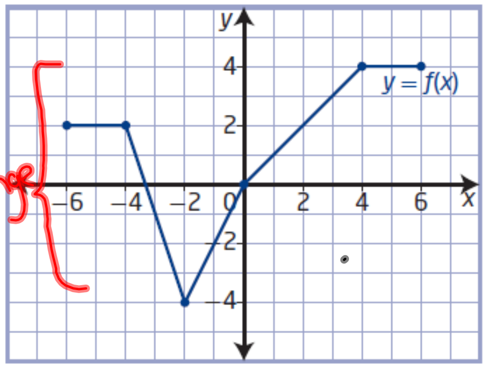
Questions from Homework

6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a=2$



Range

$(x,y) \longrightarrow (x, 2y)$

D: $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$

or $[-6,6]$

R: $\{y \mid -8 \leq y \leq 8, y \in \mathbb{R}\}$

or $[-8,8]$

2. a) Copy and complete the table of values for the given functions.

$a=3$ $a=1/3$

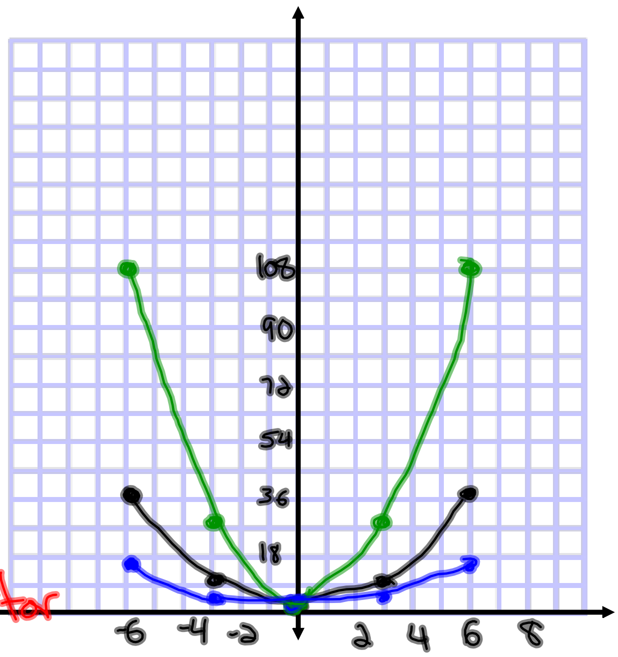
x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

stretched vertically by a factor of 3

$(x,y) \rightarrow (x,3y)$

compressed vertically by a factor of $1/3$

$(x,y) \rightarrow (x,\frac{1}{3}y)$



Questions from Homework

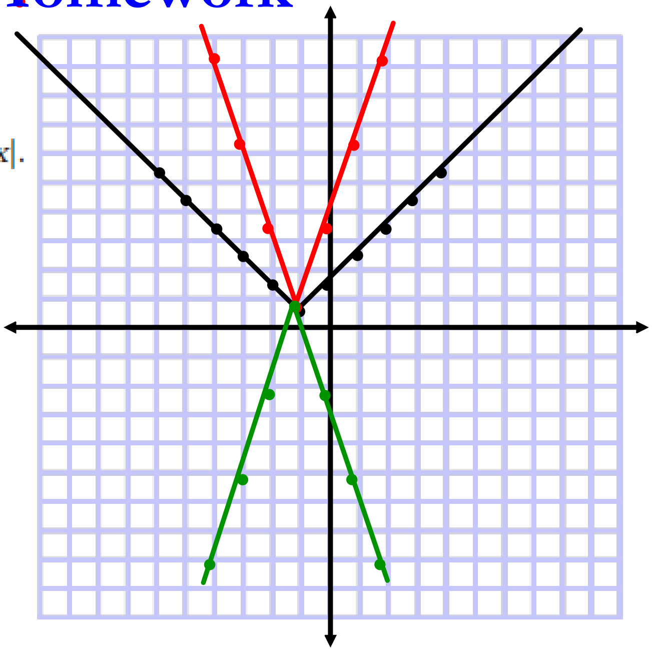
10. Thomas and Sharyn discuss the order of the transformations of the graph of $y = -3|x|$ compared to the graph of $y = |x|$. Thomas states that the reflection must be applied first. Sharyn claims that the vertical stretch should be applied first.

a) Sketch the graph of $y = -3|x|$ by applying the reflection first.

b) Sketch the graph of $y = -3|x|$ by applying the stretch first.

c) Explain your conclusions. Who is correct? Neither . . . .

It does not matter what order the reflections and stretchs are done in.



Extend

14. Consider the function $f(x) = (x + 4)(x - 3)$. Without graphing, determine the zeros of the function after each transformation.

quadratic
$f(x) = x^2 + x - 12$

x intercepts

$x+4=0$ | $x-3=0$
$x=-4$ | $x=3$
$(-4,0)$ | $(3,0)$

a) $y = 4f(x)$ $a=4$ $(x,y) \to (x,4y)$

b) $y = f(-x)$ horizontal reflection $(x,y) \to (-x,y)$

c) $y = f\left(\frac{1}{2}x\right)$ $b=\frac{1}{2}$ $(x,y) \to (2x,y)$

d) $y = f(2x)$ $b=2$ $(x,y) \to (\frac{1}{2}x,y)$

a) $(-4,0)$ + $(3,0)$

b) $(4,0)$ + $(-3,0)$

c) $(-8,0)$ + $(6,0)$

d) $(-2,0)$ + $(\frac{3}{2},0)$

Summary of Transformations...

Transformations of the graphs of functions		
$f(x)+k$	shift $f(x)$ up c units	vertical translation
$f(x)-k$	shift $f(x)$ down c units	
$f(x+h)$	shift $f(x)$ left c units	horizontal "
$f(x-h)$	shift $f(x)$ right c units	
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal reflection
$-f(x)$	reflect $f(x)$ about the x-axis	vertical "
$af(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ **Multiply the y values by c**	vertical stretch
$f(bx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by c**	horizontal stretch

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right) + k$$

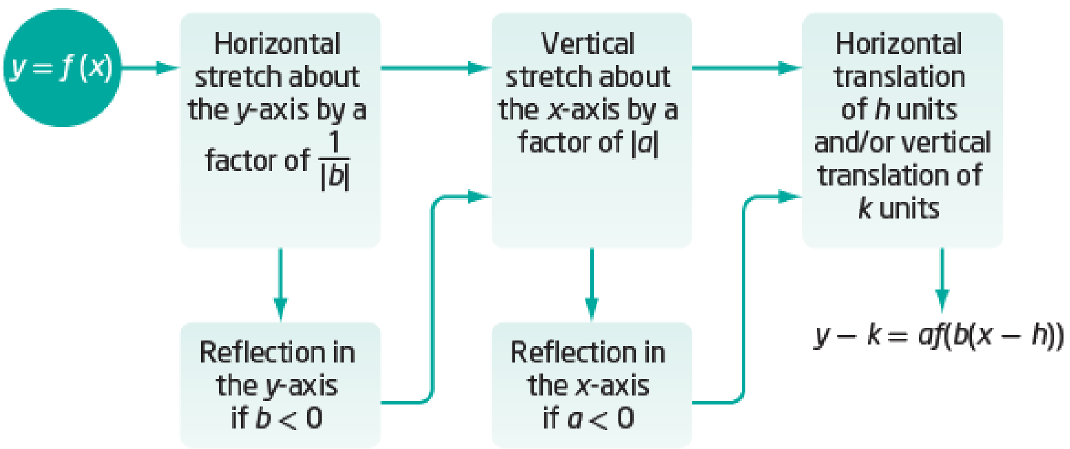
Mapping Rule: $$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

$a=3$ $b=1$ $h=0$ $k=0$

(1) $y = 3f(x)$

$(x,y) \rightarrow (\frac{1}{1}x+0, 3y+0)$

$(x,y) \rightarrow (x, 3y)$

$(-2,5) \rightarrow (-2,15)$

$a=1$ $b=\frac{-1}{3}$ $h=0$ $k=0$

(2) $y = f\left(-\frac{1}{3}x\right)$

$(x,y) \rightarrow (-3x, y)$

$(-2,5) \rightarrow (6,5)$

$a=4$ $b=\frac{1}{2}$ $h=-5$ $k=-3$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right]-3$

$(x,y) \rightarrow [2x-5, 4y-3]$

$(-2,5) \rightarrow (-9,17)$

$a=-2$ $b=-2$ $h=3$ $k=5$

(4) $y-5=-2f(-2x+6)$

$y=-2f(-2(x-3))+5$

$(x,y) \rightarrow (-\frac{1}{2}x+3, -2y+5)$

$(-2,5) \rightarrow (4,-5)$

Transformations:

$g(x) = -3f(4(x-4))-10$

$a=-3$
$b=4$
$h=4$
$k=-10$

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

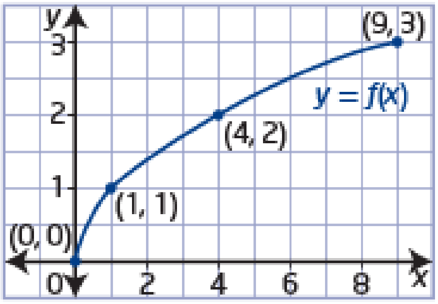
The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

a) y axis
b) 1/4
c) x axis
d) 3
e) x axis
f) 4
g) 10

Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.



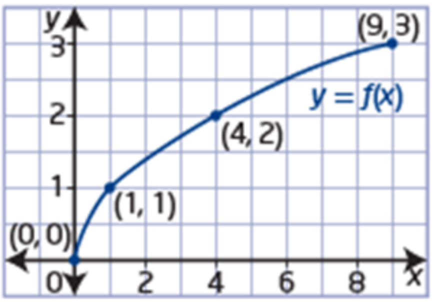
a) $y = 3f(2x)$

b) $y = f(3x + 6)$

a) $y = 3f(2x)$ $a=3$ $b=2$ $h=0$ $k=0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

Base: $y=\sqrt{x}$



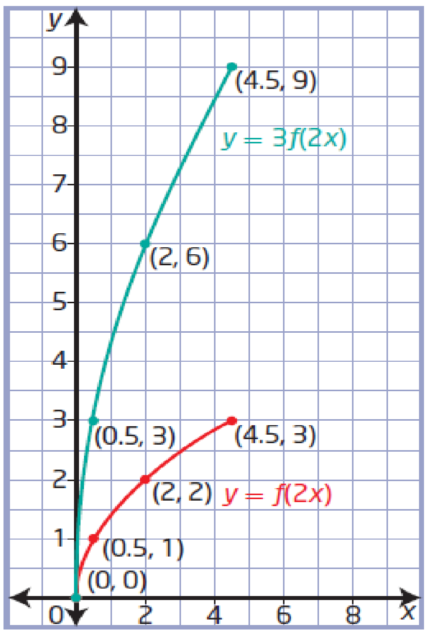
$(x,y) \rightarrow (\frac{1}{2}x, 3y)$

$(0,0) \rightarrow (0,0)$

$(1,1) \rightarrow (\frac{1}{2}, 3)$

$(4,2) \rightarrow (2,6)$

$(9,3) \rightarrow (\frac{9}{2}, 9)$

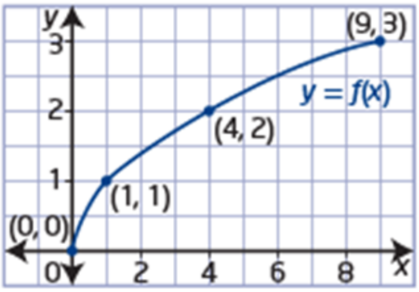


Factor

b) $y = f(3x + 6)$ $a = 1$ $b = 3$ $h = -2$ $k = 0$

$y = f(3(x+2))$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.



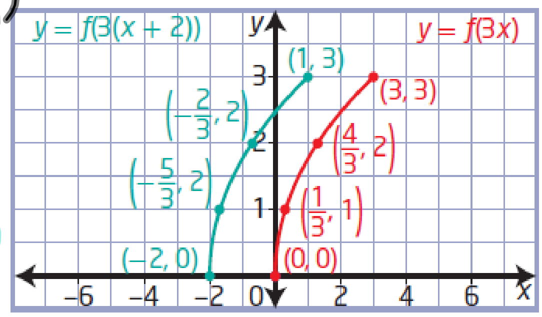
$(x,y) \rightarrow (\frac{1}{3}x-2, y)$

$(0,0) \rightarrow (-2, 0)$

$(1,1) \rightarrow (-5/3, 1)$

$(4,2) \rightarrow (-2/3, 2)$

$(9,3) \rightarrow (1, 3)$



Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

Function	Reflections	Vertical Stretch Factor (a)	Horizontal Stretch Factor ($1/b$)	Vertical Translation (k)	Horizontal Translation (h)
$y - 4 = f(x - 5)$	—	—	—	4	5
$y + 5 = 2f(3x)$	—	2	$\frac{1}{3}$	−5	—
$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	—	$\frac{1}{2}$	2	—	4
$y + 2 = -3f(2(x + 2))$	↑ vertical	3	$\frac{1}{2}$	−2	−2

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

d) $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$ Factor First

$y = -2f\left(-\frac{2}{3}(x + 9)\right) + 4$

$a = -2$ $b = -\frac{2}{3}$ $h = -9$ $k = 4$

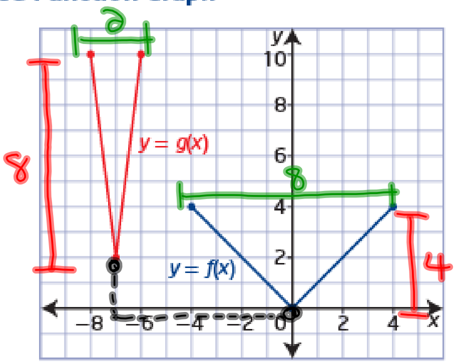
$(x, y) \longrightarrow \left(-\frac{3}{2}x - 9, -2y + 4\right)$

$\boxed{(-12, 18) \longrightarrow (9, -32)}$

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Reflections: None

② Vertical Stretch Factor: $\frac{8}{4} = 2$ $a = 2$
(Compare Range $\frac{\text{New}}{\text{Old}}$)

③ Horizontal Stretch Factor: $\frac{2}{8} = \frac{1}{4}$ $b = 4$
(Compare Domain $\frac{\text{New}}{\text{Old}}$)

④ Horizontal Translation: $(0,0) \rightarrow (-7,2)$ Left 7 $h = -7$

⑤ Vertical Translation: $(0,0) \rightarrow (-7,2)$ Up 2 $K = 2$

⑥ Equation: $g(x) = 2f(4(x+7)) + 2$

* Check using Key Points:

$(x,y) \longrightarrow (\frac{1}{4}x - 7, 2y + 2)$

$(-4,4) \longrightarrow (-8,10)$

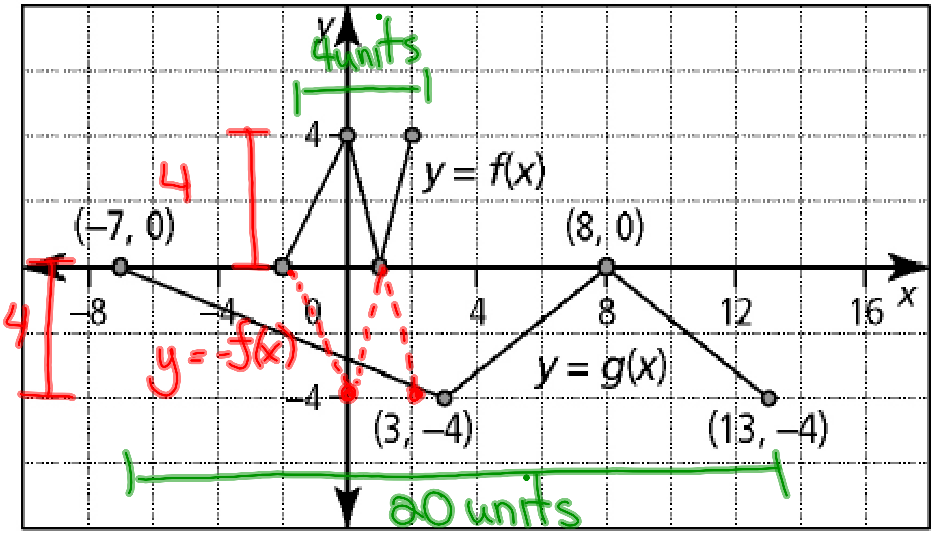
$(0,0) \longrightarrow (-7,2)$

$(4,4) \longrightarrow (-6,10)$

How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

$y = -f\left(\frac{1}{5}(x-3)\right)$



① vertical reflection (−a)

② vertical stretch factor: $\frac{\text{New}}{\text{Old}} = \frac{4}{4} = 1$ $a = -1$

③ horizontal " " : $\frac{\text{New}}{\text{Old}} = \frac{20}{4} = 5$ $b = \frac{1}{5}$

④ horizontal translation: right 3 → $h = 3$

Pick a point where $x=0$

$(0,4) \longrightarrow (3,-4)$

⑤ vertical translation: no change $k=0$

pick a point where $y=0$

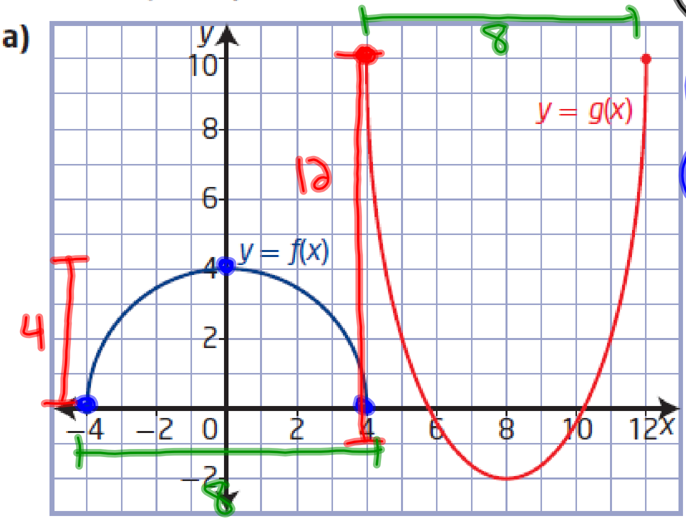
$(1,0) \longrightarrow (8,0)$

Homework

Page 38 # 3-6
Plus 7, 8, 9 (a, c, e) and 10

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

a)



$(x,y) \to (1x+8, -3y+10)$

$(-4,0) \to (4,10)$

$(0,4) \to (8,-2)$

$(4,0) \to (12,10)$

① VSF: $\frac{12}{4} = 3$ $(a=3)$

② HSF: $\frac{8}{8} = 1$ $(b=1)$ ← Reciprocal of 1 is 1

③ Reflection: Vertical reflection in the x-axis $(-a)$

④ VT: $(-4,0) \to (4,10)$ Up 10 → $k=10$

⑤ HT: $(0,4) \to (8,-2)$ Right 8 → $h=8$

⑥ Equation: $y = -3f(1(x-8)) + 10$

$y = -3f(x-8) + 10$

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.