x + 6)$



a)  $y = 3f(2x)$      $a=3$      $b=2$      $h=0$      $k=0$

The graph of  $y = f(x)$  is horizontally stretched about the  $y$ -axis by a factor of  $\frac{1}{2}$  and then vertically stretched about the  $x$ -axis by a factor of 3.

Base:  $y = \sqrt{x}$

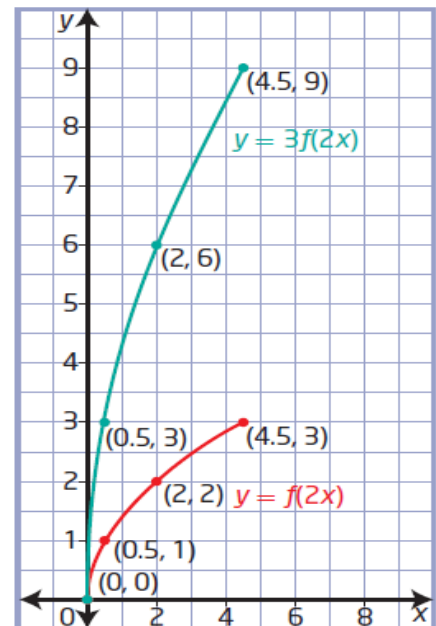
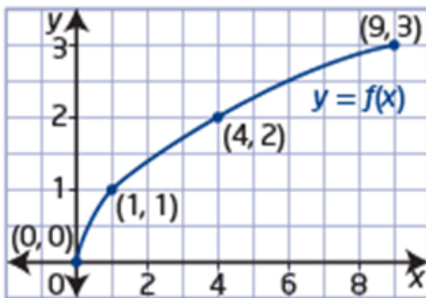
$(x, y) \rightarrow (\frac{1}{2}x, 3y)$

$(0, 0) \rightarrow (0, 0)$

$(1, 1) \rightarrow (\frac{1}{2}, 3)$

$(4, 2) \rightarrow (2, 6)$

$(9, 3) \rightarrow (\frac{9}{4}, 9)$

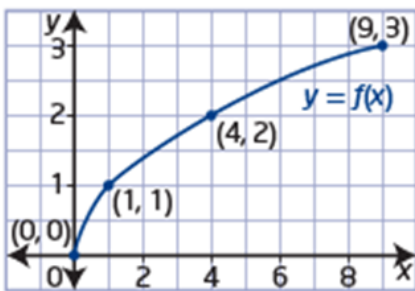




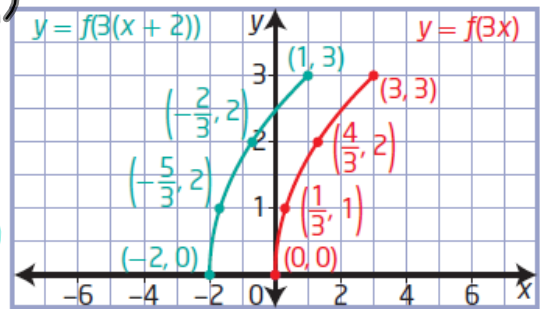
Factor

b)  $y = f(3x + 6)$      $a=1$      $b=3$      $h=-2$      $k=0$   
 $y = f(3(x+2))$

The graph of  $y = f(x)$  is horizontally stretched about the y-axis by a factor of  $\frac{1}{3}$  and then horizontally translated 2 units to the left.



$(x,y) \rightarrow (\frac{1}{3}x - 2, y)$   
 $(0,0) \rightarrow (-2, 0)$   
 $(1,1) \rightarrow (-\frac{5}{3}, 1)$   
 $(4,2) \rightarrow (-\frac{2}{3}, 2)$   
 $(9,3) \rightarrow (1, 3)$



## Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function  $y = f(x)$ .

Function	Reflections	Vertical Stretch Factor $a$	Horizontal Stretch Factor $b$	Vertical Translation $k$	Horizontal Translation $h$
$y - 4 = f(x - 5)$	-	-	-	4	5
$y + 5 = 2f(3x)$	-	2	$\frac{1}{3}$	-5	-
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	-	$\frac{1}{2}$	2	-	4
$y + 2 = -3f(2(x + 2))$	↑	3	$\frac{1}{2}$	-2	-2

vertical

6. The key point  $(-12, 18)$  is on the graph of  $y = f(x)$ . What is its image point under each transformation of the graph of  $f(x)$ ?

d)  $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$  Factor First

$$y = -2f\left(-\frac{2}{3}(x + 9)\right) + 4$$

$$a = -2 \quad b = -\frac{2}{3} \quad h = -9 \quad k = 4$$

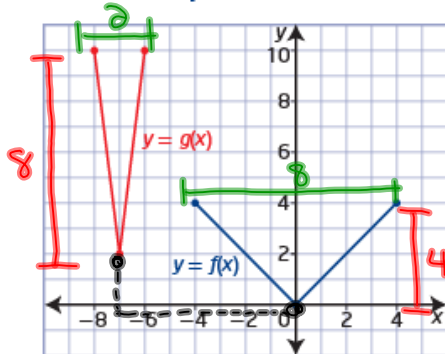
$$(x, y) \longrightarrow \left(-\frac{3}{2}x - 9, -2y + 4\right)$$

$$\boxed{(-12, 18) \longrightarrow (9, -32)}$$

**Example 3**

**Write the Equation of a Transformed Function Graph**

The graph of the function  $y = g(x)$  represents a transformation of the graph of  $y = f(x)$ . Determine the equation of  $g(x)$  in the form  $y = af(b(x - h)) + k$ . Explain your answer.



**Solution**

Locate key points on the graph of  $f(x)$  and their image points on the graph of  $g(x)$ .

- $(-4, 4) \rightarrow (-8, 10)$
- $(0, 0) \rightarrow (-7, 2)$
- $(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is  $g(x) = 2f(4(x + 7)) + 2$ .

① Reflections: None

② Vertical Stretch Factor:  $\frac{8}{4} = 2$   $a = 2$   
 (Compare Range  $\frac{\text{New}}{\text{Old}}$ )

③ Horizontal Stretch Factor:  $\frac{2}{8} = \frac{1}{4}$   $b = 4$   
 (Compare Domain  $\frac{\text{New}}{\text{Old}}$ )

④ Horizontal Translation:  $(0, 0) \rightarrow (-7, 2)$  Left 7  $h = -7$

⑤ Vertical Translation:  $(0, 0) \rightarrow (-7, 2)$  Up 2  $k = 2$

⑥ Equation:  $g(x) = 2f(4(x + 7)) + 2$

\* Check using Key Points:

How could you use the mapping  $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$  to verify this equation?

$(x, y) \longrightarrow \left(\frac{1}{4}x - 7, 2y + 2\right)$

$(-4, 4) \longrightarrow (-8, 10)$

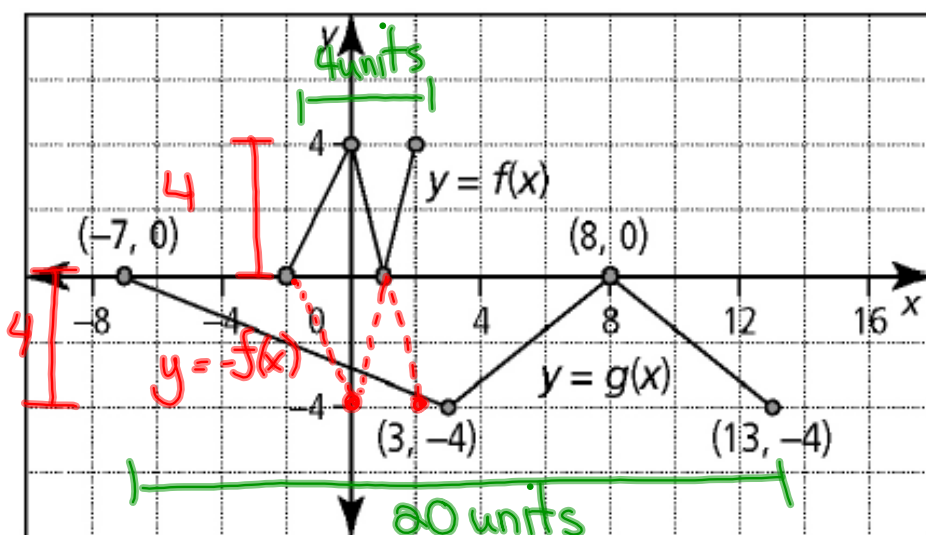
$(0, 0) \longrightarrow (-7, 2)$

$(4, 4) \longrightarrow (-6, 10)$

The graph of the function  $y = g(x)$  represents a transformation of the graph of  $y = f(x)$ .

Determine the equation of  $g(x)$  in the form  $y = af(b(x - h)) + k$ .

$$y = -f\left(\frac{1}{5}(x-3)\right)$$



① vertical reflection ( $-a$ )

② vertical stretch factor:  $\frac{\text{New}}{\text{Old}} = \frac{4}{4} = 1$   $a = -1$

③ horizontal " " :  $\frac{\text{New}}{\text{Old}} = \frac{20}{4} = 5$   $b = \frac{1}{5}$

④ horizontal translation: right 3  $\rightarrow h = 3$

Pick a point where  $x = 0$

$$(0, 4) \rightarrow (3, -4)$$

⑤ vertical translation: no change  $k = 0$

pick a point where  $y = 0$

$$(1, 0) \rightarrow (8, 0)$$

## Homework

Page 38 # 3-6  
Plus 7, 8, 9 (a, c, e) and 10

17. The graph of the function  $y = 2x^2 + x + 1$  is stretched vertically about the  $x$ -axis by a factor of 2, stretched horizontally about the  $y$ -axis by a factor of  $\frac{1}{3}$ , and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the  $x$ -axis by a factor of 2, stretched horizontally about the  $y$ -axis by a factor of  $\frac{1}{3}$ , and translated 2 units to the right and 4 units down. Write the equation of the transformed function.