

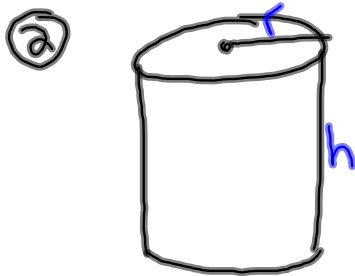
$$\textcircled{1} \text{ c) } y = x^5 + 8x^3 + x$$

$$y' = 5x^4 + 24x^2 + 1$$

increasing on  $(-\infty, \infty)$

←  $y'$  is always positive.

The function is always increasing



Let  $r$  = the radius

$$A = 2\pi r^2 + 2\pi r h$$

Express with a single variable

$$V = \pi r^2 h$$

$$1000 = \pi r^2 h$$

$$h = \frac{1000}{\pi r^2}$$

$$A = 2\pi r^2 + 2\pi r \left[ \frac{1000}{\pi r^2} \right]$$

$$A = 2\pi r^2 + 2000r^{-1}$$

$$A' = 4\pi r - \frac{2000}{r^2}$$

$$\frac{2000}{r^2} = 4\pi r$$

$$4\pi r^3 = 2000$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}} \approx 5.41 \text{ cm}$$

$$\textcircled{5} f(x) = 3x^4 - 16x^3 + 18x^2 - 1$$

$$f'(x) = 12x^3 - 48x^2 + 36x$$

$$f'(x) = 12x(x^2 - 4x + 3)$$

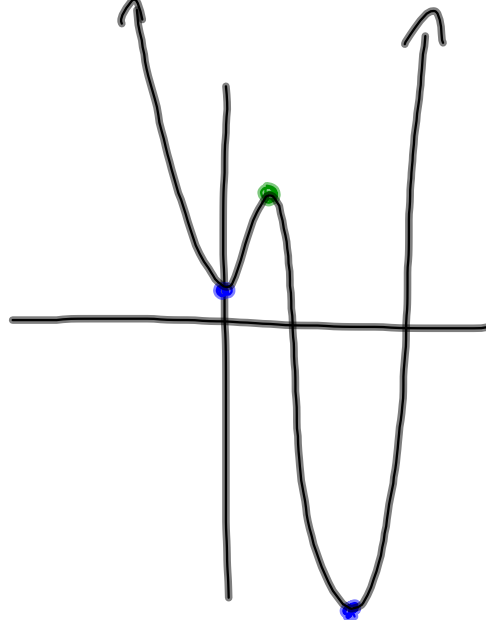
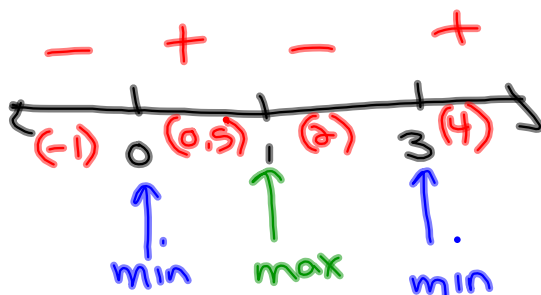
$$f'(x) = 12x(x-1)(x-3)$$

$$\text{CV: } x = 0, 1, 3$$

$$f(0) = 1 \quad (0, 1)$$

$$f(1) = 6 \quad (1, 6)$$

$$f(3) = -26 \quad (3, -26)$$

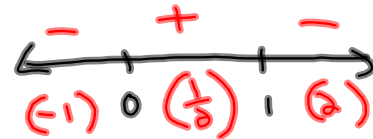


$$\textcircled{5} \text{ b) } f(x) = 1 + 3x^2 - 2x^3$$

$$f'(x) = 6x - 6x^2$$

$$f'(x) = 6x(1-x)$$

$$\text{CV: } x=0, 1$$

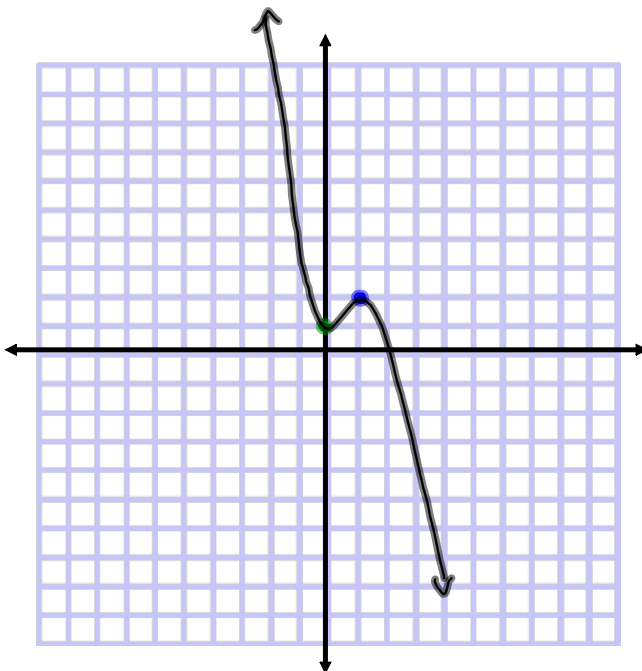


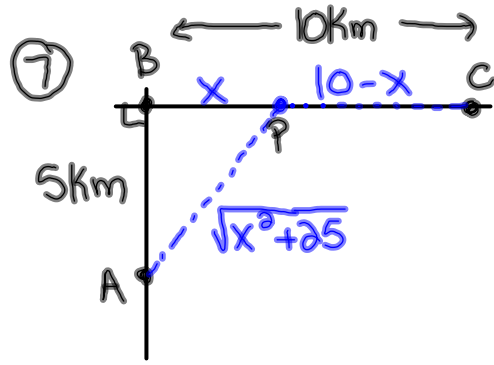
$$f(0) = 1 + 3(0)^2 - 2(0)^3 = 1$$

$(0, 1)$  local min

$$f(1) = 1 + 3(1)^2 - 2(1) = 2$$

$(1, 2)$  local max





Let  $x$  = the distance from B to P

$$T = \frac{d}{s}$$

$$T = \frac{\sqrt{x^2 + 25}}{3} + \frac{10-x}{5}$$

$$T = \frac{1}{3}(x^2 + 25)^{1/2} + \frac{10}{5} - \frac{1}{5}x$$

$$T' = \frac{1}{3} \cdot \frac{1}{2}(x^2 + 25)^{-1/2} (2x) - \frac{1}{5}$$

or

$$T' = \frac{5x - 3\sqrt{x^2 + 25}}{15\sqrt{x^2 + 25}}$$

denominator always +

$$T' = \frac{x}{3(x^2 + 25)^{1/2}} - \frac{1}{5}$$

$$\frac{1}{5} = \frac{x}{3\sqrt{x^2 + 25}}$$

$$(5x)^2 = (3\sqrt{x^2 + 25})^2$$

square both sides

$$25x^2 = 9(x^2 + 25)$$

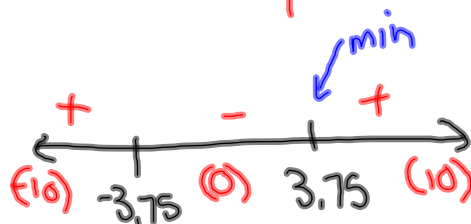
$$25x^2 = 9x^2 + 225$$

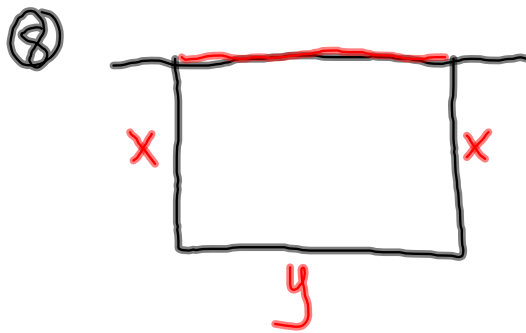
$$16x^2 = 225$$

$$x^2 = \frac{225}{16}$$

$$x = \pm \frac{15}{4} = \pm 3.75$$

∴ You should head to a point 3.75km east of B.





$$P = 2x + y$$

$$500 = 2x + y$$

$$500 - 2x = y$$

$$500 - 2(125) = y$$

$$500 - 250 = y$$

$$250\text{m} = y$$

$$\therefore 125\text{m} \times 250\text{m}$$

$$A = xy$$

Express as a single variable

$$A = x(500 - 2x)$$

$$A = 500x - 2x^2$$

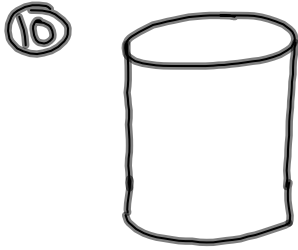
$$A' = 500 - 4x$$

$$4x = 500$$

$$x = 125\text{m}$$

|             |       |       |
|-------------|-------|-------|
| +           | ↓ max | -     |
| ————— ————— |       |       |
| (1)         | 125   | (200) |

Ex. 3.2



$$A = 169.56 \text{ cm}^2$$

$$2\pi r^2 + 2\pi r h = 169.56$$

$$2\pi r h = 169.56 - 2\pi r^2$$

$$h = \frac{169.56 - 2\pi r^2}{2\pi r}$$

$$h = \frac{169.56 - 2\pi(3)^2}{2\pi(3)}$$

$$h = 6 \text{ cm}$$

$$V = \pi r^2 h$$

$$V = \pi r^2 \left[ \frac{169.56 - 2\pi r^2}{2\pi r} \right]$$

$$V = \frac{169.56r - 2\pi r^3}{2}$$

$$V = 84.78r - \pi r^3$$

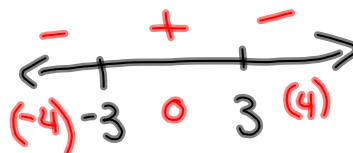
$$V' = 84.78 - 3\pi r^2$$

$$0 = 84.78 - 3\pi r^2$$

$$3\pi r^2 = 84.78$$

$$r^2 = 9$$

$$r = \pm 3$$



↑  
max

$$r = 3 \text{ cm}$$

$$\begin{aligned} \text{Max Volume} &= \pi r^2 h \\ &= \pi (3)^2 (6) \\ &= 54\pi \\ &= 169.56 \text{ cm}^3 \end{aligned}$$