Radical Functions and Transformations

Focus on...

- investigating the function $y = \sqrt{x}$ using a table of values and a graph
- graphing radical functions using transformations
- identifying the domain and range of radical functions

radical function

- a function that involves a radical with a variable in the radicand
- $y = \sqrt{3x}$ and $y = 4\sqrt[3]{5 + x}$ are radical functions.

Example 1

Graph Radical Functions Using Tables of Values

$y = a\sqrt{b(x-h)} + k$

Use a table of values to sketch the graph of each function. Then, state the domain and range of each function.

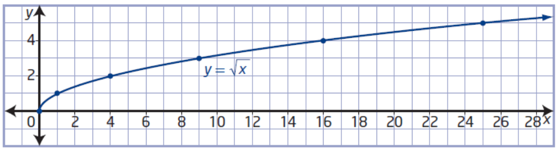
a) $y = \sqrt{x}$ **b)** $y = \sqrt{x - 2}$ **c)** $y = \sqrt{x} - 3$

(base function)

a) For the function $y = \sqrt{x}$, the radicand x must be greater than or equal to zero, $x \geq 0$. (cannot take the square root of a negative)

x	y
0	0
1	1
4	2
9	3
16	4
25	5

How can you choose values of x that allow you to complete the table without using a calculator? → use perfect squares



The graph has an endpoint at (0, 0) and continues up and to the right. The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$. The range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

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b) For the function $y = \sqrt{x - 2}$, the value of the radicand must be greater than or equal to zero.

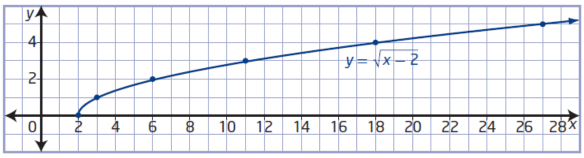
$x - 2 \geq 0$
$x \geq 2$

h = 2
Translated 2 units to the right

x	y
2	0
3	1
6	2
11	3
18	4
27	5

How is this table related to the table for $y = \sqrt{x}$ in part a)?

How does the graph of $y = \sqrt{x - 2}$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 2, x \in \mathrm{R}\}$. The range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

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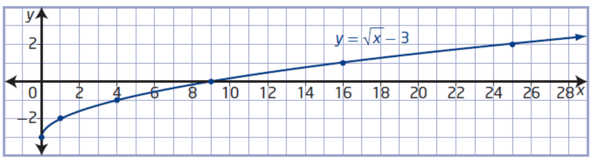
c) The radicand of $y = \sqrt{x} - 3$ must be non-negative.
$x \geq 0$

k = -3
Translated 3 units down

x	y
0	−3
1	−2
4	−1
9	0
16	1
25	2

How does the graph of $y = \sqrt{x} - 3$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -3, y \in \mathrm{R}\}$.

Graphing Radical Functions Using Transformations

You can graph a radical function of the form $y = a\sqrt{b(x - h)} + k$ by transforming the graph of $y = \sqrt{x}$ based on the values of a, b, h, and k. The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

- Parameter a results in a vertical stretch of the graph of $y = \sqrt{x}$ by a factor of $|a|$. If $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x-axis.
- Parameter b results in a horizontal stretch of the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{|b|}$. If $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y-axis.
- Parameter h determines the horizontal translation. If $h > 0$, the graph of $y = \sqrt{x}$ is translated to the right h units. If $h < 0$, the graph is translated to the left $|h|$ units.
- Parameter k determines the vertical translation. If $k > 0$, the graph of $y = \sqrt{x}$ is translated up k units. If $k < 0$, the graph is translated down $|k|$ units.

Example 2

Graph Radical Functions Using Transformations

$y = a\sqrt{b(x-h)} + k$

Sketch the graph of each function using transformations. Compare the domain and range to those of $y = \sqrt{x}$ and identify any changes.

a) $y = 3\sqrt{-(x - 1)}$

b) $y - 3 = -\sqrt{2x}$

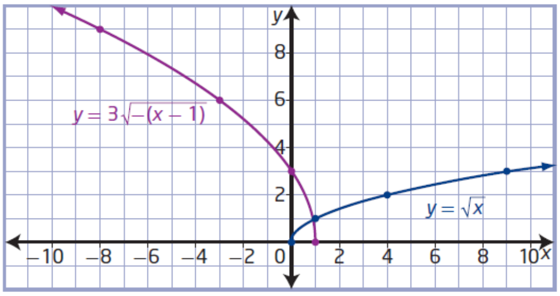
a) $y = 3\sqrt{-(x-1)}$

$a=3$ $b=-1$ $h=1$ $k=0$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x,y) \longrightarrow [-1x+1, 3y+0]$

x	y
1	0
0	3
-3	6
-8	9
-15	12
-24	15



Domain:

$\{x \mid x \le 1, x \in R\}$

$-(x-1) \ge 0$

$-x+1 \ge 0$

$-x \ge -1$

$x \le 1$

Range:

$\{y \mid y \ge 0, y \in R\}$

b) $y - 3 = -\sqrt{2x}$

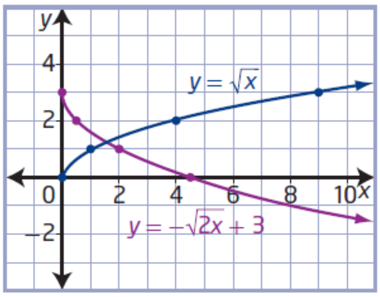
$y = -\sqrt{2x} + 3$

$a = -1$ $b = 2$ $h = 0$ $k = 3$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x, y) \longrightarrow \left[\frac{x}{2} + 0, -1y + 3\right]$

x	y
0	3
0.5	2
2	1
4.5	0
8	-1
12.5	-2



Domain: $\{x \mid x \geq 0, x \in R\}$

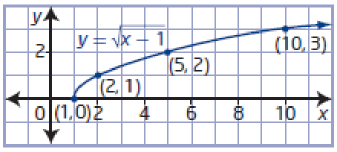
Range: $\{y \mid y \leq 3, y \in R\}$

Homework

#2-5

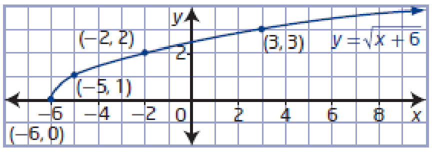
2.1 Radical Functions and Transformations, pages 72 to 77

1. a)



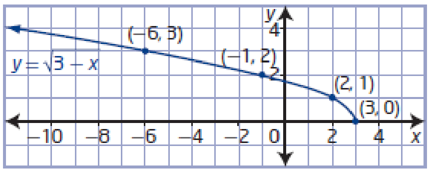
domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b)



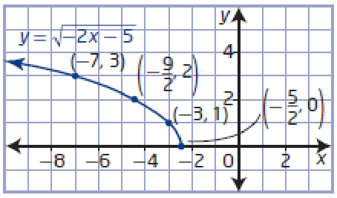
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)



domain $\{x \mid x \leq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d)



domain $\left\{x \mid x \leq -\frac{5}{2}, x \in \mathrm{R}\right\}$,
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

2. a) $a = 7 \rightarrow$ vertical stretch by a factor of 7
$h = 9 \rightarrow$ horizontal translation 9 units right
domain $\{x \mid x \geq 9, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) $b = -1 \rightarrow$ reflected in y-axis
$k = 8 \rightarrow$ vertical translation up 8 units
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 8, y \in \mathrm{R}\}$

c) $a = -1 \rightarrow$ reflected in x-axis
$b = \frac{1}{5} \rightarrow$ horizontal stretch factor of 5
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

d) $a = \frac{1}{3} \rightarrow$ vertical stretch factor of $\frac{1}{3}$
$h = -6 \rightarrow$ horizontal translation 6 units left
$k = -4 \rightarrow$ vertical translation 4 units down
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$,
range $\{y \mid y \geq -4, y \in \mathrm{R}\}$

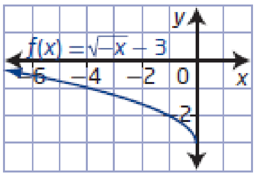
3. a) B **b)** A **c)** D **d)** C

4. a) $y = 4\sqrt{x + 6}$ **b)** $y = \sqrt{8x} - 5$

c) $y = \sqrt{-(x - 4)} + 11$ or $y = \sqrt{-x + 4} + 11$

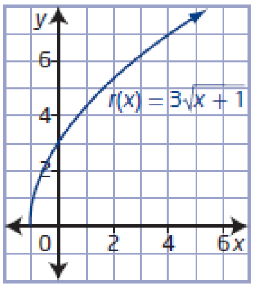
d) $y = -0.25\sqrt{0.1x}$ or $y = -\frac{1}{4}\sqrt{\frac{1}{10}x}$

5. a)



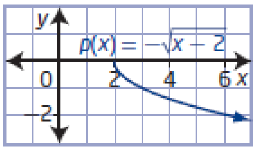
domain
$\{x \mid x \le 0, x \in \mathrm{R}\}$,
range
$\{y \mid y \ge -3, y \in \mathrm{R}\}$

b)



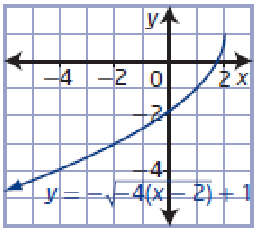
domain
$\{x \mid x \ge -1, x \in \mathrm{R}\}$,
range
$\{y \mid y \ge 0, y \in \mathrm{R}\}$

c)



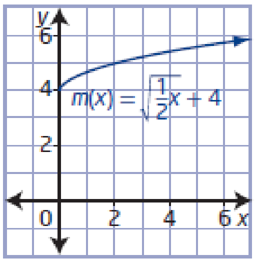
domain
$\{x \mid x \ge 2, x \in \mathrm{R}\}$,
range
$\{y \mid y \le 0, y \in \mathrm{R}\}$

d)



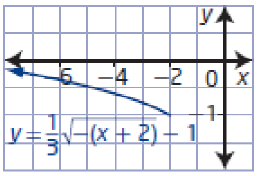
domain
$\{x \mid x \le 2, x \in \mathrm{R}\}$,
range
$\{y \mid y \le 1, y \in \mathrm{R}\}$

e)



domain
$\{x \mid x \ge 0, x \in \mathrm{R}\}$,
range
$\{y \mid y \ge 4, y \in \mathrm{R}\}$

f)



domain
$\{x \mid x \le -2, x \in \mathrm{R}\}$
range
$\{y \mid y \ge -1, y \in \mathrm{R}\}$