Prove the identity:

$$\frac{\csc\theta}{1+\csc\theta}+\frac{\csc\theta}{1-\csc\theta}=-\frac{2\sin\theta}{\cos^2\theta}$$

$$\frac{\csc\theta(1-\csc\theta)}{(1+\csc\theta)(1-\csc\theta)}+\frac{\csc\theta(1+\csc\theta)}{(1+\csc\theta)(1-\csc\theta)}$$

$$\frac{\csc\theta-\csc^2\theta+\csc\theta+\csc^2\theta}{(1+\csc\theta)(1-\csc\theta)}$$

$$\frac{2\csc\theta}{1-\csc^2\theta}$$

$$\frac{2\csc\theta}{-\cot^2\theta}$$

$$2\left(\frac{1}{\sin\theta}\right)\div\left(\frac{-\cos^2\theta}{\sin^2\theta}\right)$$

$$\left(\frac{2}{\sin\theta}\right)\left(-\frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$\frac{-2\sin^2\theta}{\sin\theta\cos^2\theta}$$

$$\frac{-2\sin\theta}{\cos^2\theta}$$

$$-\frac{2\sin\theta}{\cos^2\theta}$$

Prove the identity:

$$\frac{1}{\sec^2\theta\cot\theta} = \frac{\sin\theta - \sin^3\theta}{\cos\theta}$$

$\frac{1}{\sec^2\theta} \cdot \frac{1}{\cot\theta}$

$\cos^2\theta \cdot \frac{\sin\theta}{\cos\theta}$

$\sin\theta\cos\theta$

$\frac{\sin\theta(1-\sin^2\theta)}{\cos\theta}$

$\frac{\sin\theta\cos^2\theta}{\cos\theta}$

$\sin\theta\cos\theta$

# Trig Identities #4

④ $\tan^2\theta - \cot^2\theta = \sec^2\theta - \csc^2\theta$

$\sec^2\theta - 1 - (\csc^2\theta - 1)$

$\sec^2\theta - 1 - \csc^2\theta + 1$

$\sec^2\theta - \csc^2\theta$

⑤ $\sec^2\theta - \sin^2\theta = \cos^2\theta + \tan^2\theta$

$\tan^2\theta + 1 - (1 - \cos^2\theta)$

$\tan^2\theta + 1 - 1 + \cos^2\theta$

$\cos^2\theta + \tan^2\theta$

⑥ $\dfrac{\sin^4\theta - \cos^4\theta}{\sin^2\theta\cos^2\theta - \cos^4\theta} = \dfrac{\csc^2\theta}{\cot^2\theta}$

$\dfrac{(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)}{\cos^2\theta(\sin^2\theta - \cos^2\theta)}$

$\dfrac{1}{\cos^2\theta}$

$\dfrac{1}{\sin^2\theta} \div \dfrac{\cos^2\theta}{\sin^2\theta}$

$\dfrac{1}{\sin^2\theta} \times \dfrac{\sin^2\theta}{\cos^2\theta}$

$\dfrac{1}{\cos^2\theta}$

# Review

(32) $\dfrac{\cos y}{(1+\sin y)} + \dfrac{1+\sin y}{(\cos y)} = 2\sec y$

$\dfrac{\cos^2 y + (1+\sin y)^2}{\cos y(1+\sin y)}$

$\dfrac{\cos^2 y + 1 + 2\sin y + \sin^2 y}{\cos y(1+\sin y)}$

Factor $\rightarrow$ $\dfrac{2+2\sin y}{\cos y(1+\sin y)}$

$\dfrac{2(1+\sin y)}{\cos y(1+\sin y)}$

$\dfrac{2}{\cos y}$

$2\left(\dfrac{1}{\cos y}\right)$

$\dfrac{2}{\cos y}$

(33) $\sin(x+y)+\sin(x-y) = 2\sin x\cos y$

$\sin x\cos y + \cos x\sin y + \sin x\cos y - \cos x\sin y$

$2\sin x\cos y$

$2\sin x\cos y$

(28) $\sin 2\theta(1-\cos 2\theta) = 4\sin^3\theta\cos\theta$

$2\sin\theta\cos\theta\left(1-(\cos^2\theta-\sin^2\theta)\right)$

$2\sin\theta\cos\theta\left(1-\cos^2\theta+\sin^2\theta\right)$

$2\sin\theta\cos\theta\left(\sin^2\theta+\sin^2\theta\right)$

$(2\sin\theta\cos\theta)(2\sin^2\theta)$

$4\sin^3\theta\cos\theta$

$4\sin^3\theta\cos\theta$

(15) $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\cos^2\theta - (1 - \cos^2\theta)$

$\cos^2\theta - 1 + \cos^2\theta$

$2\cos^2\theta - 1$

$2\cos^2\theta - 1$

(16) Diff. of Squares

$\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$(1)(\cos^2\theta - \sin^2\theta)$

$(\cos^2\theta - \sin^2\theta)$

(20) $\frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)} = 2\sec^2\theta$

$\frac{1-\sin\theta + 1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$\frac{2}{(1-\sin^2\theta)}$

$\frac{2}{\cos^2\theta}$

$2\left(\frac{1}{\cos^2\theta}\right)$

$\frac{2}{\cos^2\theta}$

(a) $\tan\theta + \frac{1}{\tan\theta} = \csc\theta\sec\theta$

$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\frac{1}{\sin\theta\cos\theta}$

$\left(\frac{1}{\sin\theta}\right)\left(\frac{1}{\cos\theta}\right)$

$\frac{1}{\sin\theta\cos\theta}$

(29) $\cos^4\theta = 1 - 2\sin^2\theta + \sin^4\theta$

$(\cos^2\theta)(\cos^2\theta)$

$(1-\sin^2\theta)(1-\sin^2\theta)$

$1 - 2\sin^2\theta + \sin^4\theta$

$1 - 2\sin^2\theta + \sin^4\theta$

(17) $\dfrac{1-\sin^2\theta}{\csc^2\theta-1} = \sin^2\theta$

$\dfrac{\cos^2\theta}{\cot^2\theta}$

$\dfrac{\cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta}}$

$\cancel{\cos^2\theta} \cdot \dfrac{\sin^2\theta}{\cancel{\cos^2\theta}}$

$\sin^2\theta$

# Review Period 2

(15) $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\cos^2\theta - (1 - \cos^2\theta)$

$\cos^2\theta - 1 + \cos^2\theta$

$2\cos^2\theta - 1$

(6) $\cos\theta \sin\theta \cot\theta = \cos^2\theta$

$(\cos\theta)(\sin\theta)\left(\frac{\cos\theta}{\sin\theta}\right)$

$\cos^2\theta$

(16) Diff of Squares

$\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$ (the $\cos^2\theta + \sin^2\theta = 1$)

$(\cos^2\theta - \sin^2\theta)(1)$

$\cos^2\theta - \sin^2\theta$

(a) $\frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)} = 2\sec^2\theta$

$\frac{1-\sin\theta + 1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)} = 2\left(\frac{1}{\cos^2\theta}\right)$

$\frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$

$\frac{2}{\cos^2\theta}$

(29) $\cos^4\theta = 1 - 2\sin^2\theta + \sin^4\theta$

$(\cos^2\theta)(\cos^2\theta)$

$(1-\sin^2\theta)(1-\sin^2\theta)$

$1 - 2\sin^2\theta + \sin^4\theta$

(24) $\dfrac{\sec\theta - 1}{\sec\theta + 1} = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

$(\sec\theta - 1)(1 + \cos\theta)$

$\sec\theta + \cos\theta\sec\theta - 1 - \cos\theta$

$\sec\theta + \cancel{\cos\theta}\left(\dfrac{1}{\cancel{\cos\theta}}\right) - 1 - \cos\theta$

$\sec\theta + 1 - 1 - \cos\theta$

$\sec\theta - \cos\theta$

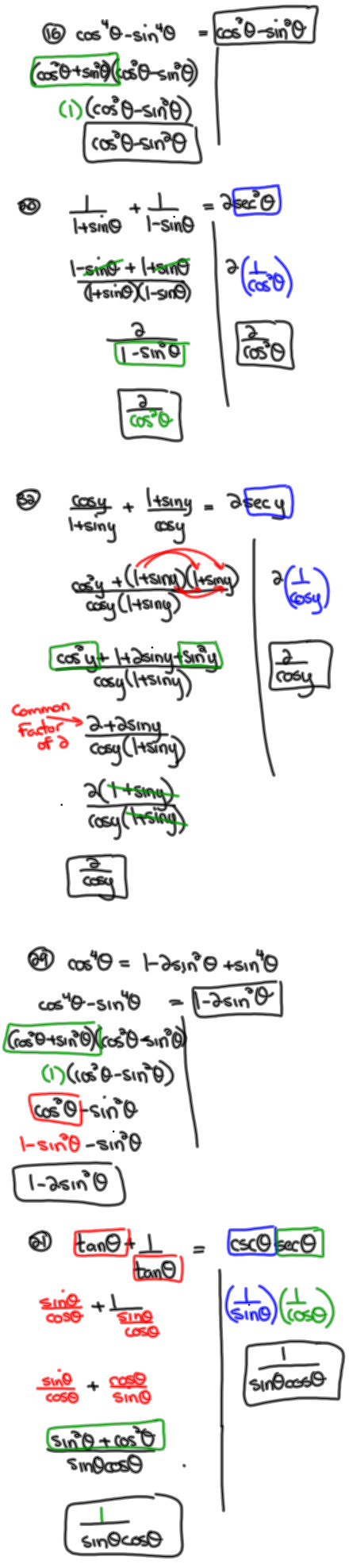
$(\sec\theta + 1)(1 - \cos\theta)$

$\sec\theta - \cos\theta\sec\theta + 1 - \cos\theta$

$\sec\theta - \cancel{\cos\theta}\left(\dfrac{1}{\cancel{\cos\theta}}\right) + 1 - \cos\theta$

$\sec\theta - 1 + 1 - \cos\theta$

$\sec\theta - \cos\theta$


16 $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$
$(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)$
$(1)(\cos^2\theta-\sin^2\theta)$
$\cos^2\theta-\sin^2\theta$
20 $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta} = 2\sec^2\theta$
$\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$
$2\left(\frac{1}{\cos^2\theta}\right)$
$\frac{2}{1-\sin^2\theta}$
$\frac{2}{\cos^2\theta}$
$\frac{2}{\cos^2\theta}$
23 $\frac{\cos y}{1+\sin y}+\frac{1+\sin y}{\cos y} = 2\sec y$
$\frac{\cos^2 y+(1+\sin y)(1+\sin y)}{\cos y(1+\sin y)}$
$2\left(\frac{1}{\cos y}\right)$
$\frac{\cos^2 y+1+2\sin y+\sin^2 y}{\cos y(1+\sin y)}$
$\frac{2}{\cos y}$
Common Factor of 2
$\frac{2+2\sin y}{\cos y(1+\sin y)}$
$\frac{2(1+\sin y)}{\cos y(1+\sin y)}$
$\frac{2}{\cos y}$
29 $\cos^4\theta = 1-2\sin^2\theta+\sin^4\theta$
$\cos^4\theta-\sin^4\theta = 1-2\sin^2\theta$
$(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)$
$(1)(\cos^2\theta-\sin^2\theta)$
$\cos^2\theta-\sin^2\theta$
$1-\sin^2\theta-\sin^2\theta$
$1-2\sin^2\theta$
31 $\tan\theta+\frac{1}{\tan\theta} = \csc\theta\sec\theta$
$\frac{\sin\theta}{\cos\theta}+\frac{1}{\frac{\sin\theta}{\cos\theta}}$
$\left(\frac{1}{\sin\theta}\right)\left(\frac{1}{\cos\theta}\right)$
$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$
$\frac{1}{\sin\theta\cos\theta}$
$\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$
$\frac{1}{\sin\theta\cos\theta}$

(24) $\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{1 - \cos\theta}{1 + \cos\theta}$

$(\sec\theta - 1)(1 + \cos\theta)$

$\sec\theta + \sec\theta\cos\theta - 1 - \cos\theta$

$\sec\theta + \left(\frac{1}{\cos\theta}\right)(\cos\theta) - 1 - \cos\theta$

$\sec\theta + 1 - 1 - \cos\theta$

$\sec\theta - \cos\theta$

$= (1 - \cos\theta)(\sec\theta + 1)$

$\sec\theta + 1 - \cos\theta\sec\theta - \cos\theta$

$\sec\theta + 1 - \cos\theta\left(\frac{1}{\cos\theta}\right) - \cos\theta$

$\sec\theta + 1 - 1 - \cos\theta$

$\sec\theta - \cos\theta$

(16) $\cos^4\theta - \sin^4\theta =$ $\cos^2\theta - \sin^2\theta$

$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$(1)(\cos^2\theta - \sin^2\theta)$

$\cos^2\theta - \sin^2\theta$

(15) $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\cos^2\theta - (1 - \cos^2\theta)$

$\cos^2\theta - 1 + \cos^2\theta$

$2\cos^2\theta - 1$

(15) $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$1 - \sin^2\theta = 2\cos^2\theta - \cos^2\theta$

$\cos^2\theta$ → Pythagorean

$1 - \sin^2\theta$

(15) $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta - (\sin^2\theta + \cos^2\theta)$

$2\cos^2\theta - \sin^2\theta - \cos^2\theta$

$\cos^2\theta - \sin^2\theta$

# Final Review

③ $\sin(x+y)\sin(x-y) = \cos^2 y - \cos^2 x$

$(\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$

$\sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$(1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)$

$\cos^2 y - \cos^2 x\cos^2 y - \cos^2 x + \cos^2 x\cos^2 y$

$\cos^2 y - \cos^2 x$

$\cos^2 y - \cos^2 x$

Final Review

④ $\cos^4\theta = 1 - 2\sin^2\theta + \sin^4\theta$ ← Factor

$= (1-\sin^2\theta)(1-\sin^2\theta)$ ← Pythagorean Identity

$= (\cos^2\theta)(\cos^2\theta)$

$= \cos^4\theta$

⑤ $\tan^4\theta = \sec^4\theta(1 - 2\cos^2\theta + \cos^4\theta)$ ← Factor

$\frac{\sin^4\theta}{\cos^4\theta}$

$\frac{1}{\cos^4\theta}(1-\cos^2\theta)(1-\cos^2\theta)$ ← Pythag

$\frac{1}{\cos^4\theta}(\sin^2\theta)(\sin^2\theta)$

$\frac{\sin^4\theta}{\cos^4\theta}$

⑥ $\tan\theta = \frac{1+\sin\theta-\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$\frac{\sin\theta}{\cos\theta}$

$\frac{\sin^2\theta+\sin\theta}{\cos\theta(1+\sin\theta)}$ ← Factor

$\frac{\sin\theta(\sin\theta+1)}{\cos\theta(1+\sin\theta)}$

(7) $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2\sec\theta$

$\frac{\cos^2\theta + (1+\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

Pythag

$\frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$

Factor

$\frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$

$\frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$\frac{2}{\cos\theta}$

$2\left[\frac{1}{\cos\theta}\right]$

$\frac{2}{\cos\theta}$

(12) $\sin(x+y) - \sin(x-y) = 2\cos x\sin y$

$\sin x\cos y + \cos x\sin y - (\sin x\cos y - \cos x\sin y)$

$2\cos x\sin y$

$2\cos x\sin y$

(16) $\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta\cos^2\theta - \cos^4\theta} = \frac{\csc^2\theta}{\cot^2\theta}$

$\frac{(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)}{\cos^2\theta(\sin^2\theta-\cos^2\theta)}$

$\frac{1}{\cos^2\theta}$

$\frac{1}{\sin^2\theta} \div \frac{\cos^2\theta}{\sin^2\theta}$

$\frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$

$\frac{1}{\cos^2\theta}$

(30) $\cos(x+y) - \cos(x-y) = -2\sin x \sin y$

$\cos x\cos y - \sin x\sin y - (\cos x\cos y + \sin x\sin y) = -2\sin x\sin y$

$\cos x\cos y - \sin x\sin y - \cos x\cos y - \sin x\sin y$

$\boxed{-2\sin x\sin y}$

(31) $\dfrac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$

$\dfrac{1+(\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta}$ | $\dfrac{\cos\theta}{\sin\theta}$

$\dfrac{1+\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$

$\dfrac{\cos^2\theta + \cos^2\theta}{2\sin\theta\cos\theta}$

$\dfrac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$\boxed{\dfrac{\cos\theta}{\sin\theta}}$

(32) $\dfrac{\cos y}{1+\sin y} + \dfrac{1+\sin y}{\cos y} = 2\sec y$

FOIL

$\dfrac{\cos^2 y + (1+\sin y)(1+\sin y)}{\cos y(1+\sin y)}$

$\dfrac{\cos^2 y + 1 + 2\sin y + \sin^2 y}{\cos y(1+\sin y)}$

$\dfrac{2+2\sin y}{\cos y(1+\sin y)}$

$\dfrac{2(1+\sin y)}{\cos y(1+\sin y)}$

$2\left(\dfrac{1}{\cos y}\right)$

$\dfrac{2}{\cos y}$


8
cosx/(secx-1) - cosx/tan²x = cot²x
cosx tan²x / tan²x(secx-1) - cosx(secx-1) / tan²x(secx-1)
1/tan²x
C.F.
cosx tan²x - cosx secx + cosx / tan²x(secx-1)
Pyth.
cosx(tan²x - secx + 1) / tan²x(secx-1)
C.F
cosx(sec²x - secx) / tan²x(secx-1)
cosx secx(secx-1) / tan²x(secx-1)
cosx (1/cosx) / tan²x
1/tan²x

⑤ $\tan^4\theta = \sec^4\theta(1-2\cos^2\theta+\cos^4\theta)$

$\frac{\sin^4\theta}{\cos^4\theta}$ | $\frac{1}{\cos^4\theta}(1-\cos^2\theta)(1-\cos^2\theta)$

$\frac{1}{\cos^4\theta}(\sin^2\theta)(\sin^2\theta)$

$\frac{\sin^4\theta}{\cos^4\theta}$