Warm-Up

Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \leq 0$$

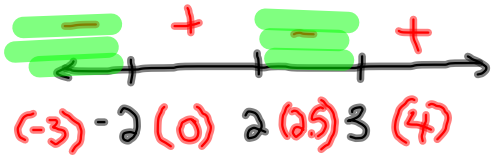
$y = (x^3 - 3x^2)(4x + 12)$

$y = x^2(x-3) - 4(x-3)$

$y = (x^2 - 4)(x-3)$

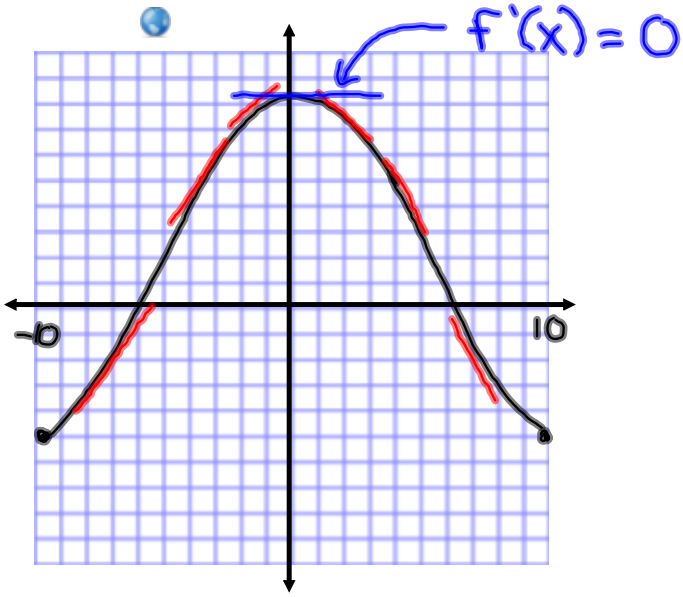
$y = (x+2)(x-2)(x-3)$

Roots: $x = -2, 2, 3$



$x \in (-\infty, -2] \cup [2, 3]$

Increasing and Decreasing Functions



Test for Increasing and Decreasing Functions

1. If $f'(x) > 0$ for all x in an interval I, then f is increasing on I. $x \in (-10, 0)$

2. If $f'(x) < 0$ for all x in an interval I, then f is decreasing on I $x \in (0, 10)$

Example 1

Find the intervals on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing.

Solution

First we find the derivative of $f(x) = 1 - 5x + 4x^2$ and get

$f'(x) = -5 + 8x$

The function f will be increasing when $-5 + 8x > 0$

$8x > 5$

$x > \frac{5}{8}$

Thus f will be increasing on the interval $x \in (5/8, \infty)$

Similarly,

The function f will be decreasing when $-5 + 8x < 0$

$8x < 5$

$x < \frac{5}{8}$

Thus f will be decreasing on the interval $x \in (-\infty, 5/8)$

Example 2

Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

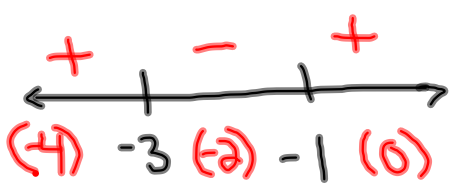
Solution

First we compute the derivative and factor it:

$y' = 3x^2 + 12x + 9$
$= 3(x^2 + 4x + 3)$
$= 3(x+1)(x+3)$

Critical Numbers:
$x = -3, -1$

The function f will be increasing when $y' > 0$, so we have to solve the quadratic ineqaulity $(x+1)(x+3)>0$



Increasing on $(-\infty,-3)+(-1,\infty)$
decreasing on $(-3,-1)$

or

Interval	(x+1)	(x+3)	$f'(x)$	f
$(-\infty,-3)$ (-4)	−	−	+	increasing
$(-3,-1)$ (-2)	−	+	−	decreasing
$(-1,\infty)$ (0)	+	+	+	increasing

Example 3

Find the intervals on which the function $f(x) = x^4 - 4x^3 - 8x^2 - 1$ is increasing and decreasing.

Solution

First we compute the derivative and factor it:

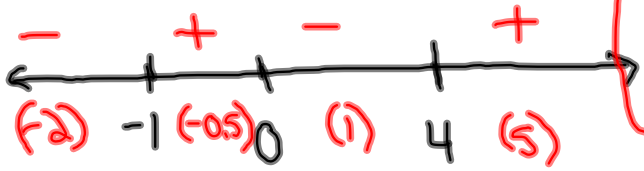
$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x+1)(x-4)$

Critical Numbers:

$x = -1, 0, 4$



increasing on $(-1,0) + (4,\infty)$

decreasing on $(-\infty,-1) + (0,4)$

or

Interval	$4x$	$(x+1)$	$(x-4)$	$f'(x)$	
$(-\infty,-1)$ (-2)	−	−	−	−	decreasing
$(-1,0)$ (-0.5)	−	+	−	+	increasing
$(0,4)$ (1)	+	+	−	−	decreasing
$(4,\infty)$ (5)	+	+	+	+	increasing

Homework