Exam Review
Answer Section

COMPLETION

1. ANS: –6

DIF: Average

2. ANS: 6

DIF: Average

3. ANS: $\{x \mid x \le -\sqrt{7} \text{ or } x \ge \sqrt{7}, x \in \mathbb{R}\}$

DIF: Average

MATCHING

1.	ANS: A	DIF:	Average
2.	ANS: C	DIF:	Average
3.	ANS: E	DIF:	Average
4.	ANS: D	DIF:	Average
5.	ANS: C	DIF:	Easy
6.	ANS: A	DIF:	Easy
7.	ANS: D	DIF:	Easy
8.	ANS: E	DIF:	Easy
9.	ANS: B	DIF:	Easy

SHORT ANSWER

1. ANS:

a) $g(x) = (x-4)^2 + 3$

$= x^2 - 8x + 16 + 3$

$= x^2 - 8x + 19$

b) $g(x) = (x+2)^2 + 1$

$= x^2 + 4x + 4 + 1$

$= x^2 + 4x + 5$

c) $g(x) = (x+7)^2 - 2$

$= x^2 + 14x + 49 - 2$

$= x^2 + 14x + 47$

DIF: Average

2. ANS:

a) $g(x) = -f(x)$

$= -(x-1)^2 - 2$

b) $g(x) = f(-x)$

$= |-x| + 1$

$= |x| + 1$

DIF: Average

3. ANS:

a) a reflection in the *x*-axis, a horizontal compression by a factor of $\frac{1}{2}$, and then a translation of 1 unit to the left and 2 units down

b) a vertical stretch by a factor of 2, and then a translation of 3 units to the right and 4 units down

c) reflections in the *x*-axis and the *y*-axis, a vertical compression by a factor of $\frac{1}{2}$, and then a translation of 5 units to the right and 1 unit up

DIF: Difficult +

4. ANS:

a) i)

$$y = \frac{5}{2}x - 3$$

$$x = \frac{5}{2}y - 3$$

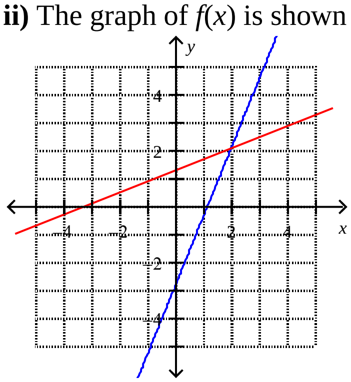
$$x + 3 = \frac{5}{2}y$$

$$2x + 6 = 5y$$

$$y = \frac{2}{5}x + \frac{6}{5}$$

$$f^{-1}(x) = \frac{2}{5}x + \frac{6}{5}$$

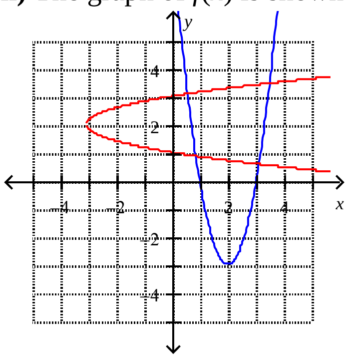
ii) The graph of *f*(*x*) is shown in blue and the graph of $f^{-1}(x)$ is shown in red.



b) i)

$$y = 3(x-2)^2 - 3$$
$$x = 3(y-2)^2 - 3$$
$$x + 3 = 3(y-2)^2$$
$$\frac{x+3}{3} = (y-2)^2$$
$$\pm\sqrt{\frac{x+3}{3}} = y - 2$$
$$y = 2 \pm \sqrt{\frac{x+3}{3}}$$
$$f^{-1}(x) = 2 \pm \sqrt{\frac{x+3}{3}}$$

ii) The graph of *f*(*x*) is shown in blue and the graph of $f^{-1}(x)$ is shown in red.



DIF: Difficult

5. ANS:

a) First, rewrite the quadratic function in vertex form.

$$f(x) = x^2 - 4x + 5$$
$$= (x^2 - 4x + 4 - 4) + 5$$
$$= (x-2)^2 + 1$$

Then, determine the inverse.

$$x = (y-2)^2 + 1$$
$$x - 1 = (y-2)^2$$
$$\pm\sqrt{x-1} = y - 2$$
$$2 \pm \sqrt{x-1} = y$$
$$f^{-1}(x) = 2 \pm \sqrt{x-1}$$

b) For *f*(*x*): domain $\{x \in \mathbb{R}\}$, range $\{y \geq 1,\ y \in \mathbb{R}\}$
For $f^{-1}(x)$: domain $\{x \geq 1,\ x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$
c) The inverse is not a function because it fails the vertical line test.

DIF: Average

6. ANS:

$$a = \theta r$$

$$= \frac{\pi}{9}(3)$$

$$a = \frac{\pi}{3}$$

The child travels through an arc length of $\frac{\pi}{3}$ m.

DIF: Easy

7. ANS:

$$\frac{\cot\left(\frac{\pi}{3}\right)+\csc\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{4}\right)}=\frac{\dfrac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)}+\dfrac{1}{\sin\left(\frac{\pi}{3}\right)}}{\cos\left(\frac{\pi}{4}\right)}$$

$$=\frac{\dfrac{\cos\left(\frac{\pi}{3}\right)+1}{\sin\left(\frac{\pi}{3}\right)}}{\cos\left(\frac{\pi}{4}\right)}$$

$$=\frac{\cos\left(\frac{\pi}{3}\right)+1}{\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)}$$

$$=\frac{\frac{1}{2}+1}{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$$

$$=\frac{\frac{3}{2}}{\frac{\sqrt{6}}{4}}$$

$$=\left(\frac{3}{2}\right)\left(\frac{4}{\sqrt{6}}\right)$$

$$=\frac{6}{\sqrt{6}}$$

$$\frac{\cot\left(\frac{\pi}{3}\right)+\csc\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{4}\right)}=\sqrt{6}$$

DIF: Average

8. ANS:

$\sin x = \cos\left(\frac{\pi}{5}\right)$

$\sin x = \cos\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$

$\sin x = \sin\left(\frac{3\pi}{10}\right)$

$x = \frac{3\pi}{10}$

DIF: Average

9. ANS:

Since $\csc\theta = -\frac{2}{\sqrt{3}}$, $\sin\theta = -\frac{\sqrt{3}}{2}$. Since $\sin 60° = \frac{\sqrt{3}}{2}$, the reference angle is 60°. The ratio is negative in quadrants III and IV.This means that the angle can be found by looking for reflections of 60° that lie in these quadrants.

quadrant III: 180° + 60° = 240°

quadrant IV: 360° – 60° = 300°

DIF: Average

10. ANS:

$r = \frac{d}{2}$

$= \frac{21}{2}$

$= 10.5$

$a = r\theta$

$a = (10.5)(1.2)$

$= 12.6$

The arc length is 12.6 cm.

DIF: Average

11. ANS:

$$\left[\cos\left(\frac{5\pi}{6}\right)\right]^2 - \left[\sin\left(\frac{5\pi}{6}\right)\right]^2 = \left(-\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{1}{2}$$

DIF: Difficult

12. ANS:

$\sin A = \frac{\sqrt{2}}{2}$ $\cos B = \frac{\sqrt{3}}{2}$

$\angle A = \frac{\pi}{4}$ $\angle B = \frac{\pi}{6}$

$$\begin{aligned}
\sec A + \sec B &= \sec\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{6}\right) \\
&= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{3}} \\
&= \frac{2\sqrt{3} + 2\sqrt{2}}{\sqrt{6}} \\
&= \frac{2\sqrt{18} + 2\sqrt{12}}{6} \\
&= \frac{2 \times 3\sqrt{2} + 2 \times 2\sqrt{3}}{6} \\
&= \frac{3\sqrt{2} + 2\sqrt{3}}{3}
\end{aligned}$$

DIF: Average

13. ANS:

The tangent ratio is negative in quadrants II and IV. In quadrant II for the domain $0° \le \theta \le 180°$, $\theta = 120°$. In quadrant IV for the domain $-180° \le \theta \le 0°$, $\theta = -60°$.

DIF: Difficult

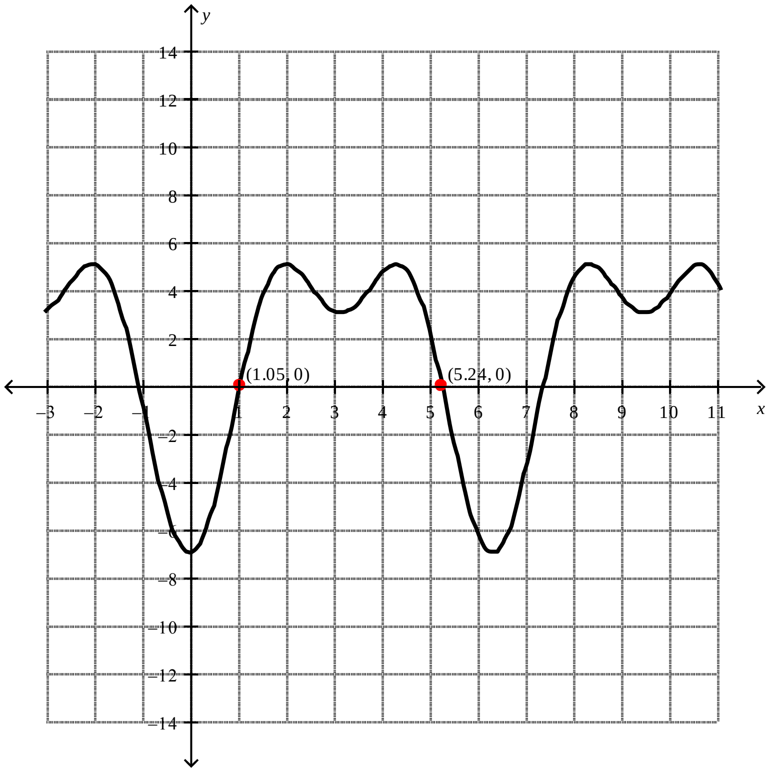
14. ANS:

$\frac{5\pi}{6}$

DIF: Easy

15. ANS:

Graph $6\sin^2 x - 5\cos x - 2 = 0$ using graphing technology.



The zeros occur at approximately 1.0 and 5.2 or exactly $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

DIF: Difficult +

16. ANS:

The amplitude is half the diameter of the Ferris wheel, or 9.25 m. The highest point is 18.5 + 3 or 21.5 m above the ground, so the vertical displacement is 21.5 – 9.25 or 12.25 m. The wheel rotates at a rate of 0.2 rad/s, which means it takes 5 s/rad. One full rotation of the wheel is equivalent to 2π radians. Therefore, it will take $2\pi\left(5\,\frac{\text{s}}{\text{rad}}\right) = 10\pi$ seconds for the Ferris wheel to do one full rotation. The period is 10π.

$$\text{period} = \frac{2\pi}{|b|}$$

$$10\pi = \frac{2\pi}{|b|}$$

$$|b| = \frac{2\pi}{10\pi}$$

$$|b| = \frac{1}{5}$$

The value of *b* is thus $\frac{1}{5}$.

There is no phase shift if a cosine function with a negative value for *a* is used. Thus, the sinusoidal function that models the height of the car is

$$h = -9.25\cos\left(\frac{t}{5}\right) + 12.25$$

DIF: Average

17. ANS:

Substitute t = 2.5 into the equation, since 2:30 a.m. is 2.5 hours after midnight.

$$d = 5\sin\left[2\pi\frac{(t-4)}{12.4}\right] + 6$$

$$= 5\sin\left(2\pi\frac{(2.5-4)}{12.4}\right) + 6$$

$$\approx 2.56$$

The water depth is approximately 2.56 m.

DIF: Easy

18. ANS:

a reflection in the x-axis, a vertical stretch by a factor of 2, a horizontal stretch by a factor of 8, a phase shift of $\frac{\pi}{3}$ to the right, and a vertical translation of 1 unit up

DIF: Average

19. ANS:

Solutions may vary. Sample solution:

The amplitude is half the diameter, or 30 cm. The maximum height of the pebble is 60 cm, so the vertical displacement must be 30 cm. The wheel rotates at 4 revolutions per second, so the period is $\frac{1}{4}$ s. Thus, the value of b is $\frac{2\pi}{\frac{1}{4}}$ or 8π.

Thus, the relationship between the height of the pebble above the ground and time is

$h = 30\sin(8\pi t) + 30$

DIF: Easy

20. ANS:

a) 0.7 m

b) Since b = 72 and period = $\frac{2\pi}{b}$, then period = $\frac{2\pi}{72}$ or $\frac{\pi}{36}$ s. The number of revolutions of the rope is the reciprocal of the period, or $\frac{36}{\pi}$, or 11.46 rev/s. Multiply by 60 to get 688 revolutions/min.

DIF: Difficult

21. ANS:

$$\cos\frac{2\pi}{9}\cos\frac{\pi}{18}+\sin\frac{2\pi}{9}\sin\frac{\pi}{18}=\cos\left(\frac{2\pi}{9}-\frac{\pi}{18}\right)$$
$$=\cos\left(\frac{4\pi}{18}-\frac{\pi}{18}\right)$$
$$=\cos\left(\frac{3\pi}{18}\right)$$
$$=\cos\left(\frac{\pi}{6}\right)$$
$$=\frac{\sqrt{3}}{2}$$

DIF: Average

22. ANS:

L.S. $=\tan^2\theta-\sin^2\theta$ R.S. $=\sin^2\theta\tan^2\theta$

$$=\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta$$
$$=\frac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\cos^2\theta}$$
$$=\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$
$$=\frac{\sin^2\theta(\sin^2\theta)}{\cos^2\theta}$$
$$=\sin^2\theta\tan^2\theta$$

L.S. = R.S.

DIF: Average

23. ANS:

$$2\cos x-\sqrt{3}=0$$
$$2\cos x=\sqrt{3}$$
$$\cos x=\frac{\sqrt{3}}{2}$$
$$x=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$
$$=\frac{\pi}{6}$$

Since cosine is also positive in quadrant IV, another solution is $2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$.

DIF: Easy

24. ANS:

$$\sec^2\theta - 2\tan\theta - 3 = 0$$

$$(1 + \tan^2\theta) - 2\tan\theta - 3 = 0$$

$$\tan^2\theta - 2\tan\theta - 2 = 0$$

Use the quadratic formula.

$$\tan\theta = \frac{2 \pm \sqrt{12}}{2}$$

$$= 1 \pm \sqrt{3}$$

$$\approx 2.732 \text{ or } -0.732$$

$$\theta = \tan^{-1}(2.732) \text{ or } \tan^{-1}(-0.732)$$

$$\approx 70° \text{ or } -36°$$

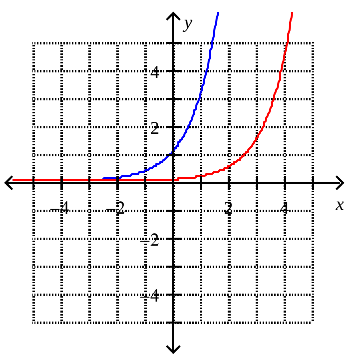
Since the period for $\tan\theta$ is 180°, a positive solution corresponding to –36° is –36° + 180° or 144°. The general solution is $70° + 180°n$ and $144° + 180°n$, where $n \in I$.

DIF: Difficult

25. ANS:

a) a vertical compression by a factor of $\frac{1}{2}$ and a translation of 2 units to the right

b) The graph of $y = 3^x$ is shown in blue and the graph of $y = \frac{1}{2}(3)^{x-2}$ is shown in red.



c) domain $\{x|x \in R\}$, range $\{y|y > 0, y \in R\}$, $y = 0$

DIF: Average

26. ANS:

a) $y = 5^{-x}$

b) $y = 5^{x-3}$

c) $y = 5^{x+4} - 1$

d) $y = -5^x - 2$

DIF: Average

27. ANS:

$$9^{n-1} = \left(\frac{1}{3}\right)^{4n-1}$$

$$\left(3^2\right)^{n-1} = \left(3^{-1}\right)^{4n-1}$$

$$3^{2n-2} = 3^{1-4n}$$

Equate the exponents:

$$2n - 2 = 1 - 4n$$

$$6n = 3$$

$$n = \frac{1}{2}$$

DIF: Average

28. ANS:

$$3^x = 9^{x^2 - \frac{1}{2}}$$

$$3^x = 3^{2\left(x^2 - \frac{1}{2}\right)}$$

Equate the exponents:

$$x = 2x^2 - 1$$

$$0 = 2x^2 - x - 1$$

$$0 = (2x+1)(x-1)$$

$$x = -\frac{1}{2}, \qquad x = 1$$

DIF: Difficult

29. ANS:

$$\begin{aligned} \log_2 64 + \log_3 27 \times \log_4\left(\frac{1}{256}\right) &= 6 + 3(-4) \\ &= 6 - 12 \\ &= -6 \end{aligned}$$

DIF: Difficult

30. ANS:

Solve the system of equations:

$m - n = 4^0$ and $m + n = 4^2$

$$m - n = 1$$

$$m + n = 16$$

Add these equations to find *m*:

$$2m = 17$$

$$m = 8.5$$

Subtract the first equation from the second to find *n*:
$2n = 15$

$n = 7.5$

DIF: Difficult

31. ANS:
First solve for *k*.

$$N = N_0 10^{kt}$$

$$2N_0 = N_0 10^{k(25)}$$

$$2 = 10^{25k}$$

$$\log 2 = 25k \log 10$$

$$k = \frac{\log 2}{25}$$

Solve for *t* when $N = 3N_0$.

$$3N_0 = N_0 10^{\left(\frac{\log 2}{25}\right)t}$$

$$3 = 10^{\left(\frac{\log 2}{25}\right)t}$$

$$\log 3 = \left(\frac{\log 2}{25}\right)t$$

$$t = \left(\frac{25}{\log 2}\right)\log 3$$

$$t \approx 39.62$$

It will take 39.6 years for the population to triple.

DIF: Difficult +

32. ANS:

$$6^{3x+1} = 2^{2x-3}$$

$$\log(6^{3x+1}) = \log(2^{2x-3})$$

$$(3x+1)\log 6 = (2x-3)\log 2$$

$$3x\log 6 + \log 6 = 2x\log 2 - 3\log 2$$

$$x(3\log 6 - 2\log 2) = -3\log 2 - \log 6$$

$$x = \frac{-(3\log 2 + \log 6)}{3\log 6 - 2\log 2}$$

DIF: Average

33. ANS:

$$2\log_4(x+4)-\log_4(x+12)=1$$
$$\log_4(x+4)^2-\log_4(x+12)=1$$
$$\log_4\frac{(x+4)^2}{(x+12)}=\log_4 4^1$$
$$\frac{(x+4)^2}{(x+12)}=4$$
$$(x+4)^2=4x+48$$
$$x^2+8x+16=4x+48$$
$$x^2+4x-32=0$$
$$(x+8)(x-4)=0$$
$$x=-8,\ x=4$$

Since $x = –8$ is an extraneous root, the solution is $x = 4$.

DIF: Average

34. ANS:

$$h(x)=f(x)+g(x)$$
$$=x+1+x^2+3x+1$$
$$=x^2+4x+2$$

DIF: Easy

35. ANS:

$$g(f(x))=g(\cos x)$$
$$=2^{\cos x}+5$$
$$g(f(\pi))=2^{(\cos \pi)}+5$$
$$=2^{-1}+5$$
$$=\frac{1}{2}+5$$
$$=\frac{11}{2}\text{ or }5.5$$

DIF: Difficult

36. ANS:

$$g(2)=2-(2)^3$$
$$=-6$$
$$f(-6)=(-6)^2-7$$
$$=29$$
$$f(g(2))=29$$

DIF: Average

PROBLEM

1. ANS:

a) i) $-f(x) = -[2(x-1)^2 - 3]$

$= -2(x-1)^2 + 3$

ii) $f(-x) = 2(-x-1)^2 - 3$

$= 2(-1)^2(x+1)^2 - 3$

$= 2(x+1)^2 - 3$

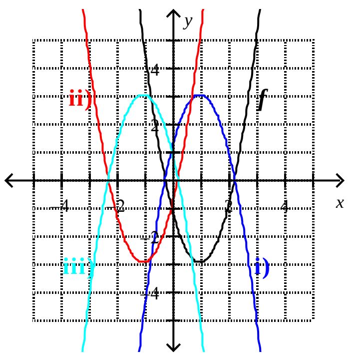
iii) $-f(-x) = -[2(-x-1)^2 - 3]$

$= -[2(-1)^2(x+1)^2 - 3]$

$= -[2(x+1)^2 - 3]$

$= -2(x+1)^2 + 3$

b)



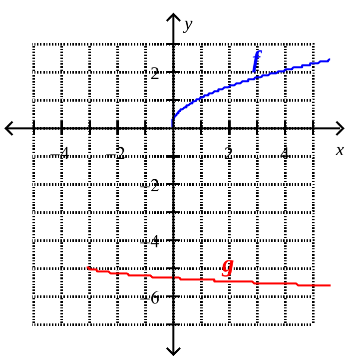
c) The graphs of $f(x)$ and $f(-x)$ are horizontal translations of each other. This is also true of $-f(x)$ and $-f(-x)$.
d) In both pairs, one curve is a horizontal translation of 2 units left or right of the other.

DIF: Difficult

2. ANS:

a) $g(x) = -\frac{1}{3}\sqrt{\frac{1}{2}(x+3)} - 5$

b)



c) The reflection, horizontal stretch, and vertical compression must be done first, but can be done in any order.
d) The translations to the left and down must be done last, but can be done in any order.

DIF: Difficult +

3. ANS:
a) domain $\{x \geq -1,\ x \in \mathrm{R}\}$, range $\{y \leq -4,\ y \in \mathrm{R}\}$
b) domain $\{x \leq -4,\ x \in \mathrm{R}\}$, range $\{y \geq -1,\ y \in \mathrm{R}\}$
c)

$$y = -2\sqrt{x+1} - 4$$

$$x = -2\sqrt{y+1} - 4$$

$$x + 4 = -2\sqrt{y+1}$$

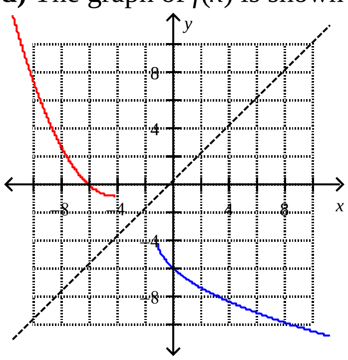
$$-\frac{(x+4)}{2} = \sqrt{y+1}$$

$$\left(-\left(\frac{x+4}{2}\right)\right)^2 = y + 1$$

$$y = \left(\frac{x+4}{2}\right)^2 - 1$$

$$f^{-1}(x) = \frac{1}{4}(x+4)^2 - 1$$

d) The graph of $f(x)$ is shown in blue and the graph of $f^{-1}(x)$ is shown in red.



DIF: Difficult

4. ANS:

a)

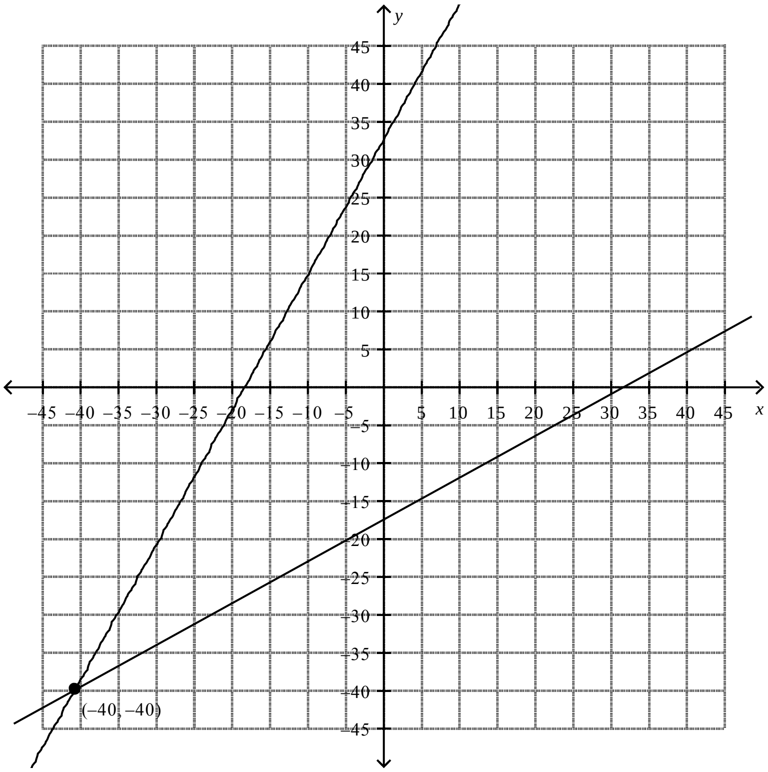
$$x = \frac{9}{5}y + 32$$

$$x - 32 = \frac{9}{5}y$$

$$y = \frac{5}{9}(x - 32)$$

The inverse represents the equation to convert a temperature from degrees Fahrenheit to degrees Celsius. The variable *x* represents the temperature in degrees Fahrenheit and the variable *y* represents the temperature in degrees Celsius.

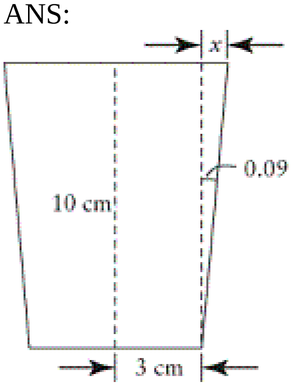
b)



c) The temperature that is the same in Celsius and Fahrenheit is −40°. This is the invariant point of the original function and its inverse.

DIF: Average

5. ANS:



First, find *x*.

$$\frac{x}{10} = \tan 0.09$$

$$x = 10\tan 0.09$$

$$A_{\text{bottom}} = \pi r^2 = \pi(3)^2 = 9\pi$$

$$A_{\text{top}} = \pi r^2 = \pi(3+x)^2 = \pi(3+10\tan 0.09)^2 \approx 15.23\pi$$

$$V = \left(\frac{A_{\text{top}} + A_{\text{bottom}}}{2}\right)h$$
$$= \left(\frac{15.23\pi + 9\pi}{2}\right)(10)$$
$$\approx 381$$

The volume of the cup is approximately 381 cm^3, or 381 mL.

DIF: Difficult

6. ANS:

∠A is in quadrant II. Therefore, only the sine ratio will be positive.
Use the Pythagorean theorem.

$$r^2 = x^2 + y^2$$
$$= (-5)^2 + 7^2$$
$$= 25 + 49$$
$$= 74$$
$$r = \sqrt{74}$$

Therefore, $\sin A = \frac{7}{\sqrt{74}}$, $\cos A = -\frac{5}{\sqrt{74}}$, and $\tan A = -\frac{7}{5}$.

b) The quadrant in which the sine ratio is still positive, but the cosine and tangent ratios change from negative to positive, is quadrant I. In this quadrant, all three primary trigonometric ratios are positive.

$\sin B = \frac{7}{\sqrt{74}}$, $\cos B = \frac{5}{\sqrt{74}}$, and $\tan B = \frac{7}{5}$.

c) Use the fact that ∠B is the reference angle for ∠A.

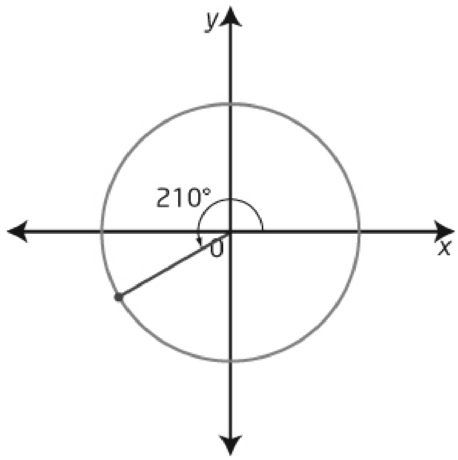
$$\angle B = \sin^{-1}\frac{7}{\sqrt{74}}$$
$$\angle B \approx 54°$$
$$\angle A = 180° - 54°$$
$$= 126°$$

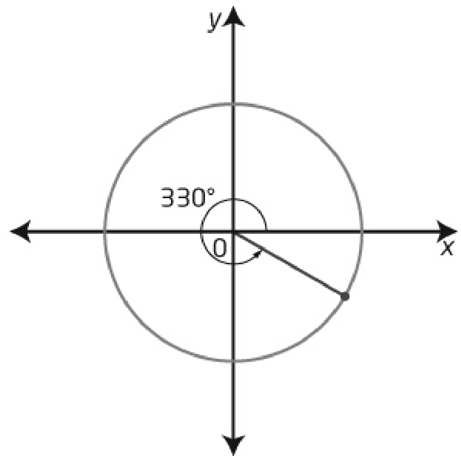
DIF: Average

7. ANS:

a) Since sin 30° = $\frac{1}{2}$, the reference angle is 30°. The sine ratio is negative in quadrants III and IV. Look for reflections of the 30° angle in these quadrants.
quadrant III: 180° + 30° = 210°
quadrant IV: 360° – 30° = 330°

b) Using a calculator, sin 210° = $-\frac{1}{2}$ and sin 330° = $-\frac{1}{2}$.





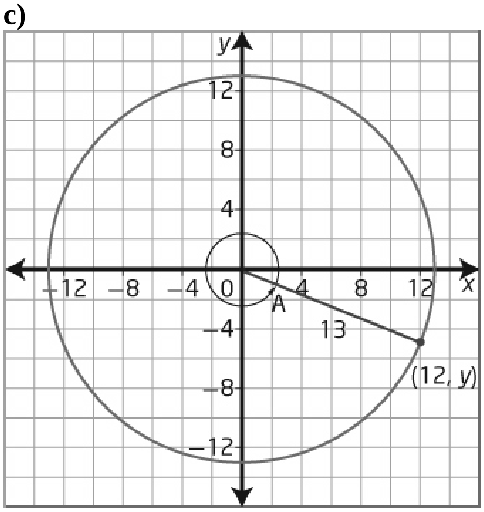
DIF: Average

8. ANS:

a) Since the cosine ratio is positive, the angle is in quadrant I or IV.

b) If the sine ratio is negative, the angle is in quadrant IV.

c)



d) Use the Pythagorean theorem.

$$r^2 = x^2 + y^2$$

$$13^2 = 12^2 + y^2$$

$$y^2 = 169 - 144$$

$$y^2 = 25$$

$$y = \pm 5$$

Therefore, a point on the terminal arm is (12, –5).

e) $\sin A = -\dfrac{5}{13}$, $\tan A = -\dfrac{5}{12}$

DIF: Average

9. ANS:

a) Answers may vary. Sample answers:

i) –315° and 405°
ii) –60° and 120°
iii) 240° and –240°
b) In all cases, locate the given angle on a Cartesian plane, and then identify the reference angle. Then, locate the other quadrant where the given trigonometric ratio has the same sign as the given ratio. Then, any angle that is co-terminal to the two angles in the diagram has the same trigonometric ratio as that given.

DIF: Average

10. ANS:
a) Since the point on the terminal arm lies in quadrant III, the tangent ratio is positive, and the sine and cosine ratios are negative.
b) The cotangent ratio is positive, and the cosecant and secant ratios are negative.
c) $r^2 = x^2 + y^2$

$$\begin{aligned} &= (-3)^2 + (-6)^2 \\ &= 9 + 36 \\ &= 45 \\ r &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

$\sin P = \frac{y}{r} = \frac{-6}{3\sqrt{5}} = -\frac{2}{\sqrt{5}}$ $\quad \cos P = \frac{x}{r} = \frac{-3}{3\sqrt{5}} = -\frac{1}{\sqrt{5}}$ $\quad \tan P = \frac{y}{x} = \frac{-6}{-3} = 2$

d) Using the answers in part c), take the reciprocal of each primary trigonometric ratio to write the reciprocal trigonometric ratios.
$\csc P = -\frac{\sqrt{5}}{2}$, $\sec P = -\sqrt{5}$, and $\cot P = \frac{1}{2}$

DIF: Average

11. ANS:
$$\frac{150 \text{ revolutions}}{\text{minute}} = \frac{150(2\pi)}{60 \text{ s}} \approx 15.71 \text{ rad/s}$$
The angular velocity is 15.71 rad/s.

DIF: Average

12. ANS:
a) Solve the equation $11.6 = 9.2 + 2.4\cos\left[\frac{\pi}{6}(t-6)\right]$ for *t*.

$$11.6 = 9.2 + 2.4\cos\left[\frac{\pi}{6}(t-6)\right]$$

$$2.4 = 2.4\cos\left[\frac{\pi}{6}(t-6)\right]$$

$$1 = \cos\left[\frac{\pi}{6}(t-6)\right]$$

$$\frac{\pi}{6}(t-6) = 0 \text{ or } \frac{\pi}{6}(t-6) = 2\pi$$

$$t = 6 \text{ or } t = 18$$

These both represent June. Sales will be 11 600 in June. Since the cosine function is equal to 1 at this point, this is the maximum number of sales.

b) The minimum sales will occur when the cosine function is equal to −1, which occurs when

$$\frac{\pi}{6}(t-6) = \pi$$

$$t - 6 = 6$$

$$t = 12$$

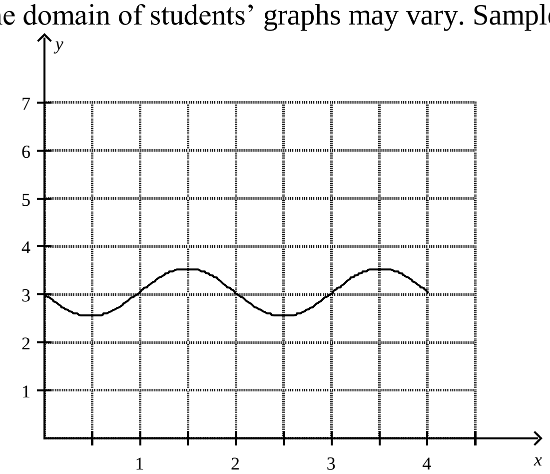
The minimum sales are in December.

c) Yes. You would expect the most sales to occur at the beginning of summer and the least sales in a cold month like December.

DIF: Average

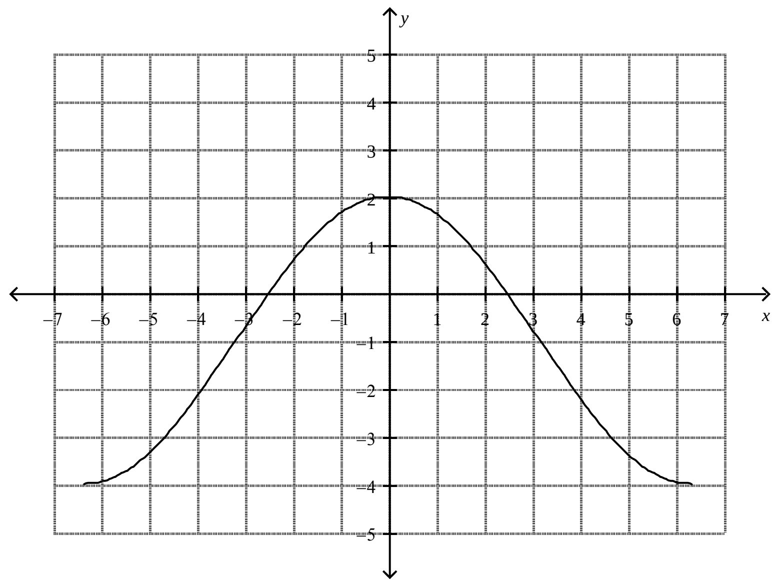
13. ANS:

The domain of students' graphs may vary. Sample graph:



DIF: Average

14. ANS:



DIF: Average

15. ANS:

The period is 12 h 24 min, or 12.4 h.

$$12.4 = \frac{2\pi}{b}$$

$$b = \frac{\pi}{6.2}$$

An equation that models the depth of the water is $d = 4\cos\left(\frac{\pi}{6.2}t\right)$.

At 10:00 a.m., t = 2. Substitute t = 2 into the equation.

$$d = 4\cos\left(\frac{\pi}{6.2}t\right)$$

$$= 4\cos\left[\frac{\pi}{6.2}(2)\right]$$

$$\approx 2.1$$

The depth of the water at 10 a.m. is 2.1 m.

DIF: Average

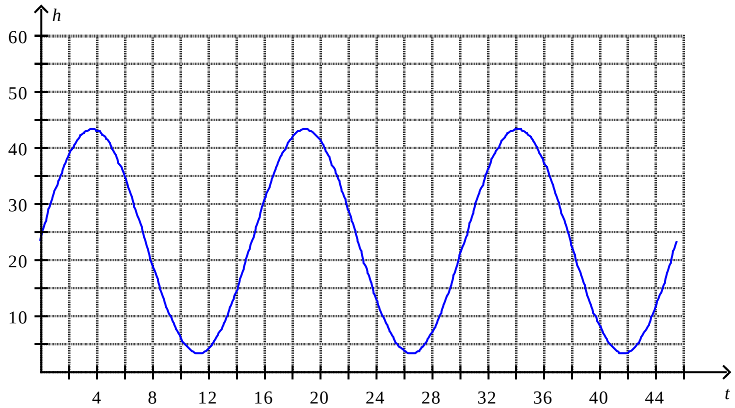
16. ANS:

Solutions may vary. Sample solution:

a) The amplitude is 20 m, and the vertical displacement is 23 m. The frequency of the blades is 4 revolutions per minute, so the period is 0.25 min or 15 s. Thus, $b = \frac{2\pi}{15}$, and the sinusoidal function is

$$h = 20\sin\left(\frac{2\pi}{15}t\right) + 23.$$

b)



c) Substitute t = 10 into the equation.

$$h = 20 \sin\left(\frac{2\pi}{15}t\right) + 23$$

$$= 20 \sin\left(\frac{2\pi}{15}(10)\right) + 23$$

$$\approx 5.7$$

The tip of the blade is approximately 5.7 m above the ground at t = 10 s.

DIF: Average

17. ANS:

a) The function $y = 2 \sin 2x + 2$ has an amplitude of 2, a period of 180°, and a maximum of 4, but the maximum occurs at x = 45°. For the maximum to occur at (0, 4), shift the function 45° to the left:

$y = 2 \sin [2(x + 45°)] + 2$

b) The function $y = 2 \cos 2x + 2$ has an amplitude of 2, a period of 180°, and a maximum of 4 at x = 0.

c) A phase shift of a sine function of 45° to the left will create a coincident cosine function, as long as the period of the sine function is 180°.

DIF: Average

18. ANS:

a) amplitude: $\frac{1}{2}$; period: $\frac{360°}{3}$ or 120°; phase shift: 30° to the right; vertical shift: 4 units up

b) minimum: $-\frac{1}{2} + 4$ or $\frac{7}{2}$, maximum: $\frac{1}{2} + 4$ or $\frac{9}{2}$

c) The function does not cross the x-axis, so there are no x-intercepts.

d) Substitute x = 0 in the function:

$f(x) = \frac{1}{2}\sin[3(x - 30°)] + 4$

$f(0) = \frac{1}{2}\sin[3(0 - 30°)] + 4$

$= \frac{1}{2}\sin(-90°) + 4$

$= -\frac{1}{2} + 4$

$= \frac{7}{2}$

The *y*-intercept is $\frac{7}{2}$.

DIF: Average

19. ANS:

L.S. $= \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

R.S. $= \sec 2\theta - \tan 2\theta$

$= \dfrac{1}{\cos 2\theta} - \dfrac{\sin 2\theta}{\cos 2\theta}$

$= \dfrac{1 - \sin 2\theta}{\cos 2\theta}$

$= \dfrac{1 - 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$

$= \dfrac{(\cos\theta - \sin\theta)(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$

$= \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

L.S. = **R.S.**

Therefore, $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \sec 2\theta - \tan 2\theta$.

DIF: Difficult

20. ANS:

$$\text{L.S.} = \frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta} \qquad \text{L.S.} = \tan\theta$$

$$= \frac{1-(1-2\sin^2\theta)+2\sin\theta\cos\theta}{1+(2\cos^2\theta-1)+2\sin\theta\cos\theta}$$

$$= \frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta(\sin\theta+\cos\theta)}{2\cos\theta(\sin\theta+\cos\theta)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

L.S. = R.S

Therefore, $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta} = \tan\theta.$

DIF: Average

21. ANS:

$$\text{L.S.} = \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin\theta\cos\theta} \qquad \text{R.S.} = 1-\tan\theta$$

$$= \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\cos\theta(\cos\theta+\sin\theta)} \qquad = 1-\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta-\sin\theta}{\cos\theta} \qquad = \frac{\cos\theta-\sin\theta}{\cos\theta}$$

L.S. = R.S.

DIF: Average

22. ANS:

$$\text{L.S.} = \frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta} \qquad \text{R.S.} = \frac{2}{\sin^2\theta}$$

$$= \frac{1-\cos\theta+1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{2}{1-\cos^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

L.S. = R.S.

DIF: Average

23. ANS:

$$\cos 2x + 2 = \sin x$$
$$1 - 2\sin^2 x + 2 = \sin x$$
$$2\sin^2 x + \sin x - 3 = 0$$
$$(2\sin x + 3)(\sin x - 1) = 0$$
$$\sin x = -\frac{3}{2} \text{ or } \sin x = 1$$
$$\text{no solution or } x = 90°$$
The solution is $x = 90°$.

DIF: Easy

24. ANS:

$$4\sin^4 x + 3\sin^2 x - 1 = 0$$
$$(4\sin^2 x - 1)(\sin^2 x + 1) = 0 \quad \text{Divide both sides by } \sin^2 x + 1 \text{ because it is always positive.}$$
$$4\sin^2 x - 1 = 0$$
$$\sin x = \pm\frac{1}{2}$$
sin x is positive in quadrants I and II and negative in quadrants III and IV. The solution is $x = 30°, 150°, 210°, 330°$.

DIF: Average

25. ANS:

a) $V = 35\ 000(0.80)^t$

b) $V = 35\ 000(0.80)^t$
$$= 35\ 000(0.80)^2$$
$$= 22\ 400$$
The value of the vehicle after 2 years is $22 400.

c) $V = 35\ 000(0.80)^t$
$$3000 = 35\ 000(0.80)^t$$
Use systematic trial. When $t = 11$, $V = 3006.48$. Therefore, after approximately 11 years, the vehicle will be worth $3000.

DIF: Average

26. ANS:

a) $A = 60\left(\frac{1}{2}\right)^n$, where A is the amount of cobalt-60 remaining, in milligrams, and n is the number of half-life periods.

b) 10.6 years equals 2 half-life periods, since $5.3 \times 2 = 10.6$.

$$A = 60\left(\frac{1}{2}\right)^n$$

$$= 60\left(\frac{1}{2}\right)^2$$

$$= \frac{60}{4}$$

$$= 15$$

15 mg will be present in 10.6 years.

c) $12.5\% = 0.125$

$$= \frac{1}{8}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$$

$$3 = n$$

It will take 5.3 × 3, or 15.9 years, for the amount of cobalt-60 to decay to 12.5% of its initial amount.

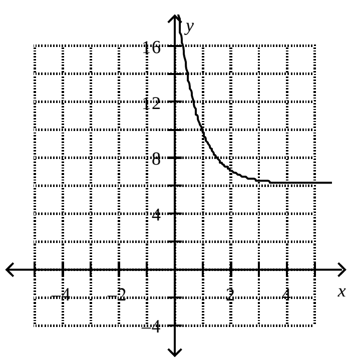
DIF: Difficult

27. ANS:

a) $y = 2^{-2(x-2)} + 6$

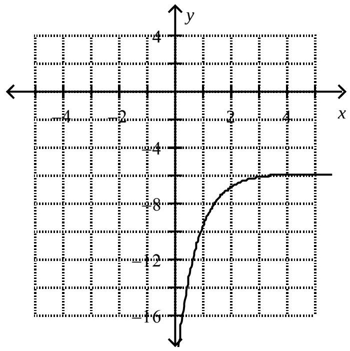
b) Reflect in the *y*-axis, compress horizontally by a factor of $\frac{1}{2}$, and translate 2 units to the right and 6 units up.

c)



d) $y = -2^{-2(x-2)} - 6$

e)



DIF: Average

28. ANS:

$$\sqrt[3]{256^2} \times 16^x = 64^{x-3}$$

$$\left(2^8\right)^{\frac{2}{3}} \times 2^{4x} = 2^{6x-18}$$

$$2^{4x+\frac{16}{3}} = 2^{6x-18}$$

$$4x + \frac{16}{3} = 6x - 18$$

$$-2x = -\frac{70}{3}$$

$$x = \frac{35}{3}$$

DIF: Average

29. ANS:

Substitute $V = 50$ into the formula $R = -236\log\left(\frac{V}{80}\right)$ and solve for R.

$R \doteq 48.17$

DIF: Average

30. ANS:

a) $M = \log\left(\frac{10^{5.1}A_0}{A_0}\right)$

$M = \log 10^{5.1}$

$M = 5.1$

The earthquake in Norman Wells measured 5.1 on the Richter scale.

b) $$\frac{\text{Vancouver Island amplitude}}{\text{Norman Wells amplitude}} = \frac{10^{7.3}A_0}{10^{5.1}A_0}$$

$$= \frac{10^{7.3}}{10^{5.1}}$$

$$\approx 158$$

The earthquake off Vancouver Island was about 158 times as strong as the earthquake off Norman Wells.

DIF: Average

31. ANS:

Let m_1 represent the apparent magnitude of Sirius, b_1 represent the brightness of Sirius, m_2 represent the apparent magnitude of the Sun, and b_2 represent the brightness of the Sun.

a) $$m_2 - m_1 = \log\left(\frac{b_1}{b_2}\right)$$

$$0.12 + 1.5 = \log\left(\frac{b_1}{b_2}\right)$$

$$1.62 = \log\left(\frac{b_1}{b_2}\right)$$

$$10^{1.62} = \frac{b_1}{b_2}$$

$$\frac{b_1}{b_2} \approx 41.69$$

Sirius is approximately 42 times as bright as the Sun.

b) $$m_1 - m_2 = \log\left(\frac{b_2}{b_1}\right)$$

$$-1.5 - m_2 = \log(1.3 \times 10^{10})$$

$$m_2 \approx -1.5 - 10.11$$

$$m_2 \approx -11.61$$

The apparent magnitude of the Sun is –11.6.

DIF: Difficult

32. ANS:

$\mathbf{L.S.} = \dfrac{1}{\log_a b}$ $\mathbf{R.S.} = \log_b a$

$= 1 \div \log_a b$

$= 1 \div \dfrac{\log b}{\log a}$

$= 1 \times \dfrac{\log a}{\log b}$

$= \log_b a$

L.S. = **R.S.**

Thus, $\dfrac{1}{\log_a b} = \log_b a$.

DIF: Difficult +

33. ANS:

$\mathbf{L.S.} = 3\log\sqrt{x} + 2\log x - \log\sqrt{x}$ $\mathbf{R.S.} = 3\log x$

$= \log x^{\frac{3}{2}} + \log x^2 - \log x^{\frac{1}{2}}$

$= \log\left[\dfrac{x^{\frac{3}{2}} x^2}{x^{\frac{1}{2}}}\right]$

$= \log x^{\frac{3}{2}+2-\frac{1}{2}}$

$= \log x^3$

$= 3\log x$

L.S. = **R.S.**

Thus, $3\log\sqrt{x} + 2\log x - \dfrac{1}{2}\log x = 3\log x$.

DIF: Average

34. ANS:

a) Let *h* represent the half-life of the substance.

$$140 = 200\left(\frac{1}{2}\right)^{\frac{3}{h}}$$

$$0.7 = 0.5^{\frac{3}{h}}$$

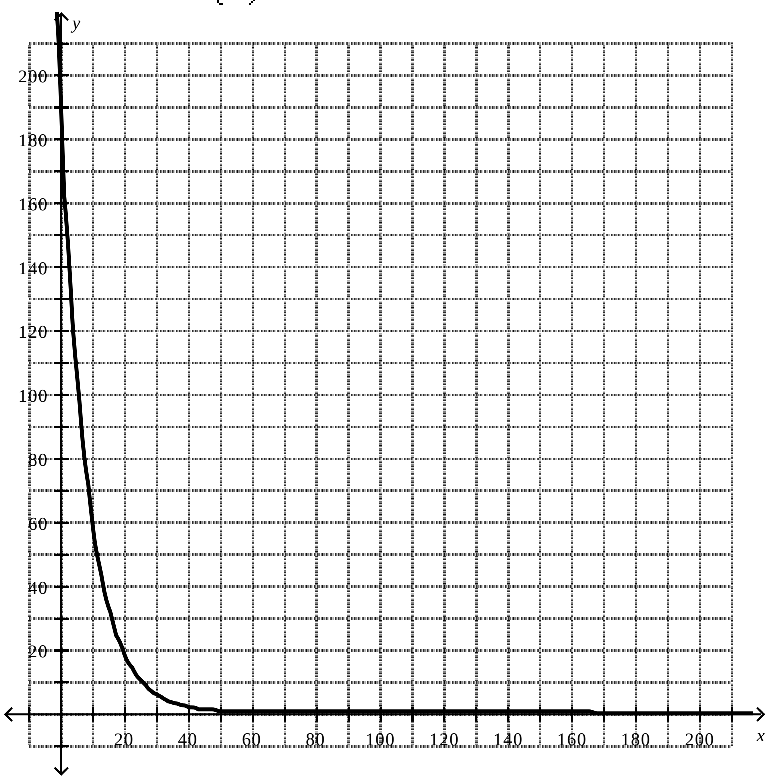
$$\log 0.7 = \frac{3}{h}\log 0.5$$

$$h = \frac{3\log 0.5}{\log 0.7}$$

$$h \approx 5.83$$

The half-life is 5.83 h.

b) Graph $y = 200\left(\frac{1}{2}\right)^{\frac{x}{5.83}}$.



DIF: Difficult

35. ANS:

$\log_2 \sqrt{x^2 - 8x} = \log_2 3$

$\sqrt{x^2 - 8x} = 3$	Since the bases are the same, equate the logarithmic arguments.
$x^2 - 8x = 9$	Square both sides.
$x^2 - 8x - 9 = 0$	Express the quadratic equation in standard form.
$(x - 9)(x + 1) = 0$	Solve for x.

$x = 9$ or $x = -1$

Check the values for extraneous roots.
In this case, both values are possible.

DIF: Difficult

36. ANS:

$$\log \sqrt[3]{x^2 + 48x} = \frac{2}{3}$$

$$\log(x^2 + 48x)^{\frac{1}{3}} = \frac{2}{3}$$

$$\frac{1}{3}\log(x^2 + 48x) = \frac{2}{3}$$

$$\log(x^2 + 48x) = 2$$

$$\log(x^2 + 48x) = \log 100$$

$$x^2 + 48x = 100$$

$$x^2 + 48x - 100 = 0$$

$$(x + 50)(x - 2) = 0$$

$$x = -50 \text{ or } x = 2$$

Check the values for extraneous roots.
In this case, both values are possible and solve the equation, so they are both valid.

DIF: Difficult

37. ANS:

For the first 5 years, the investment is compounded monthly for a total of 5 × 12, or 60, periods.

$$A = 18\ 000(1 + 0.0065)^{60}$$

$$= 26\ 552.12$$

Solve for the remaining time—compounded daily for *n* years is 365*n* periods.

$$35\ 000 = 26\ 552.12\left(1 + \frac{0.05}{365}\right)^{365n}$$

$$\frac{35\ 000}{26\ 552.12} = \left(\frac{365.05}{365}\right)^{365n}$$

$$\log\left(\frac{35\ 000}{26\ 552.12}\right) = 365n\log\left(\frac{365.05}{365}\right)$$

$$n \approx 5.53$$

Bruce will need invest for approximately 5.5 more years.

DIF: Difficult

38. ANS:

$$k(x) = (x+1)^2 + 3(x+1) + 2$$
$$= x^2 + 2x + 1 + 3x + 3 + 2$$
$$= x^2 + 5x + 6$$

DIF: Average

39. ANS:

$$h(x) = \frac{1}{1 - \sin^2 x}$$
$$= \frac{1}{\cos^2 x}$$
$$= \sec^2 x$$

DIF: Average

40. ANS:

$$f(g(x)) = g(f(x))$$
$$f(x+1) = g\left(\sqrt{x}\right)$$
$$\sqrt{x+1} = \sqrt{x} + 1$$
$$\left(\sqrt{x+1}\right)^2 = \left(\sqrt{x} + 1\right)^2$$
$$x + 1 = x + 2\sqrt{x} + 1$$
$$0 = 2\sqrt{x}$$
$$x = 0$$

DIF: Difficult