# Questions From Homework

The sum of two numbers is 12. Find the numbers so that their product is a maximum?

A rectangle has a perimeter of 150cm. What length and width should it have so that its area is a maximum?

Find the point on the graph of $\boxed{y = 2x + 6}$ that is the minimum distance from the point (1, 2).

$x_1, y_1$

Let $x = x$

Let $y = y$

Remember ***d is smallest when $d^2$ is smallest***

$$d = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

$$d = \sqrt{(x-1)^2 + (\boxed{y}-2)^2}$$

Express with a single variable

$$d = \sqrt{(x-1)^2 + (2x+6-2)^2}$$

$$d = \sqrt{(x-1)^2 + (2x+4)^2} = \left[(x-1)^2 + (2x+4)^2\right]^{1/2}$$

$$f(x) = (x-1)^2 + (2x+4)^2$$

$$f'(x) = 2(x-1)(1) + 2(2x+4)(2)$$

$$f'(x) = 2x - 2 + 8x + 16$$

$$f'(x) = 10x + 14$$

$$0 = 10x + 14$$

$$-10x = 14$$

$$x = -\frac{7}{5} \longrightarrow y = 2\left(-\frac{7}{5}\right) + 6$$

$$y = -\frac{14}{5} + 6 = \frac{16}{5}$$

The point closest to (1, 2) is $\left(-\frac{7}{5}, \frac{16}{5}\right)$

Remember ***d* is smallest when *d*² is smallest ...** here is the proof!

$$d = \left[(x-1)^2 + (2x+4)^2\right]^{1/2}$$

$$d' = \frac{1}{2}\left[(x-1)^2+(2x+4)^2\right]^{-1/2}\left[2(x-1)(1)+2(2x+4)(2)\right]$$

$$= \frac{1}{2}\left[(x-1)^2+(2x+4)^2\right]^{-1/2}\left[2x-2+8x+16\right]$$

$$= \frac{1}{2}\left[(x-1)^2+(2x+4)^2\right]^{-1/2}(10x+14)$$

$$= \frac{5x+7}{\sqrt{(x-1)^2+(2x+4)^2}}$$

← denominator is always positive

So $d' = 0$ when $5x+7=0$

$$5x=-7$$

$$x=\frac{-7}{5}$$ ← is the only critical number

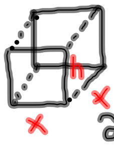
$$y = 2x+6$$
$$= 2\left(\frac{-7}{5}\right)+6$$
$$= \frac{-14}{5}+\frac{30}{5}$$
$$= \frac{16}{5}$$

∴ The point on the curve $y=2x+6$, that is closest to $(1,2)$, would be $\left(\frac{-7}{5}, \frac{16}{5}\right)$

If 2700 $cm^2$ (Area) of material is available to make a box with square base and an open top, find the largest possible volume of the box.

Let $x$ = *the length of the base*
Let $h$ = *the height*

We want to maximize the *volume*.




$$A = x^2 + 4xh$$
$$2700 = x^2 + 4xh$$
$$2700 - x^2 = 4xh$$
$$\boxed{h = \frac{2700 - x^2}{4x}}$$

$V = lwh$

$V = x^2 h$

We want to eliminate $h$ from the *volume* function and we do so by finding a relationship between $x$ *and* $h$. We use the area of the available material

$$V = x^{\cancel{2}}\left[\frac{2700 - x^2}{4\cancel{x}}\right]$$ ← Expressed with only one variable.

$$V = \frac{2700x - x^3}{4}$$

$$V = 675x - \frac{1x^3}{4}$$

$$V' = 675 - \frac{3x^2}{4}$$

$$0 = 675 - \frac{3x^2}{4}$$

$$\frac{3x^2}{4} = 675$$

$$3x^2 = 2700$$
$$x^2 = 900$$
$$x = \pm 30 \text{ cm}$$

$$h = \frac{2700 - (30)^2}{4(30)}$$

$$h = \frac{1800}{120}$$

$$h = 15 \text{ cm}$$

max

| (−31) | −30 | (0) | 30 | (31) |
|---|---|---|---|---|
| − | | + | | − |

$$V = x^2 h$$
$$= (30)^2(15)$$
$$= \underline{\underline{13500 \text{ cm}^3}}$$ is the maximum volume.

Find the points on the parabola $y = 6 - x^2$ that are closest to the point (0, 3)

# Homework