② f) $y = x^5 + 8x^3 + x$

$y' = 5x^4 + 24x^2 + 1$

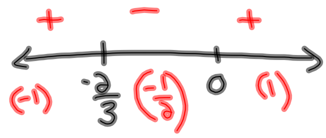
increasing on $(-\infty, \infty)$

③ a) $f(x) = x^2 + x^3$

$f'(x) = 2x + 3x^2$

$= (x)(2 + 3x)$ → $2 + 3x = 0$, $3x = -2$, $x = -\frac{2}{3}$

CV: $x = 0, -\frac{2}{3}$



Decreasing on $(-\frac{2}{3}, 0)$

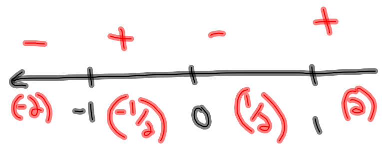
c) $h(x) = (1 - x^2)^2$

$h'(x) = 2(1 - x^2)(-2x)$

$= -4x(1 - x^2)$

$= (-4x)(1 - x)(1 + x)$

CV: $x = -1, 0, 1$



Decreasing on $(-\infty, -1)$ & $(0, 1)$

d) $F(x) = 4x + x^4$

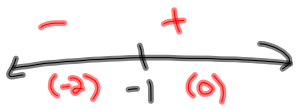
$F'(x) = 4 + 4x^3$

$= 4(1 + x^3)$

$= 4(1 + x)(1 - x + x^2)$ ← Always positive

$x^2 - x + 1$

CV: $x = -1$



Decreasing on $(-\infty, -1)$

Questions from Homework

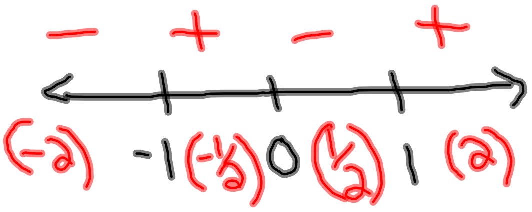
(4) c) $g(x) = x^4 - 2x^2 + 16$

$g'(x) = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x+1)(x-1)$

CV: $x = -1, 0, 1$



increasing on $(-1,0)$ + $(1,\infty)$

decreasing on $(-\infty,-1)$ + $(0,1)$

4 e) $h(x) = x^3(x-1)^4$

$$h'(x) = x^3(4)(x-1)^3(1) + 3x^2(x-1)^4$$
$$= 4x^3(x-1)^3 + 3x^2(x-1)^4$$
$$= x^2(x-1)^3\left[4x + 3(x-1)\right]$$
$$= x^2(x-1)^3(7x-3)$$

CV: $x = 0, \frac{3}{7}, 1$

$+ \quad + \quad - \quad +$

$(-1)\ 0\ (0.3)\ \frac{3}{7}\ (0.5)\ 1\ (2)$

increasing on $(-\infty, \frac{3}{7}) + (1, \infty)$

decreasing on $(\frac{3}{7}, 1)$

g) $y = x\sqrt{4-x} = x(4-x)^{1/2}$

$$y' = x\left(\tfrac{1}{2}\right)(4-x)^{-1/2}(-1) + 1(4-x)^{1/2}$$
$$= -\frac{x}{2}(4-x)^{-1/2} + (4-x)^{1/2}$$
$$= (4-x)^{-1/2}\left[\boxed{-\frac{x}{2}} + (4-\boxed{x})^1\right]$$
$$= \frac{4 - \frac{3x}{2}}{(4-x)^{1/2}}$$
$$= \frac{8-3x}{2(4-x)^{1/2}}$$

CV: $x = \frac{8}{3}, 4$

$+ \quad - \quad$ und.

$(0)\ \frac{8}{3}\ (3)\ 4\ (5)$

increasing $(-\infty, \frac{8}{3})$

decreasing $(\frac{8}{3}, 4)$

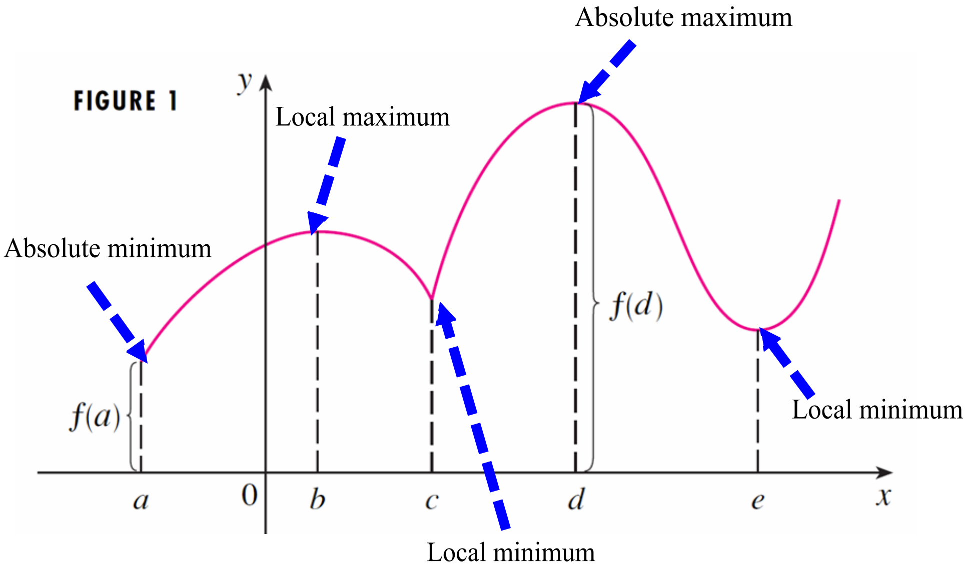
Absolute Maxima/Minima

A function *f* has an absolute (or global) maximum at *c* if $f(c) \geq f(x)$ for all *x* in the domain D of *f*.

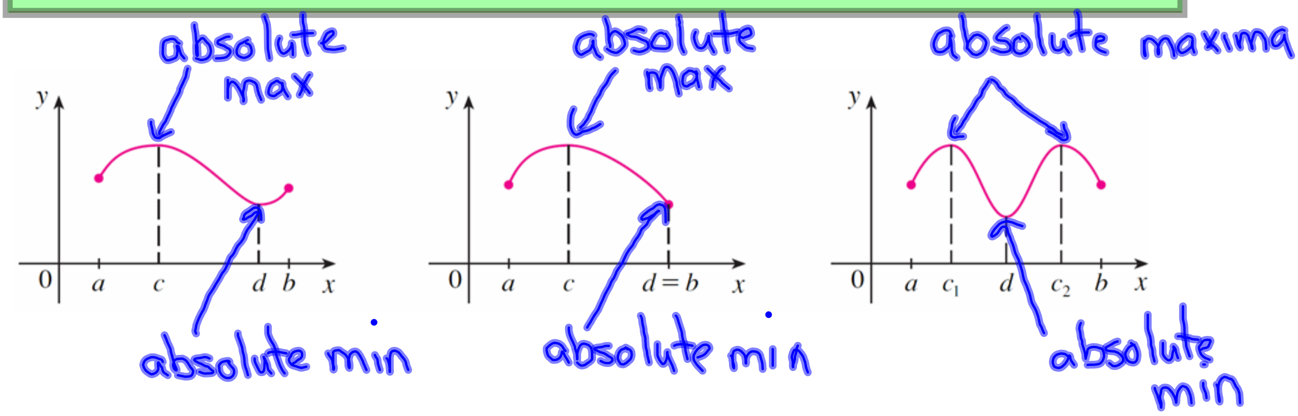
- The number *f(c)* is called the maximum value of *f* on D.

A function *f* has an absolute (or global) minimum at *c* if $f(c) \leq f(x)$ for all *x* in the domain D of *f*.

- The number *f(c)* is called the minimum value of *f* on D.



3 **The Extreme Value Theorem** If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



Here are a couple of examples to reinforce that the function must be continuous over a closed interval.

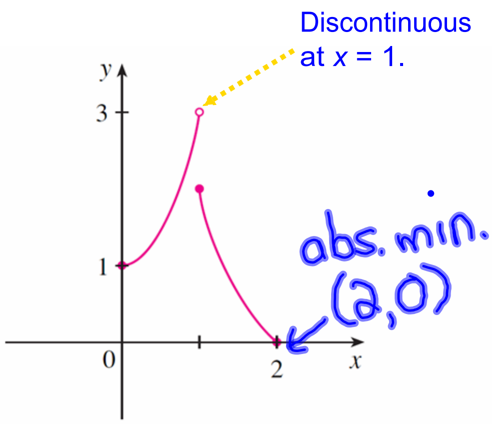


FIGURE 6
This function has minimum value $f(2) = 0$, but no maximum value.

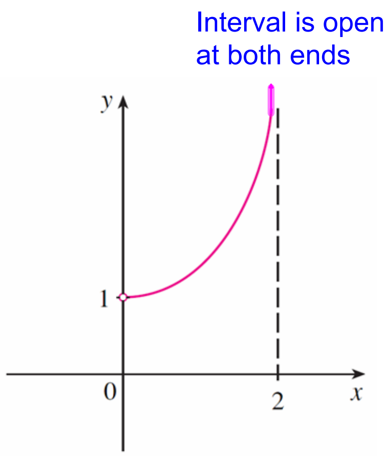


FIGURE 7
This continuous function g has no maximum or minimum.

How do we find extreme values? (max/mins)

4 **Fermat's Theorem** If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

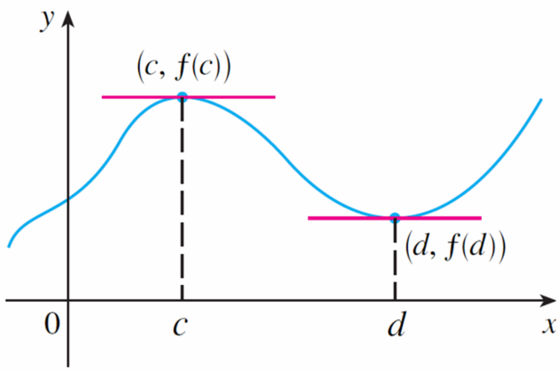


FIGURE 8

There can also be an extreme value where $f'(c)$ does not exist.

In general, functions whose graphs have "corners" or "kinks" are not differentiable there.

Look at the function $f(x) = |x|$

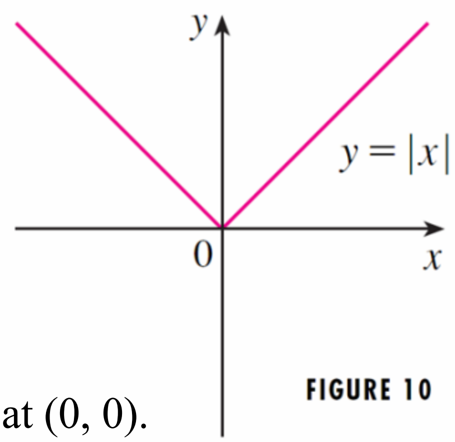


FIGURE 10

The curve does not have a tangent line at (0, 0).

Finding Absolute Maxima/Minima

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

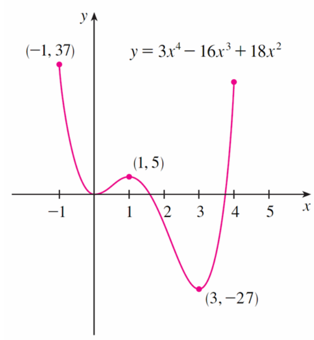
Example:

Interval $x \in [-1, 4]$

Given the function...

$$f(x) = 3x^4 - 16x^3 + 18x^2 \quad , \quad -1 \le x \le 4$$

Determine the absolute maximum and minimum values of the function.



$f'(x) = 12x^3 - 48x^2 + 36x$

$= 12x(x^2 - 4x + 3)$

$= 12x(x-1)(x-3)$

Critical Numbers: $x = 0, 1, 3$

$f(x) = 3x^4 - 16x^3 + 18x^2$

① $f(0) = 0 \rightarrow (0,0)$

$f(1) = 5 \rightarrow (1,5)$

$f(3) = 243 - 432 + 162 = -27 \rightarrow (3,-27)$ abs min

② $f(-1) = 3 + 16 + 18 = 37 \rightarrow (-1,37)$ abs max

$f(4) = 32 \longrightarrow (4,32)$

Homework