

Trig Identities # 4

$$\textcircled{4} \quad \underline{\tan^2 \theta} - \cot^2 \theta = \sec^2 \theta - \csc^2 \theta$$

$$\begin{array}{l} \sec^2 \theta - 1 - (\csc^2 \theta - 1) \\ \sec^2 \theta - 1 - \csc^2 \theta + 1 \\ \sec^2 \theta - \csc^2 \theta \end{array}$$

$$\textcircled{5} \quad \underline{\sec^2 \theta} - \underline{\sin^2 \theta} = \cos^2 \theta + \tan^2 \theta$$

$$\begin{array}{l} \tan^2 \theta + 1 - (1 - \cos^2 \theta) \\ \tan^2 \theta + 1 - 1 + \cos^2 \theta \\ \cos^2 \theta + \tan^2 \theta \end{array}$$

$$\textcircled{8} \quad \frac{\sin^4 \theta - \cos^4 \theta}{\sin^2 \theta \cos^2 \theta - \cos^4 \theta} = \frac{\csc^2 \theta}{\cot^2 \theta}$$

$$\frac{(\cancel{\sin^2 \theta - \cos^2 \theta})(\sin^2 \theta + \cos^2 \theta)}{\cos^2 \theta (\cancel{\sin^2 \theta - \cos^2 \theta})}$$

$$\frac{1}{\cos^2 \theta}$$

$$\frac{1}{\sin^2 \theta} \div \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$\frac{1}{\cancel{\sin^2 \theta}} \times \frac{\cancel{\sin^2 \theta}}{\cos^2 \theta}$$

$$\frac{1}{\cos^2 \theta}$$

Review

$$\textcircled{32} \quad \frac{\cos y}{(1+\sin y)} + \frac{1+\sin y}{(\cos y)} = 2 \boxed{\sec y}$$

$$\frac{\cos^2 y + \cancel{(1+\sin y)^2}}{\cos y(1+\sin y)}$$

$$\frac{\underline{\cos^2 y} + \underline{1} + \underline{2\sin y} + \underline{\sin^2 y}}{\cos y(1+\sin y)}$$

factor \rightarrow $\frac{2 + 2\sin y}{\cos y(1+\sin y)}$

$$\frac{\cancel{2(1+\sin y)}}{\cos y \cancel{(1+\sin y)}}$$

$$\boxed{\frac{2}{\cos y}}$$

$$2 \left(\frac{1}{\cos y} \right)$$

$$\boxed{\frac{2}{\cos y}}$$

$$(33) \quad \boxed{\sin(x+y)} + \boxed{\sin(x-y)} = 2\sin x \cos y$$

$$\sin x \cos y + \cancel{\cos x \sin y} + \sin x \cos y - \cancel{\cos x \sin y}$$

$$\boxed{2\sin x \cos y}$$

$$\boxed{2\sin x \cos y}$$

$$(28) \quad \boxed{\sin 2\theta} (1 - \boxed{\cos 2\theta}) = 4\sin^3 \theta \cos \theta$$

$$2\sin \theta \cos \theta (1 - (\cos^2 \theta - \sin^2 \theta))$$

$$2\sin \theta \cos \theta (1 - \underline{\cos^2 \theta} + \sin^2 \theta)$$

$$2\sin \theta \cos \theta (\sin^2 \theta + \sin^2 \theta)$$

$$(2\sin \theta \cos \theta)(2\sin^2 \theta)$$

$$\boxed{4\sin^3 \theta \cos \theta}$$

$$\boxed{4\sin^3 \theta \cos \theta}$$

$$\textcircled{15} \quad \cos^2\theta - \boxed{\sin^2\theta} = 2\cos^2\theta - 1$$

$$\cos^2\theta - (1 - \cos^2\theta)$$

$$\cos^2\theta - 1 + \cos^2\theta$$

$$\boxed{2\cos^2\theta - 1}$$

$$\boxed{2\cos^2\theta - 1}$$

Diff. of Squares

$$\textcircled{16} \quad \cos^4\theta - \sin^4\theta = \boxed{\cos^2\theta - \sin^2\theta}$$

$$\boxed{(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)}$$

$$(1)(\cos^2\theta - \sin^2\theta)$$

$$\boxed{(\cos^2\theta - \sin^2\theta)}$$

20

$$\frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)} = 2\sec^2\theta$$

$$\frac{\cancel{1-\sin\theta} + \cancel{1+\sin\theta}}{(1+\sin\theta)(1-\sin\theta)}$$

$$\frac{2}{(1-\sin^2\theta)}$$

$$\frac{2}{\cos^2\theta}$$

$$2 \left(\frac{1}{\cos^2\theta} \right)$$

$$\frac{2}{\cos^2\theta}$$

$$\textcircled{21} \quad \boxed{\tan\theta} + \boxed{\frac{1}{\tan\theta}} = \boxed{\csc\theta} \boxed{\sec\theta}$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\left(\frac{1}{\sin\theta}\right) \left(\frac{1}{\cos\theta}\right)$$

$$\boxed{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}}$$

$$\boxed{\frac{1}{\sin\theta\cos\theta}}$$

$$\boxed{\frac{1}{\sin\theta\cos\theta}}$$

$$\textcircled{29} \quad \underline{\cos^4 \theta} = 1 - 2\sin^2 \theta + \sin^4 \theta$$

$$\underline{(\cos^2 \theta)}(\underline{\cos^2 \theta})$$

$$(1 - \sin^2 \theta)(1 - \sin^2 \theta)$$

$$1 - 2\sin^2 \theta + \sin^4 \theta$$

$$1 - 2\sin^2 \theta + \sin^4 \theta$$

$$(17) \frac{1 - \sin^2 \theta}{\csc^2 \theta - 1} = \sin^2 \theta$$

$$\frac{\cos^2 \theta}{\cot^2 \theta}$$

$$\frac{\cos^2 \theta}{\frac{\cos^2 \theta}{\sin^2 \theta}}$$

$$\cancel{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{\cancel{\cos^2 \theta}}$$

$$\sin^2 \theta$$

Review Period 2

$$\textcircled{15} \quad \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$

$$\cos^2 \theta - (1 - \cos^2 \theta)$$

$$\cos^2 \theta - 1 + \cos^2 \theta$$

$$2\cos^2 \theta - 1$$

$$\textcircled{16} \quad \cos \theta \sin \theta \cot \theta = \cos^2 \theta$$

$$(\cos \theta) \cancel{\sin \theta} \left(\frac{\cos \theta}{\cancel{\sin \theta}} \right)$$

$$\cos^2 \theta$$

$$\textcircled{16} \quad \cos^4 \theta - \sin^4 \theta = \cos^2 \theta - \sin^2 \theta$$

D.F.F of Squares

$$(\cos^2 \theta - \sin^2 \theta) (\cos^2 \theta + \sin^2 \theta)$$

$$(\cos^2 \theta - \sin^2 \theta) (1)$$

$$\cos^2 \theta - \sin^2 \theta$$

$$\begin{aligned}
 & \textcircled{20} \quad \frac{1}{(1+\sin\theta)} + \frac{1}{(1-\sin\theta)} = 2 \sec^2\theta \\
 & \frac{\cancel{1-\sin\theta} + \cancel{1+\sin\theta}}{(1+\sin\theta)(1-\sin\theta)} \\
 & \frac{2}{1-\sin^2\theta} \\
 & \frac{2}{\cos^2\theta} \\
 & = 2 \left(\frac{1}{\cos^2\theta} \right) \\
 & \frac{2}{\cos^2\theta}
 \end{aligned}$$

$$\textcircled{29} \quad \cos^4 \theta =$$

$$1 - 2\sin^2 \theta + \sin^4 \theta$$

$$\underline{\cos^2 \theta} \underline{\cos^2 \theta}$$

$$(1 - \sin^2 \theta)(1 - \sin^2 \theta)$$

$$1 - 2\sin^2 \theta + \sin^4 \theta$$

$$\textcircled{24} \quad \frac{\sec\theta - 1}{\sec\theta + 1} \stackrel{=}{\neq} \frac{1 - \cos\theta}{1 + \cos\theta}$$

$(\sec\theta - 1)(1 + \cos\theta)$ $\sec\theta + \cos\theta \boxed{\sec\theta} - 1 - \cos\theta$ $\sec\theta + \cancel{\cos\theta} \left(\frac{1}{\cancel{\cos\theta}}\right) - 1 - \cos\theta$ $\sec\theta + 1 - 1 - \cos\theta$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\sec\theta - \cos\theta$ </div>	$(\sec\theta + 1)(1 - \cos\theta)$ $\sec\theta - \cos\theta \boxed{\sec\theta} + 1 - \cos\theta$ $\sec\theta - \cancel{\cos\theta} \left(\frac{1}{\cancel{\cos\theta}}\right) + 1 - \cos\theta$ $\sec\theta - 1 + 1 - \cos\theta$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\sec\theta - \cos\theta$ </div>
---	---

$$\textcircled{16} \cos^4 \theta - \sin^4 \theta = \boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\frac{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{(1)(\cos^2 \theta - \sin^2 \theta)} = \boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\textcircled{17} \frac{1}{1+\sin \theta} + \frac{1}{1-\sin \theta} = \boxed{2 \sec^2 \theta}$$

$$\frac{1-\sin \theta + 1+\sin \theta}{(1+\sin \theta)(1-\sin \theta)} = 2 \left(\frac{1}{\cos^2 \theta} \right)$$

$$\frac{2}{1-\sin^2 \theta} = \boxed{\frac{2}{\cos^2 \theta}}$$

$$\textcircled{18} \frac{\cos y}{1+\sin y} + \frac{1+\sin y}{\cos y} = \boxed{2 \sec y}$$

$$\frac{\cos y + (1+\sin y)(1+\sin y)}{\cos y(1+\sin y)} = 2 \left(\frac{1}{\cos y} \right)$$

$$\frac{\cos^2 y + 1 + 2\sin y + \sin^2 y}{\cos y(1+\sin y)} = \frac{2 + 2\sin y}{\cos y(1+\sin y)}$$

Common Factor of 2

$$\frac{2(1+\sin y)}{\cos y(1+\sin y)} = \boxed{\frac{2}{\cos y}}$$

$$\textcircled{19} \cos^4 \theta = 1 - 2\sin^2 \theta + \sin^4 \theta$$

$$\cos^4 \theta - \sin^4 \theta = \boxed{1 - 2\sin^2 \theta}$$

$$\frac{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{(1)(\cos^2 \theta - \sin^2 \theta)} = \boxed{1 - 2\sin^2 \theta}$$

$$\textcircled{20} \tan \theta + \frac{1}{\tan \theta} = \boxed{\csc \theta \sec \theta}$$

$$\frac{\sin \theta}{\cos \theta} + \frac{1}{\frac{\sin \theta}{\cos \theta}} = \left(\frac{1}{\sin \theta} \right) \left(\frac{1}{\cos \theta} \right)$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta}$$

$$\textcircled{a)} \quad \frac{\sec\theta - 1}{\sec\theta + 1} \quad \begin{array}{c} \swarrow \searrow \\ \nwarrow \nearrow \end{array} \quad \frac{1 - \cos\theta}{1 + \cos\theta}$$

$$(\sec\theta - 1)(1 + \cos\theta)$$

$$\sec\theta + \boxed{\sec\theta \cos\theta} - 1 - \cos\theta$$

$$\sec\theta + \left(\frac{1}{\cancel{\cos\theta}}\right) \cancel{\cos\theta} - 1 - \cos\theta$$

$$\sec\theta + 1 - 1 - \cos\theta$$

$$\boxed{\sec\theta - \cos\theta}$$

$$= (1 - \cos\theta)(\sec\theta + 1)$$

$$\sec\theta + 1 - \cos\theta \boxed{\sec\theta} - \cos\theta$$

$$\sec\theta + 1 - \cancel{\cos\theta} \left(\frac{1}{\cancel{\cos\theta}}\right) - \cos\theta$$

$$\sec\theta + 1 - 1 - \cos\theta$$

$$\boxed{\sec\theta - \cos\theta}$$

$$\textcircled{16} \cos^4 \theta - \sin^4 \theta = \boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\boxed{(\cos^2 \theta + \sin^2 \theta)} (\cos^2 \theta - \sin^2 \theta)$$

$$\textcircled{1} (\cos^2 \theta - \sin^2 \theta)$$

$$\boxed{\cos^2 \theta - \sin^2 \theta}$$

$$\textcircled{15} \cos^2 \theta - \boxed{\sin^2 \theta} = 2\cos^2 \theta - 1$$

$$\cos^2 \theta - (1 - \cos^2 \theta)$$

$$\cos^2 \theta - 1 + \cos^2 \theta$$

$$2\cos^2 \theta - 1$$

$$\textcircled{15} \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$

$$\boxed{1 - \sin^2 \theta} = 2\cos^2 \theta - \cos^2 \theta$$

$$\boxed{\cos^2 \theta} \rightarrow \text{Pythagorean}$$

$$\boxed{1 - \sin^2 \theta}$$

$$\textcircled{15} \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - \boxed{1}$$

$$2\cos^2 \theta - (\sin^2 \theta + \cos^2 \theta)$$

$$2\cos^2 \theta - \sin^2 \theta - \cos^2 \theta$$

$$\cos^2 \theta - \sin^2 \theta$$

Final Review

$$\textcircled{3} \quad \boxed{\sin(x+y)} \boxed{\sin(x-y)} = \cos^2 y - \cos^2 x$$

$$\boxed{(\sin x \cos y + \cos x \sin y)} \boxed{(\sin x \cos y - \cos x \sin y)} \quad \boxed{\cos^2 y - \cos^2 x}$$

$$\boxed{\sin^2 x \cos^2 y - \cos^2 x \sin^2 y}$$

$$\boxed{(1 - \cos^2 x) \cos^2 y - \cos^2 x (1 - \cos^2 y)}$$

$$\cos^2 y - \cancel{\cos^2 x \cos^2 y} - \cos^2 x + \cancel{\cos^2 x \cos^2 y}$$

$$\boxed{\cos^2 y - \cos^2 x}$$

Final Review

$$\begin{aligned}
 \textcircled{4} \quad \boxed{\cos^4 \theta} &= 1 - 2\sin^2 \theta + \sin^4 \theta \quad \leftarrow \text{Factor} \\
 &= (1 - \sin^2 \theta)(1 + \sin^2 \theta) \quad \leftarrow \text{Pythagorean Identity} \\
 &= (\cos^2 \theta)(\cos^2 \theta) \\
 &= \boxed{\cos^4 \theta}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \quad \boxed{\tan^4 \theta} &= \boxed{\sec^4 \theta} (1 - 2\cos^2 \theta + \cos^4 \theta) \quad \leftarrow \text{Factor} \\
 \boxed{\frac{\sin^4 \theta}{\cos^4 \theta}} & \quad \frac{1}{\cos^4 \theta} \boxed{(1 - \cos^2 \theta)(1 - \cos^2 \theta)} \quad \leftarrow \text{Pythag} \\
 & \quad \frac{1}{\cos^4 \theta} (\sin^2 \theta)(\sin^2 \theta) \\
 & \quad \boxed{\frac{\sin^4 \theta}{\cos^4 \theta}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad \boxed{\tan \theta} &= \frac{1 + \sin \theta - \cos^2 \theta}{\cos \theta (1 + \sin \theta)} \\
 \boxed{\frac{\sin \theta}{\cos \theta}} & \quad \frac{\sin^2 \theta + \sin \theta}{\cos \theta (1 + \sin \theta)} \quad \leftarrow \text{Factor} \\
 & \quad \frac{\sin \theta (\cancel{\sin \theta + 1})}{\cos \theta (\cancel{1 + \sin \theta})}
 \end{aligned}$$

$$\textcircled{7} \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = 2 \boxed{\sec\theta}$$

$$\frac{\cos^2\theta + (1+\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)} \quad \left[\frac{1}{\cos\theta} \right]$$

$$\frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)} \quad \left[\frac{2}{\cos\theta} \right]$$

Pythag

$$\frac{2 + 2\sin\theta}{\cos\theta(1+\sin\theta)} \quad \leftarrow \text{Factor}$$

$$\frac{2 \cancel{(1+\sin\theta)}}{\cos\theta \cancel{(1+\sin\theta)}}$$

$$\boxed{\frac{2}{\cos\theta}}$$

$$\textcircled{12} \sin(x+y) - \sin(x-y) = 2 \cos x \sin y$$

$$\sin x \cos y + \cos x \sin y - (\sin x \cos y - \cos x \sin y) = 2 \cos x \sin y$$

$\boxed{2 \cos x \sin y}$

$$\textcircled{16} \frac{\sin^4\theta - \cos^4\theta}{\sin^3\theta \cos^3\theta - \cos^4\theta} = \frac{\boxed{\csc^2\theta}}{\boxed{\cot^2\theta}}$$

$$\frac{(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)}{\cos^3\theta(\sin^2\theta - \cos^2\theta)}$$

$$\boxed{\frac{1}{\cos^2\theta}}$$

$$\frac{1}{\sin^2\theta} \div \frac{\cos^2\theta}{\sin^2\theta}$$

$$\frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$

$$\boxed{\frac{1}{\cos^2\theta}}$$

$$\textcircled{30} \quad \underline{\cos(x+y)} - \underline{\cos(x-y)} = -2\sin x \sin y$$

$$\cos x \cos y - \sin x \sin y - (\cos x \cos y + \sin x \sin y) = -2\sin x \sin y$$

$$\cancel{\cos x \cos y} - \sin x \sin y - \cancel{\cos x \cos y} - \sin x \sin y$$

$$\boxed{-2\sin x \sin y}$$

$$\textcircled{31} \quad \frac{1 + \boxed{\cos^2 \theta}}{\boxed{\sin^2 \theta}} = \boxed{\cot^2 \theta}$$

$$\frac{1 + (\cos^2 \theta - \sin^2 \theta)}{2\sin \theta \cos \theta} \quad \left| \quad \frac{\cos \theta}{\sin \theta}$$

$$\frac{1 + \cos^2 \theta - \sin^2 \theta}{2\sin \theta \cos \theta}$$

$$\frac{\cos^2 \theta + \cos^2 \theta}{2\sin \theta \cos \theta}$$

$$\frac{\cancel{2\cos^2 \theta}}{\cancel{2\sin \theta \cos \theta}}$$

$$\boxed{\frac{\cos \theta}{\sin \theta}}$$

$$\textcircled{3a} \quad \frac{\cos y}{1+\sin y} + \frac{1+\sin y}{\cos y} = 2 \boxed{\sec y}$$

$$\frac{\overset{\text{FOIL}}{\cos^2 y + (1+\sin y)(1+\sin y)}}{\cos y(1+\sin y)} \quad \left| \quad 2 \left(\frac{1}{\cos y} \right) \right.$$

$$\frac{\cos^2 y + 1 + 2\sin y + \sin^2 y}{\cos y(1+\sin y)} \quad \left| \quad \frac{2}{\cos y} \right.$$

$$\frac{2 + 2\sin y}{\cos y(1+\sin y)}$$

$$\frac{2 \cancel{(1+\sin y)}}{\cos y \cancel{(1+\sin y)}}$$

$$\textcircled{8} \quad \frac{\cos x}{\sec x - 1} - \frac{\cos x}{\tan^2 x} = \underline{\underline{\cot^2 x}}$$

$$\frac{\cos x \tan^2 x}{\tan^2 x (\sec x - 1)} - \frac{\cos x (\sec x - 1)}{\tan^2 x (\sec x - 1)}$$

$$\frac{1}{\tan^2 x}$$

C.F. →

$$\frac{\cos x \tan^2 x - \cos x \sec x + \cos x}{\tan^2 x (\sec x - 1)}$$

Pyth. ↙

$$\frac{\cos x (\tan^2 x - \sec x + 1)}{\tan^2 x (\sec x - 1)}$$

← C.F.

$$\frac{\cos x (\sec^2 x - \sec x)}{\tan^2 x (\sec x - 1)}$$

$$\frac{\cos x \cancel{\sec x} (\cancel{\sec x} - 1)}{\tan^2 x (\cancel{\sec x} - 1)}$$

$$\frac{\cos x \left(\frac{1}{\cos x} \right)}{\tan^2 x}$$

$$\frac{1}{\tan^2 x}$$

$$\textcircled{5} \tan^4 \theta = \sec^4 \theta (1 - 2\cos^2 \theta + \cos^4 \theta)$$
$$\frac{\sin^4 \theta}{\cos^4 \theta} \quad \left| \quad \frac{1}{\cos^4 \theta} (1 - \cos^2 \theta)(1 - \cos^2 \theta) \right.$$
$$\left. \frac{1}{\cos^4 \theta} (\sin^2 \theta)(\sin^2 \theta) \right.$$
$$\frac{\sin^4 \theta}{\cos^4 \theta}$$