

Transformations of Exponential Functions

Focus on...

- applying translations, stretches, and reflections to the graphs of exponential functions
- representing these transformations in the equations of exponential functions
- solving problems that involve exponential growth or decay

Link the Ideas

The graph of a function of the form $f(x) = a(c)^{b(x-h)} + k$ is obtained by applying transformations to the graph of the base function $y = c^x$, where $c > 0$.

Parameter	Transformation	Example
a	<ul style="list-style-type: none"> Vertical stretch about the x-axis by a factor of a For $a < 0$, reflection in the x-axis $(x, y) \rightarrow (x, ay)$ 	
b	<ul style="list-style-type: none"> Horizontal stretch about the y-axis by a factor of $\frac{1}{ b }$ For $b < 0$, reflection in the y-axis $(x, y) \rightarrow (\frac{x}{b}, y)$ 	
k	<ul style="list-style-type: none"> Vertical translation up or down $(x, y) \rightarrow (x, y + k)$ 	
h	<ul style="list-style-type: none"> Horizontal translation left or right $(x, y) \rightarrow (x + h, y)$ 	

Example 1

Apply Transformations to Sketch a Graph

Consider the base function $y = 3^x$. For each transformed function,

- state the parameters and describe the corresponding transformations
- create a table to show what happens to the given points under each transformation

$y = 3^x$
$(-1, \frac{1}{3})$
$(0, 1)$
$(1, 3)$
$(2, 9)$
$(3, 27)$

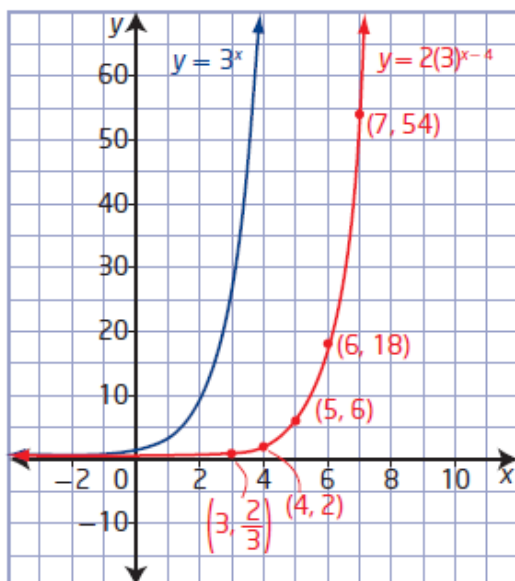
- sketch the graph of the base function and the transformed function
 - describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts
- $y = 2(3)^{x-4}$
 - $y = -\frac{1}{2}(3)^{\frac{1}{5}x} - 5$

Solution

- a) i) Compare the function $y = 2(3)^{x-4}$ to $y = a(c)^{b(x-h)} + k$ to determine the values of the parameters.
- $b = 1$ corresponds to no horizontal stretch.
 - $a = 2$ corresponds to a vertical stretch of factor 2. Multiply the y -coordinates of the points in column 1 by 2.
 - $h = 4$ corresponds to a translation of 4 units to the right. Add 4 to the x -coordinates
 - $k = 0$ corresponds to no vertical translation.
- ii) Add columns to the table representing the transformations.

$y = 3^x$	$y = 2(3)^{x-4}$
$(-1, \frac{1}{3})$	
$(0, 1)$	
$(1, 3)$	
$(2, 9)$	
$(3, 27)$	

- iii) To sketch the graph, plot the points from column 3 and draw a smooth curve through them.



- iv) The domain remains the same: $\{x \mid x \in \mathbb{R}\}$.

The range also remains unchanged: $\{y \mid y > 0, y \in \mathbb{R}\}$.

The equation of the asymptote remains as $y = 0$.

There is still no x -intercept, but the y -intercept changes to $\frac{2}{81}$ or approximately 0.025.

$$b) y = -\frac{1}{2}(3)^{\frac{1}{5}x} - 5$$

- i) state the parameters and describe the corresponding transformations
- ii) create a table to show what happens to the given points under each transformation
- iii) sketch the graph of the base function and the transformed function
- iv) describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts

$$b) y = \left(-\frac{1}{2}\right) (3)^{\frac{1}{5}x} - 5$$

(i) $a = -\frac{1}{2} \rightarrow$ vertically stretched by a factor of $\frac{1}{2}$ and reflected in the x -axis.

$b = \frac{1}{5} \rightarrow$ horizontally stretched by a factor of 5

$h = 0 \rightarrow$ No horizontal translation

$k = -5 \rightarrow$ translated 5 units down.

$$(x, y) \rightarrow \left[5x, -\frac{1}{2}y - 5\right]$$

(ii)

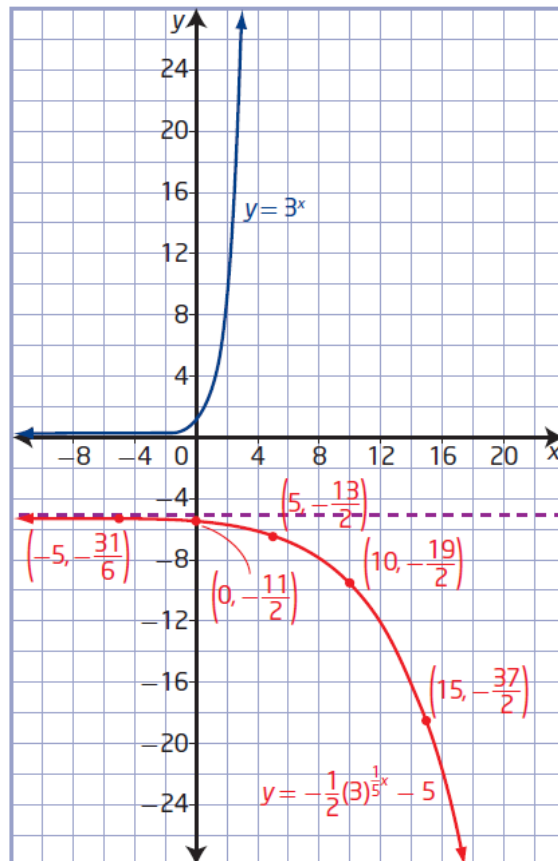
$y = 3^x$

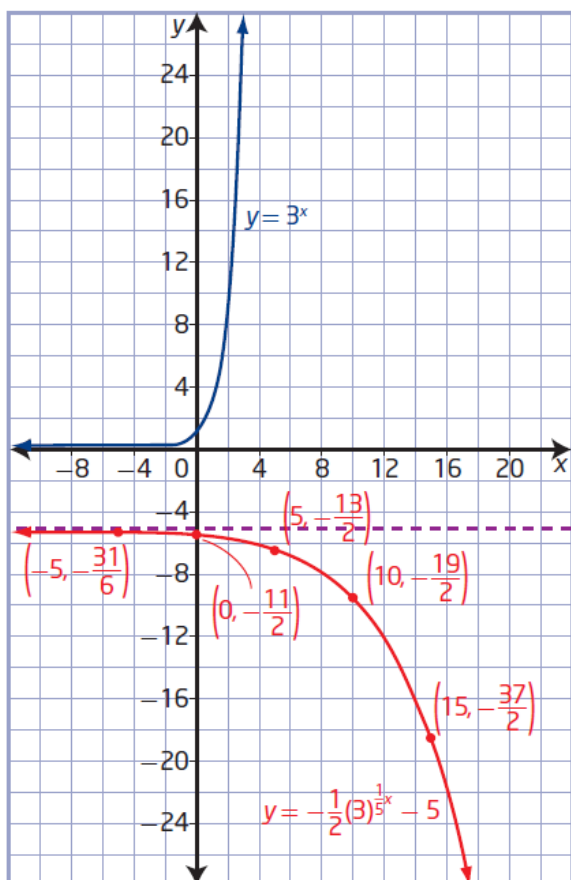
x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

$(x, y) \rightarrow [5x, -\frac{1}{2}y - 5]$

Transformed

x	y
-10	$-\frac{9}{18}$ or -5.05
-5	$-\frac{3}{6}$ or -5.16
0	$-\frac{1}{2}$ or -5.5
5	$-\frac{13}{2}$ or -6.5
10	$-\frac{19}{2}$ or -9.5





$$y = 3^x$$

$$D: \{x | x \in \mathbb{R}\}$$

$$R: \{y | y > 0, y \in \mathbb{R}\}$$

$$HA: y = 0$$

$$x\text{-int: none}$$

$$y\text{-int: } (0, 1)$$

$$y = -\frac{1}{2}(3)^{\frac{1}{5}x} - 5$$

$$D: \{x | x \in \mathbb{R}\}$$

$$R: \{y | y < -5, y \in \mathbb{R}\}$$

$$HA: y = -5$$

$$x\text{-int: none}$$

$$y\text{-int: } (x=0)$$

$$y = -\frac{1}{2}(3)^{\frac{1}{5}(0)} - 5$$

$$y = -\frac{1}{2}(3)^0 - 5$$

$$y = -\frac{1}{2}(1) - 5$$

$$y = -\frac{1}{2} - \frac{10}{2} = -\frac{11}{2} \text{ or } -5.5$$

$$(0, -5.5)$$

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Transform the graph of $y = 4^x$ to sketch the graph of $y = 4^{-2(x+5)} - 3$. Describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts.

Homework

#1-7 and #10 on page 354