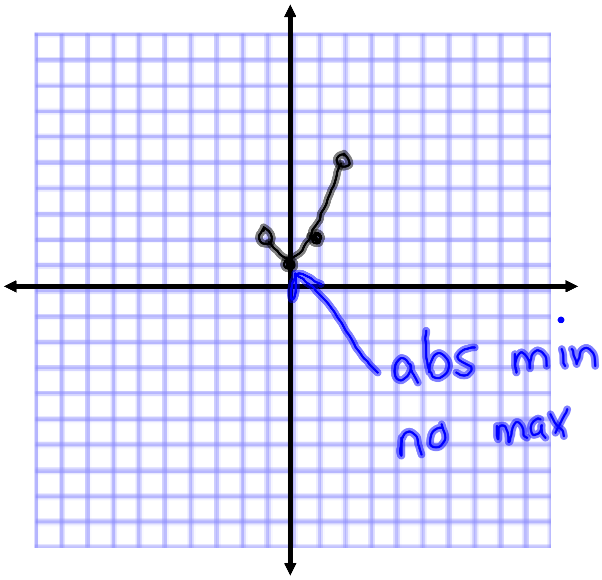
Questions from Homework

② d) $y = x^2 + 1 \quad -1 < x < 2$

x	y
∘ -1	2
0	1
1	2
∘ 2	5



③ e) $y = 2x^3 + 3x^2 - 12x + 3$

$y' = 6x^2 + 6x - 12$

$y' = 6(x^2 + x - 2)$

$y' = 6(x-1)(x+2)$

CV: $x = -2, 1$

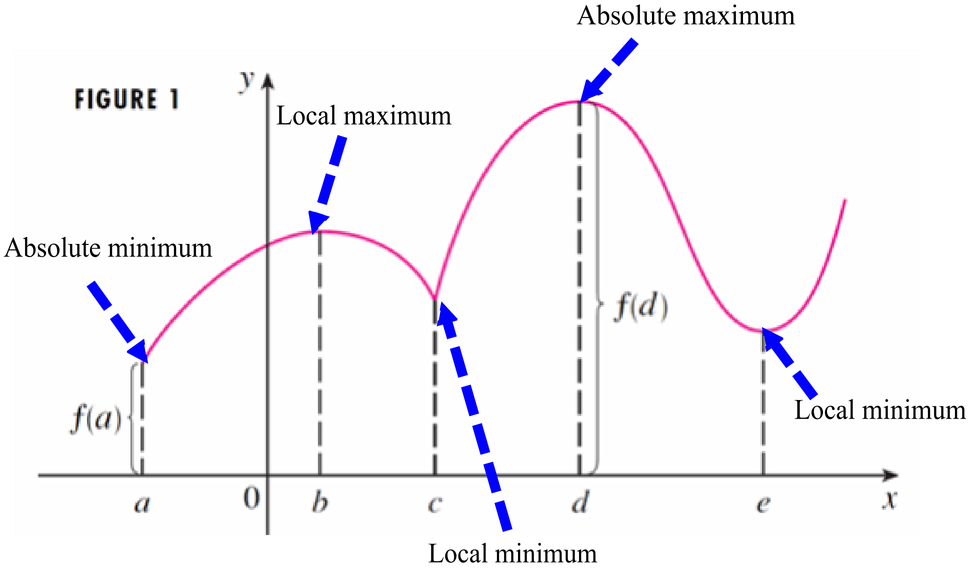
Absolute Maxima/Minima

A function *f* has an absolute (or global) maximum at *c* if $f(c) \geq f(x)$ for all *x* in the domain D of *f*.

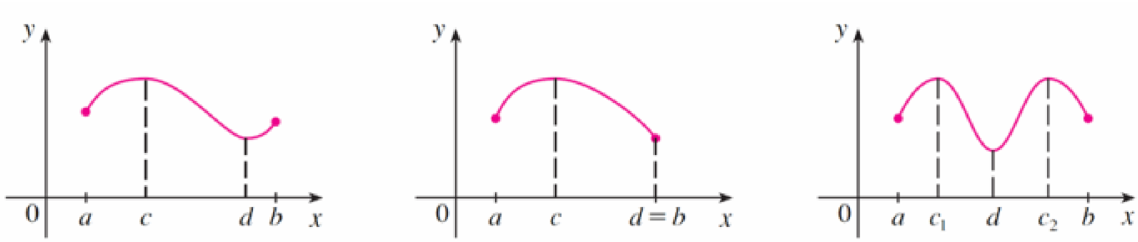
- The number *f(c)* is called the maximum value of *f* on D.

A function *f* has an absolute (or global) minimum at *c* if $f(c) \leq f(x)$ for all *x* in the domain D of *f*.

- The number *f(c)* is called the minimum value of *f* on D.



3 **The Extreme Value Theorem** If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



Here are a couple of examples to reinforce that the function must be continuous over a closed interval.

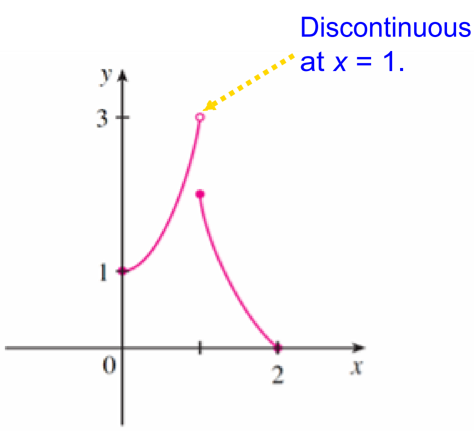


FIGURE 6
This function has minimum value $f(2) = 0$, but no maximum value.

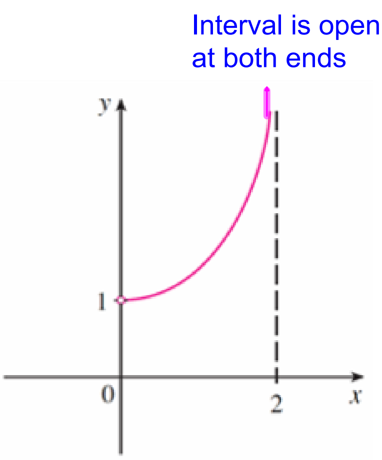


FIGURE 7
This continuous function g has no maximum or minimum.

How do we find extreme values?

4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

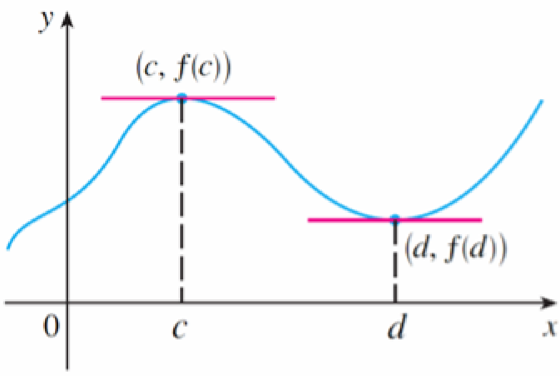


FIGURE 8

There can also be an extreme value where $f'(c)$ does not exist.

In general, functions whose graphs have "corners" or "kinks" are not differentiable there.

Look at the function $f(x) = |x|$

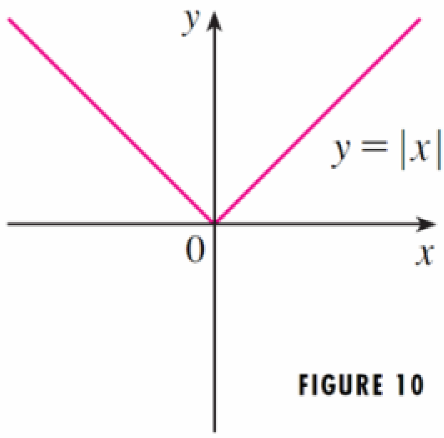


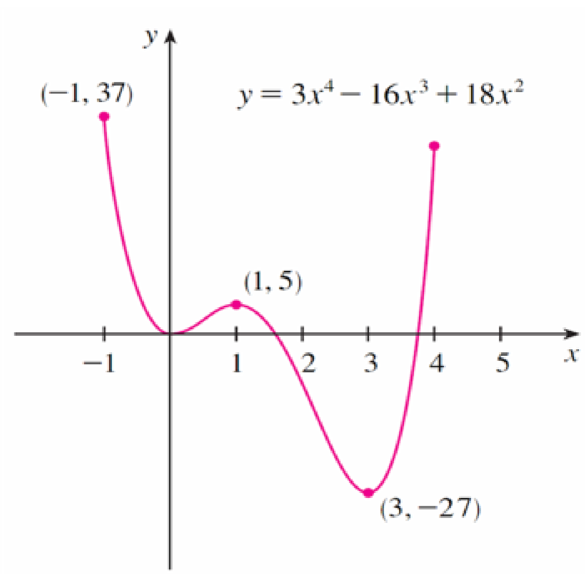
FIGURE 10

Example:

Given the function...

$$f(x) = 3x^4 - 16x^3 + 18x^2 \ , \ -1 \le x \le 4$$

Determine the absolute maximum and minimum values of the function.



Finding Absolute Maxima/Minima

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Determine the absolute max and absolute min for the following function.

$f(x) = x^3 + 6x^2 + 9x + 2 \quad -4 \leq x \leq 1$ $\quad x \in [-4, 1]$

$f'(x) = 3x^2 + 12x + 9$ $\quad$ CV: $x = -3, -1$

$f'(x) = 3(x^2 + 4x + 3)$

$f'(x) = 3(x+1)(x+3)$

$f(x) = x^3 + 6x^2 + 9x + 2$

$f(-3) = -27 + 54 - 27 + 2 = 2$ $\quad (-3, 2)$

$f(-1) = -1 + 6 - 9 + 2 = -2$ $\quad (-1, -2)$ abs min

$f(-4) = -64 + 96 - 36 + 2 = -2$ $\quad (-4, -2)$ abs min

$f(1) = 1 + 6 + 9 + 2 = 18$ $\quad (1, 18)$ abs max

Quadratics

Determine the absolute max or absolute min for the following function.

Recall Parabolas have either a max or a min at the vertex!

$$y = x^2 + 4x + 10$$

$y' = 2x + 4$

$y' = 2(x + 2)$

CV: $x = -2$

$y = (-2)^2 + 4(-2) + 10$

$= 4 - 8 + 10$

$= 6$

$(-2, 6)$ abs min because it opens up

Homework