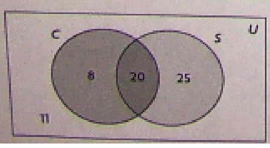
CHAPTER 3 [illegible] ANSWERS

2. a) $n(A) = 7$
   b) $n(B) = 2$
   c) $n(A \cap B) = 1$
   d) $n((A \cup B)') = 7$
   e) $n(U) = 15$

3. $A = \{2, 3, 5, 7, 11\}$, $B = \{2, 4, 6, 8, 10\}$
   $n(A) = 5$, $n(B) = 5$, $n(A \cap B) = 1$
   $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
   $n(A \cup B) = 5 + 5 - 1$
   $n(A \cup B) = 9$

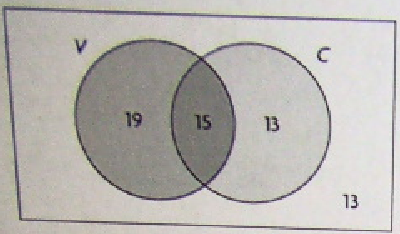
4. Let C represent using a cellphone, and let L represent using a land line:
   $n(C) + n(L) = 152$
   This is 56 more than the number of people surveyed, so 56 people use both.

5. I drew a Venn diagram and wrote 20 where the sets for canoeing and swimming overlap. I wrote 11 outside those sets, for campers who did not want to do either activity. Since 28 campers wanted to canoe, then there were 28 – 20 or 8 campers who only wanted to canoe. Since 45 campers wanted to swim, then there were 45 – 20 or 25 campers who only wanted to swim. I added the numbers in the four regions. There were 11 + 8 + 20 + 25 or 64 campers at the camp.
   Venn shows number of elements in each region:

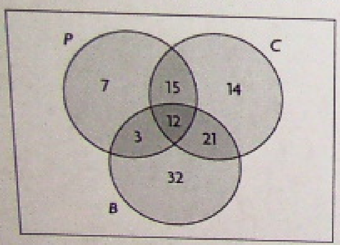


6. 15 people had vanilla ice cream and chocolate sauce.

Venn shows number of elements in each region:



7. Venn shows number of elements in each region:



Therefore, 104 students were in Grade 12 that year.

Jan 13-10:37 AM

8. a) The summer solstice marks the first day of summer and occurs on the day with the most daylight. Dominique's statement is true.

b) Converse: If the summer solstice occurs today, then today is the longest day of the year.

Since the summer solstice occurs on the longest day of the year, the converse is also true.

c) Biconditional statement: Today is the longest day of the year if and only if the summer solstice occurs today.

9. a) The statement is false. The integer could be zero. Zero is neither negative nor positive.

b) i) Converse: If an integer is positive, then it is not negative. This statement is true.

ii) Inverse: If an integer is negative, then it is not positive. This statement is true.

iii) Contrapositive: If an integer is not positive, then it is negative.

This statement is false. The integer could be zero.

CHAPTER 4 ANSWERS

1. a) For example: The table shows the possible wins and losses of one of the teams. Shaded cells indicate games that would not actually be played, since one of the teams will have already won or lost two games. That means there are only six different outcomes.

Game 1	Game 2	Game 3	Outcomes
W	W	W	1 outcome
W	W	L	
W	L	W	1 outcome
W	L	L	1 outcome
L	W	W	1 outcome
L	W	L	1 outcome
L	L	W	1 outcome
L	L	L	

b) two different ways: WLW or LWW or WWL

2. a) $2 \cdot 2 \cdot 2 \cdot 3$, or 24 ways; there are two choices for each part of the sandwich. I assumed that the sandwich has exactly one item from each of set of choices.

b) $6 \cdot 6 \cdot 6 \cdot 6$ or 1296 ways; there are six choices for each digit and repetition is allowed. I assumed that starting the password with a 0 was allowed.

c) $13 + 13$ or 26 ways; there are 13 cards in each suit and the sets are disjoint. I made no assumptions.

d) $\frac{5!}{2!\,2!}$ or 30 arrangements. I made no assumptions.

e) $\binom{25}{3}$ or 2300 different pizzas. I assumed that he would choose exactly three toppings and each would be different.

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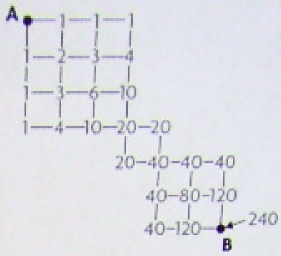
3\. a) $\binom{10}{5}$ or 252 selections

b) ${}_{10}P_5 = \frac{10!}{5!}$ or 30 240 selections

c) Order does not matter in part a) but it does matter in part b), so part a) involves combinations, whereas part b) involves permutations.

d) The answer to part a) is 5! or 120 times smaller than the answer to part b). This is because the 30 240 five-novel selections from part b) must be divided by 5! to eliminate combinations that are the same, because order does not matter.

4. She can walk 240 different ways. (The problem can be solved using a pathway diagram, as shown, or using permutations and the Fundamental Counting Principle.)



5. a) $\binom{5}{2}\binom{6}{2} = 10 \cdot 15$ or 150 ways

b) At least two girls means two, three, or four girls:
$\binom{5}{2}\binom{6}{2} + \binom{5}{1}\binom{6}{3} + \binom{5}{0}\binom{6}{4} = 10 \cdot 15 + 5 \cdot 20 + 1 \cdot 15$ or 265 committees

c) If Jim and Nanci must be on the committee, there are nine people left for the remaining two positions:
$\binom{1}{1}\binom{1}{1}\binom{9}{2} = 1 \cdot 1 \cdot 36$ or 36 committees

d) More boys than girls means three or four boys:
$\binom{5}{3}\binom{6}{1} + \binom{5}{4}\binom{6}{0} = 10 \cdot 6 + 5 \cdot 1$ or 65 committees

6. a) $$\frac{n!}{(n-4!)} = 60\left(\frac{n!}{2!(n-2)!}\right)$$

$$n(n-1)(n-2)(n-3) = 60\left(\frac{n(n-1)}{2}\right)$$

$$(n-2)(n-3) = 30$$
$$n^2 - 5n + 6 = 30$$
$$n^2 - 5n - 24 = 0$$
$$(n-8)(n+3) = 0$$
$$n = 8 \quad or \quad n = -3$$

$n = -3$ is extraneous

$n = 8$

b) $n = 8, n \geq 4$

$_nP_4 = 60(_nC_2)$, so $n \geq 4$ and $n \geq 2$, so $n \geq 4$

7. If you place the T and K as required, this can happen only one way for each position. The remaining seven letters can then be arranged in between, keeping in mind that there are repeated letters: two A's, two S's, and two O's:

$$1 \cdot 1 \cdot \frac{7!}{2!?2!?2!} = 630$$