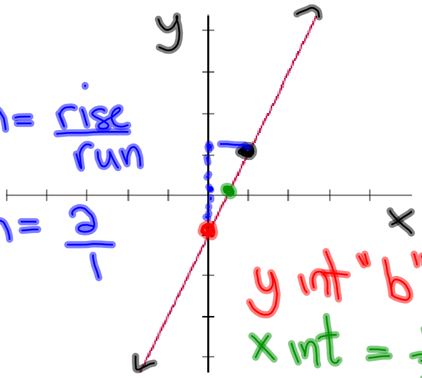
Catalog of Essential Functions

1. Linear



$m = \frac{rise}{run}$

$m = \frac{2}{1}$

y int "b" = −1 (0,−1)

x int = $\frac{1}{2}$ $(\frac{1}{2},0)$

Straight Line

Equation will be degree one

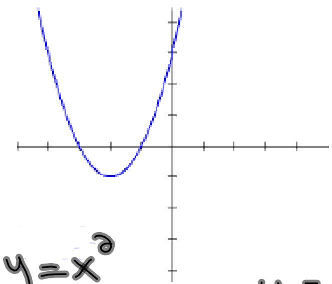
highest exponent

Should be able to identify the ***slope, intercepts, and equation*** from the graph

$y = mx + b$

$\boxed{y = 2x - 1}$

2. Quadratic



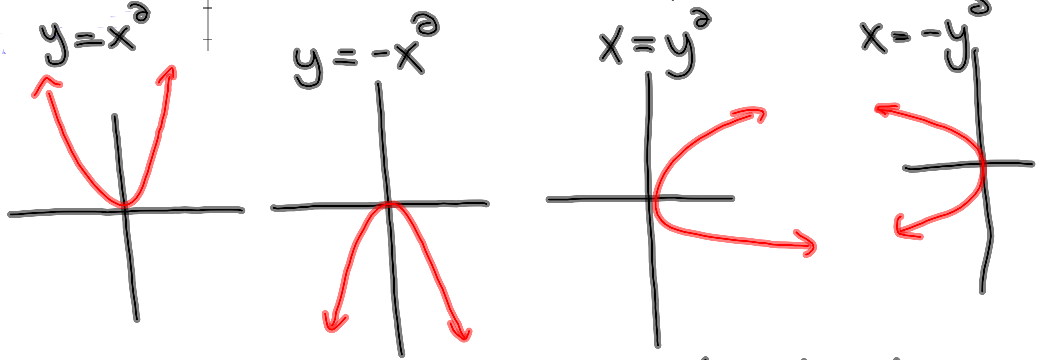
Parabola (U-Shaped)

Degree 2

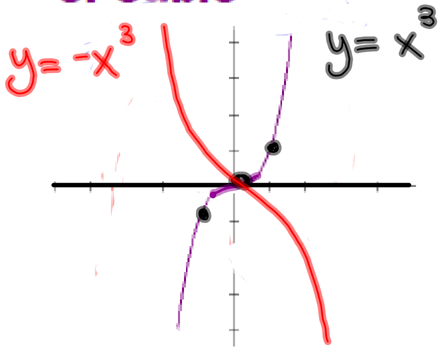
Either *y or x* will be squared (not both!)

Should know the 4 basic quadratic functions

Should be able to apply transformations to the basic quadratic functions



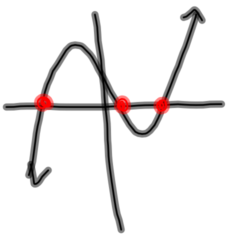
3. Cubic



S-Shaped / N-shaped

We will work with functions that are raised to the third power

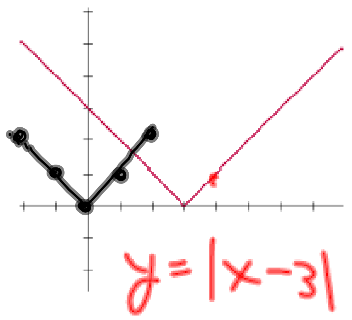
x	y
−2	−8
−1	−1
0	0
1	1
2	8



max 3 roots

Catalog of Essential Functions

4. Absolute Value



V-Shaped

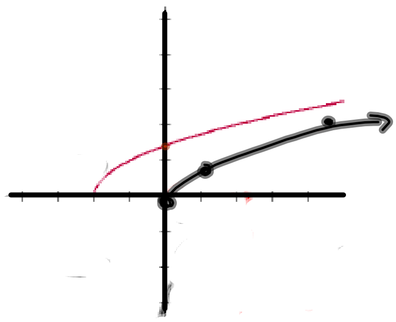
Equation will have a variable within the absolute value bars $y = |x|$

Should be able to apply transformations to the basic absolute value function

$y = |x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

5. Square Root



Half Parabola

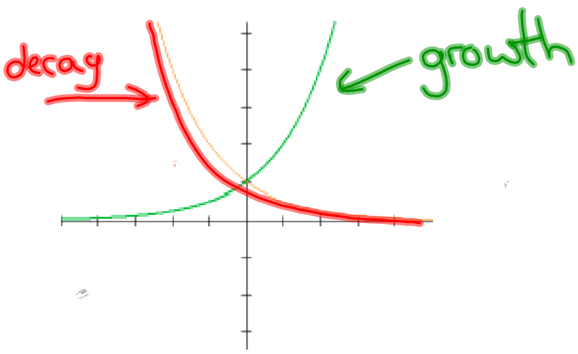
Equation will have a variable under the square root sign

Should be able to apply transformations to the basic square root function

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

6. Exponential



Steadily increasing or decreasing

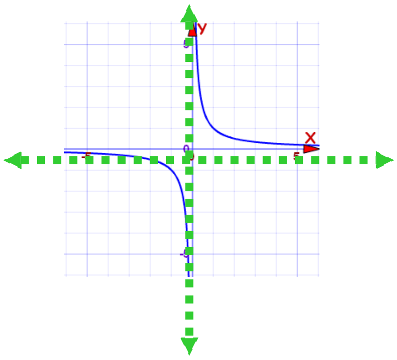
Base will be a number and variable will appear in the exponent

ex: $y = 2^x$

Should be able to identify the ***horizontal asymptote***

Catalog of Essential Functions

7. Reciprocal



Will have two branches

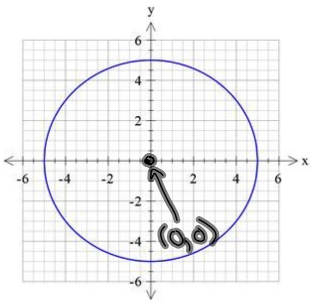
Equation will have a variable within the denominator of a rational expression

Should be able to identify the ***vertical and horizontal asymptotes***

$\boxed{y=\frac{1}{x}}$

x	y
-2	$-\frac{1}{2}$
-1	-1
0	undefined
1	1
2	$\frac{1}{2}$

8. Circle

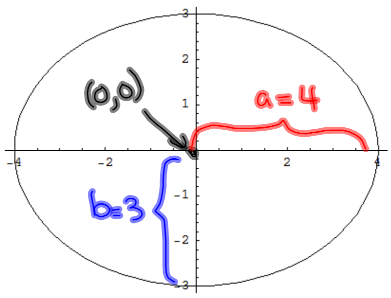


- General form: $(x-h)^2+(y-k)^2=r^2$
  - * center: (h, k) $= (0,0)$
  - * radius $= r$ $= 5$
- Be able to identify the function that would describe either just the top or bottom of the circle.

$$(x-0)^2+(y-0)^2=5^2$$

$$\boxed{x^2+y^2=25}$$

9. Ellipse



"Horizontal Major Axis"

major axis = 8

minor axis = 6

- General form: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Where...

- Center: (h, k) $= (0,0)$
- a > b
- If a is the denominator of the "y" term the ellipse will have a vertical major axis.

$$\frac{(x-0)^2}{4^2}+\frac{(y-0)^2}{3^2}=1$$

$$\boxed{\frac{x^2}{16}+\frac{y^2}{9}=1}$$

Transformations:

New Functions From Old Functions

Translations (Slide Transformation)

Stretches

Reflections

Translations

Focus on...

horizontal vertical

- determining the effects of h and k in $y - k = f(x - h)$ on the graph of $y = f(x)$ or $y = f(x-h)+k$
- sketching the graph of $y - k = f(x - h)$ for given values of h and k, given the graph of $y = f(x)$
- writing the equation of a function whose graph is a vertical and/or horizontal translation of the graph of $y = f(x)$

Base: $y = x^2$

Ex: ① $y = (x-\underline{3})^2 + \underline{2}$

$h = 3 \rightarrow$ Right

$k = 2 \rightarrow$ Up

Base: $y = |x|$

② $y - \underline{4} = |x + \underline{3}|$

$h = -3 \rightarrow$ left

$k = 4 \rightarrow$ Up

Function Notation

③ $g(x) = f(x+2) - 1$

$h = -2 \rightarrow$ Left

$k = -1 \rightarrow$ Down

Translation

- To *translate* or *shift* a graph is to move it up, down, left, or right without changing its shape.
- Translation is summarized by the following table and illustration:

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward
$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward
$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right
$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

Translations illustrated...

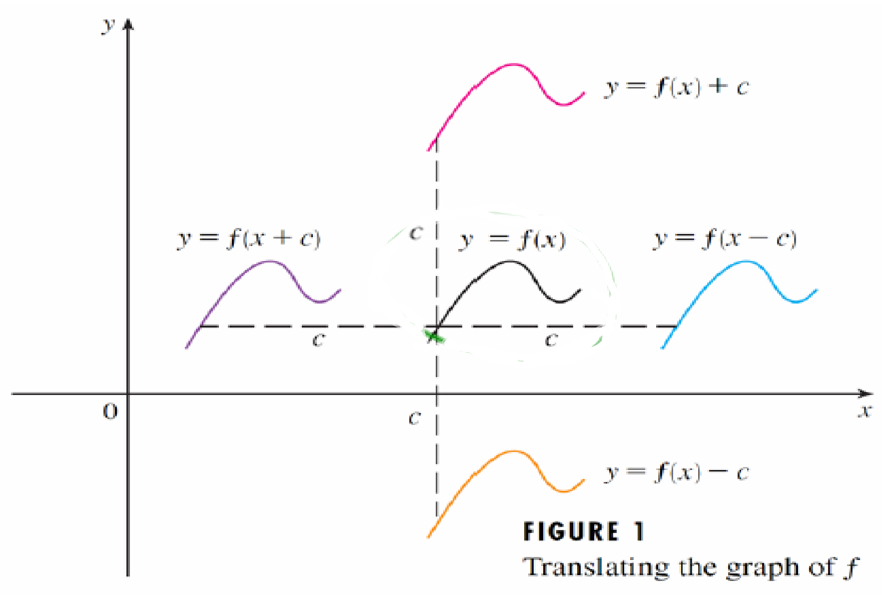


FIGURE 1
Translating the graph of f

Using Mapping Notation to Describe Transformations:

*Think of this as a set of instructions to follow to transform a graph.

Base $y=x^2$

x	$y = x^2$
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

$k=2$

x	$y = x^2 + 2$
−3	11
−2	6
−1	3
0	2
1	3
2	6
3	11

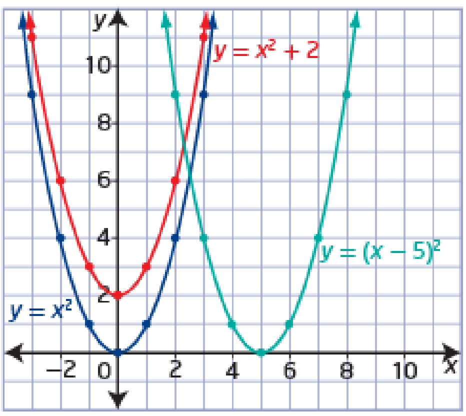
$(x,y)\rightarrow(x,y+2)$

$h=5$

x	$y = (x - 5)^2$
2	9
3	4
4	1
5	0
6	1
7	4
8	9

$(x,y)\rightarrow(x+5,y)$

Mapping Notation

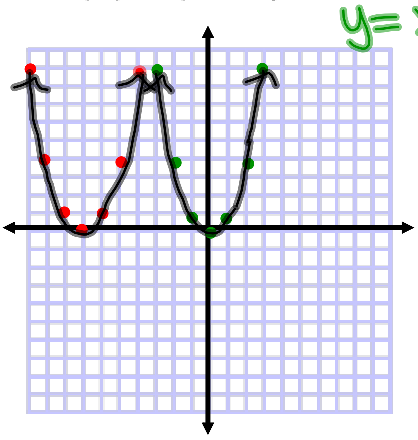


Identify the translations for each of the following...

$h = -7$

$f(x) = (x+7)^2$

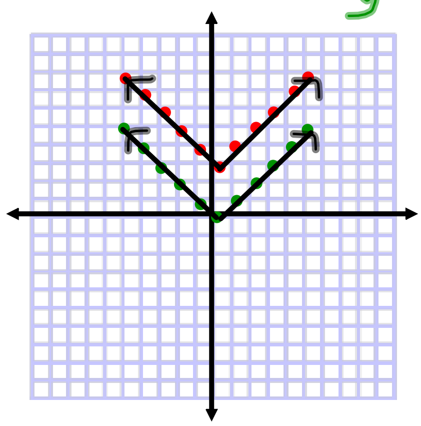
base:
$y = x^2$



$k = 3$

$f(x) = |x| + 3$

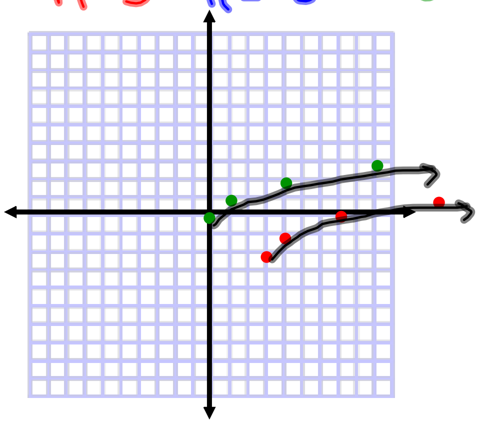
Base:
$y = |x|$



$f(x) = \sqrt{x-3} - 2$

Base:
$y = \sqrt{x}$

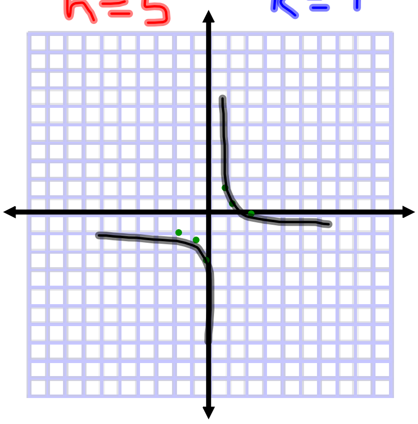
$h = 3$ $k = -2$



$f(x) = \dfrac{1}{x-5} + 7$

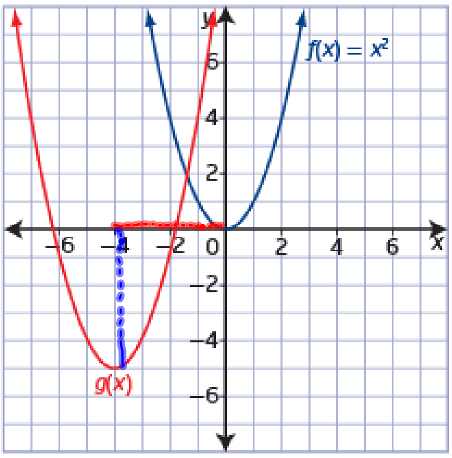
Base:
$y = \dfrac{1}{x}$

$h = 5$ $k = 7$



Determine the Equation of a Translated Function:

a)



Left 4 $h=-4$

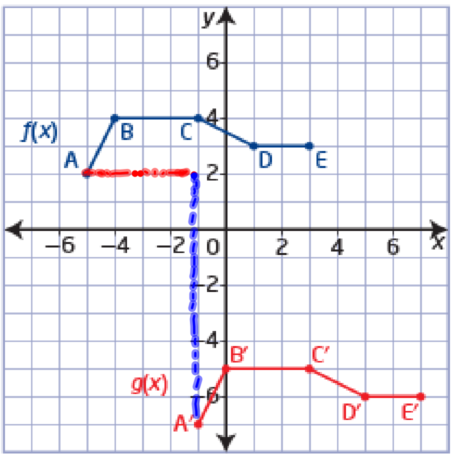
Down 5 $k=-5$

$g(x) = (x+4)^2 - 5$

or

$g(x)+5 = (x+4)^2$

b)

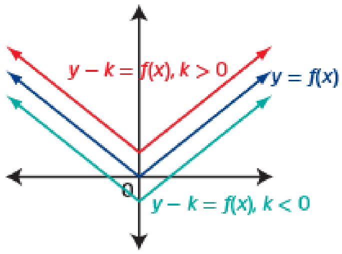
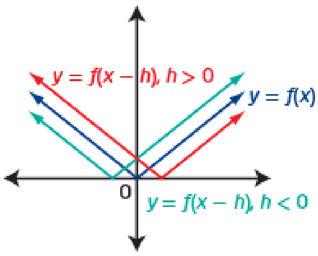


Right 4 $h=4$

Down 9 $k=-9$

$g(x) = f(x-4) - 9$

- Translations are transformations that shift all points on the graph of a function up, down, left, and right without changing the shape or orientation of the graph.
- The table summarizes translations of the function $y = f(x)$.

Function	Transformation from $y = f(x)$	Mapping	Example
$y - k = f(x)$ or $y = f(x) + k$	A vertical translation If $k > 0$, the translation is up. If $k < 0$, the translation is down.	$(x, y) \rightarrow (x, y + k)$	
$y = f(x - h)$	A horizontal translation If $h > 0$, the translation is to the right. If $h < 0$, the translation is to the left.	$(x, y) \rightarrow (x + h, y)$	

- A sketch of the graph of $y - k = f(x - h)$, or $y = f(x - h) + k$, can be created by translating key points on the graph of the base function $y = f(x)$.

Homework

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