

Questions From Homework

③ (ii) $y = x^4 - 8x^2$

$$y' = 4x^3 - 16x$$

$$y' = 4x(x^2 - 4)$$

$$y' = 4x(x-2)(x+2)$$

CV: $x = -2, 0, 2$

$$y'' = 12x^2 - 16$$

$$y'' = 4(3x^2 - 4)$$

$$3x^2 - 4 = 0$$

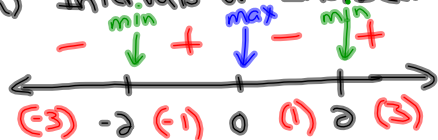
$$3x^2 = 4$$

$$x^2 = \frac{4}{3}$$

$$x = \pm \frac{2}{\sqrt{3}}$$

CV: $x = \pm \frac{2}{\sqrt{3}} \approx \pm 1.15$

a) Intervals of Inc/Dec.

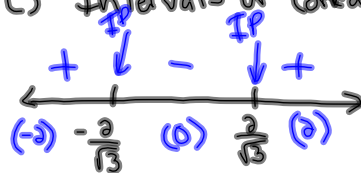


Increasing on $(-2, 0) \cup (2, \infty)$
 Decreasing on $(-\infty, -2) \cup (0, 2)$

Max Min

$f(-2) = -16$ $(-2, -16)$ min
 $f(0) = 0$ $(0, 0)$ max
 $f(2) = -16$ $(2, -16)$ min

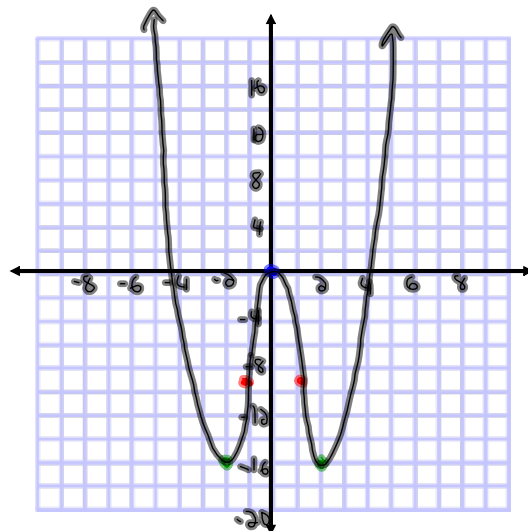
c) Intervals of Concavity



CU on $(-\infty, \frac{-2}{\sqrt{3}}) \cup (\frac{2}{\sqrt{3}}, \infty)$
 CD on $(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$

d) Inflection Points:

$f(\frac{-2}{\sqrt{3}}) = -\frac{80}{9}$ $(\frac{-2}{\sqrt{3}}, \frac{80}{9})$
 $f(\frac{2}{\sqrt{3}}) = -\frac{80}{9}$ $(\frac{2}{\sqrt{3}}, \frac{80}{9})$



Making a Complete Sketch

Example:

Examine the function $f(x) = x^4 - 4x^3$ with respect to...

- Intercepts $f(x)$
- Symmetry $f(x)$
- Asymptotes $f(x)$
- Intervals of Increase or Decrease $f'(x)$
- Local Maximum and Minimum values $f'(x)$ *Plug CV into original*
- Concavity and Points of Inflection $f''(x)$ *→ Plug CV into original*
- Sketch the Curve

① Intercepts:

x int: (y=0)

$$f(x) = x^4 - 4x^3$$

$$0 = x^3(x-4)$$

$$x = 0, 4$$

$$(0,0) \text{ and } (4,0)$$

y int: (x=0)

$$f(0) = (0)^4 - 4(0)^3$$

$$= 0$$

$$(0,0)$$

② Symmetry:

$$f(-x) = (-x)^4 - 4(-x)^3$$

$$= x^4 + 4x^3$$

No Symmetry

③ Asymptotes: No Asymptotes

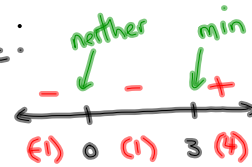
④ Intervals of Inc/Dec:

$$f(x) = x^4 - 4x^3$$

$$f'(x) = 4x^3 - 12x^2$$

$$f'(x) = 4x^2(x-3)$$

$$\text{CV: } x = 0, 3$$



Decreasing on $(-\infty, 3)$

Increasing on $(3, \infty)$

⑤ Local max/min:

$$f(3) = (3)^4 - 4(3)^3 = -27 \quad (3, -27) \text{ min}$$

⑥ Intervals of Concavity:

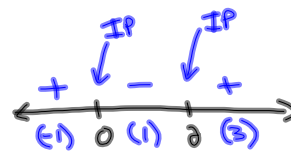
$$f(x) = x^4 - 4x^3$$

$$f'(x) = 4x^3 - 12x^2$$

$$f''(x) = 12x^2 - 24x$$

$$f''(x) = 12x(x-2)$$

$$\text{CV: } x = 0, 2$$



CU on $(-\infty, 0) \cup (2, \infty)$

CD on $(0, 2)$

⑦ Inflection Points:

$$f(0) = (0)^4 - 4(0)^3 = 0 \quad (0,0)$$

$$f(2) = (2)^4 - 4(2)^3 = -16 \quad (2, -16)$$

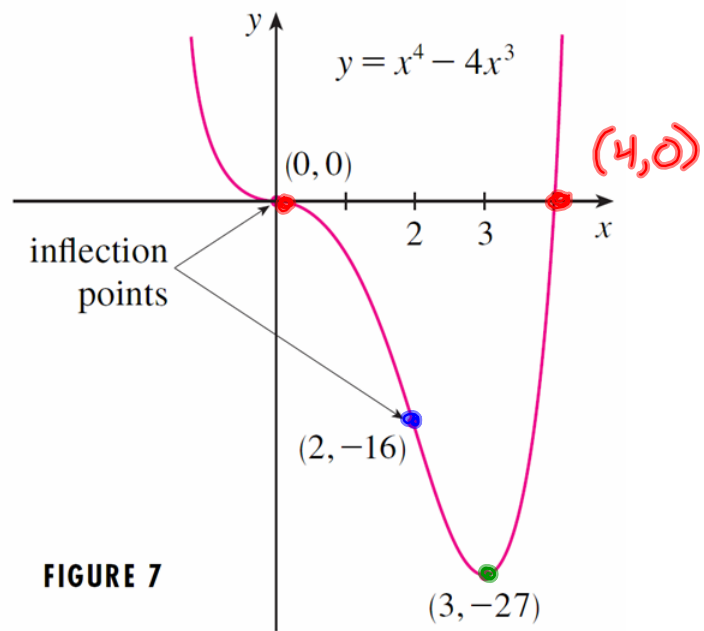


FIGURE 7

homework

Examine the function $f(x) = \frac{x^2}{1-x^2}$ with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

Examine the function $f(x) = 3x^5 - 5x^3$ with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

Examine the function $f(x) = \frac{x^2}{x-7}$ with respect to...

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve