Recall the sigma formulas from Advanced Math 120, we will need them here! They're going to help... $\sum_{i=1}^{n} i^0 = 1^0 + 2^0 + 3^0 + 4^0 + \ldots n^0 = n$

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + 4 + \ldots . n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + \ldots . n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + 4^3 + \ldots n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Riemann Sum

Area = $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$

Evaluating the area under a curve y = f(x) from a to b.

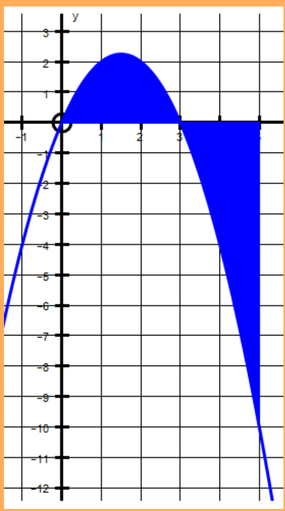
Δx (width of subinterval) will be the size of the interval [a,b] divided by n strips.

$$\Delta x = \frac{b-a}{n}$$

The i^{th} position, x_i, is: $x_i = a + i\Delta x$

Example: Find the area under the curve for $f(x) = 3x - x^2$ between x = 0 and x = 5 using the limit as $n \to \infty$ of the Riemann sum.

Area = - 25/6



① $\Delta x = \frac{5}{n}$

② $x_i^* = \frac{5i}{n}$

③ $f\left(\frac{5i}{n}\right) = 3\left(\frac{5i}{n}\right) - \left(\frac{5i}{n}\right)^2$

$= \frac{15i}{n} - \frac{25i^2}{n^2}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{15i}{n} - \frac{25i^2}{n^2}\right)\left(\frac{5}{n}\right)$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{75i}{n^2} - \frac{125i^2}{n^3}$$

$$A = \lim_{n\to\infty} \frac{75}{n^2} \cdot \frac{n(n+1)}{2} - \frac{125}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$A = \lim_{n\to\infty} \frac{75n^2 + 75n}{2n^2} - \frac{250n^3 - 375n^2 - 125n}{6n^3}$$

$$A = \frac{75}{2} - \frac{250}{6}$$

$$A = \frac{225 - 250}{6} = \boxed{-\frac{25}{6}}$$

The Definite Integral

When we computed the area under a curve by summing the areas of many rectangles, the limit took the form....

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty} [f(x_1^*)\,\Delta x + f(x_2^*)\,\Delta x + \cdots + f(x_n^*)\,\Delta x]$$

(this also arises when we try to find the distance traveled by an object)

It turns out that this same type of limit occurs in a wide variety of situations even when f is not necessarily a positive function. Limits of the this form also arise in finding lengths of curves, volumes of solids, centers of mass, force due to water pressure, and work, as well as other quantities. We therefore give this type of limit a special name and notation.

2 Definition of a Definite Integral If f is a continuous function defined for $a \leqslant x \leqslant b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0 (= a), x_1, x_2, \ldots, x_n (= b)$ be the endpoints of these subintervals and we choose **sample points** $x_1^*, x_2^*, \ldots, x_n^*$ in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the **definite integral of *f* from *a* to *b*** is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

NOTE 1 ▫ The symbol $\int$ was introduced by Leibniz and is called an **integral sign**. It is an elongated S and was chosen because an integral is a limit of sums. In the notation $\int_a^b f(x)\,dx$, $f(x)$ is called the **integrand** and a and b are called the **limits of integration**; a is the **lower limit** and b is the **upper limit**. The symbol dx has no official meaning by itself; $\int_a^b f(x)\,dx$ is all one symbol. The procedure of calculating an integral is called **integration**.

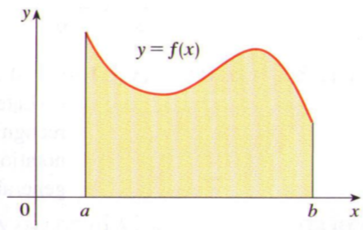


FIGURE 2
If $f(x) \geqslant 0$, the integral $\int_a^b f(x)\,dx$ is the area under the curve $y = f(x)$ from a to b.

The Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus, Part 2 If f is continuous on $[a, b]$, then

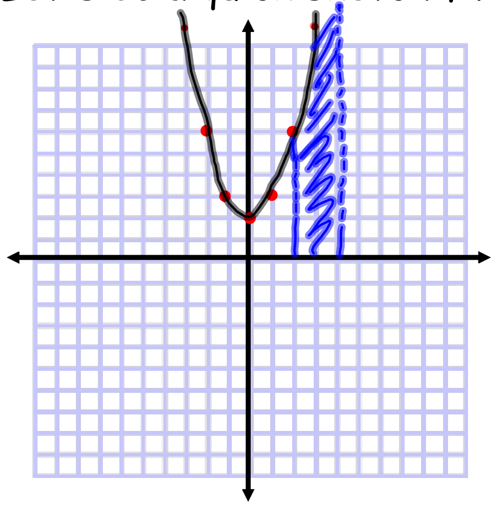
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function such that $F' = f$.

Find the area under:

$y = x^2 + 2,$ from $x = 2$ to $x = 4$

Let's do a quick sketch first



$$A = \int_2^4 x^2 + 2\,dx = \frac{x^3}{3} + 2x \Big]_2^4$$

$$= \frac{(4)^3}{3} + 2(4) - \left[\frac{(2)^3}{3} + 2(2)\right]$$

$$= \frac{64}{3} + 8 - \frac{8}{3} - 4$$

$$= \frac{56}{3} + 4$$

$$= \frac{56}{3} + \frac{12}{3}$$

$$A = \frac{68}{3}$$

$\int$ Area of a region between two curves! $\int$

Applications of Integration

Area of a Region between Two Curves

6.1 Areas between Curves

In Chapter 5 we defined and calculated areas of regions that lie under the graphs of functions. Here we use integrals to find areas of regions that lie between the graphs of two functions.

Consider the region S that lies between two curves $y = f(x)$ and $y = g(x)$ and between the vertical lines $x = a$ and $x = b$, where f and g are continuous functions and $f(x) \geq g(x)$ for all x in $[a, b]$. (See Figure 1.)

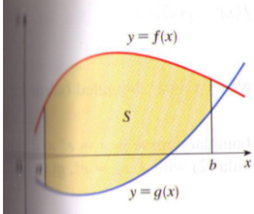
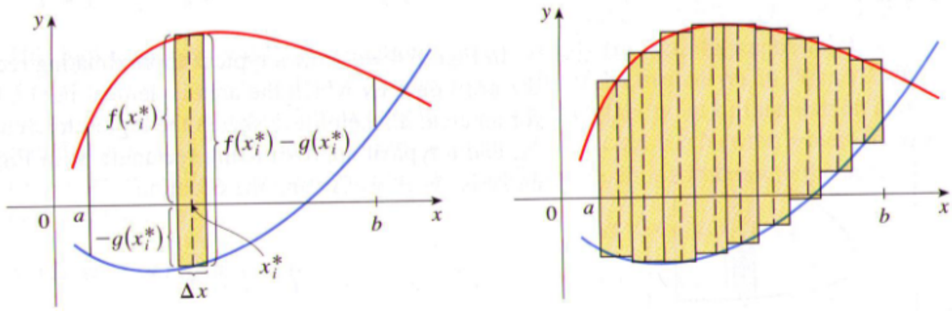


FIGURE 1
$S = \{(x, y) \mid a \leq x \leq b, g(x) \leq y \leq f(x)\}$

Just as we did for areas under curves in Section 5.1, we divide S into n strips of equal width and then we approximate the ith strip by a rectangle with base Δx and height $f(x_i^*) - g(x_i^*)$. (See Figure 2. If we like, we could take all of the sample points to be right endpoints, in which case $x_i^* = x_i$.) The Riemann sum

$$\sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)] \Delta x$$

is therefore an approximation to what we intuitively think of as the area of S.



(a) Typical rectangle (b) Approximating rectangles

This approximation appears to become better and better as $n \to \infty$. Therefore, we define the **area** A of S as the limiting value of the sum of the areas of these approximating rectangles.

$$\boxed{1} \qquad \boxed{A = \lim_{n \to \infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)] \Delta x}$$

We recognize the limit in (1) as the definite integral of $f - g$. Therefore, we have the following formula for area.

Area Between Two Curves:

If f and g are continuous on [a,b] and g (x) ≤ f (x) for all x in [a,b], then the area of the region bounded by:

$y_2 = f(x)$, $y_1 = g(x)$, $x = a$, and $x = b$ is given by:

top bottom

$$A = \int_a^b [f(x) - g(x)] dx$$

may also be stated as:

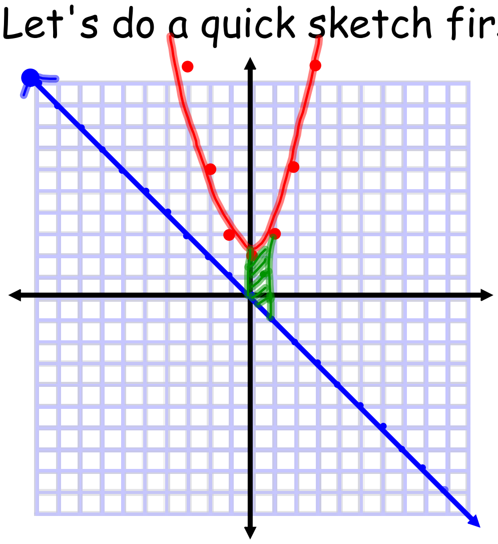
$$A = \int_a^b [y_2 - y_1] dx$$

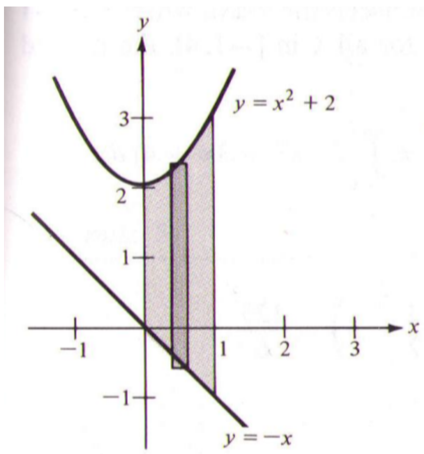
Find the area of the region bounded by:

$y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$

$f(x)$ $g(x)$

Let's do a quick sketch first





$f(x) - g(x) = x^2 + 2 - (-x)$
$= x^2 + x + 2$

$$A = \int_0^1 (x^2 + x + 2)dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 2x \Big]_0^1$$

$$= \frac{(1)^3}{3} + \frac{(1)^2}{2} + 2(1) - \left[\frac{(0)^3}{3} + \frac{(0)^2}{2} + 2(0)\right]$$

$$= \frac{1}{3} + \frac{1}{2} + 2 - 0$$

$$= \frac{2+3+12}{6}$$

$$= \frac{17}{6}$$

$$A = \frac{17}{6}$$

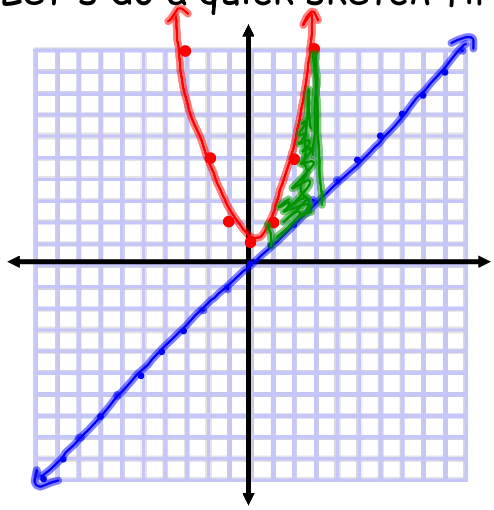
Find the area of the region bounded by:

$y = x^2 + 1$, $y = x$, $x = 1$, and $x = 3$

$f(x)$ $g(x)$

$f(x) - g(x) = x^2 + 1 - x$

Let's do a quick sketch first



$$A = \int_1^3 x^2 - x + 1\,dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x \Big]_1^3$$

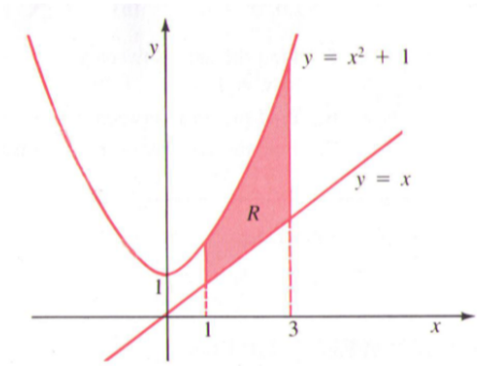
$$= \frac{(3)^3}{3} - \frac{(3)^2}{2} + (3) - \left[\frac{(1)^3}{3} - \frac{(1)^2}{2} + (1)\right.$$

$$= 9 - \frac{9}{2} + 3 - \frac{1}{3} + \frac{1}{2} - 1$$

$$= \frac{66}{6} - \frac{27}{6} - \frac{2}{6} + \frac{3}{6}$$

$$= \frac{40}{6}$$

$$A = \frac{20}{3}$$



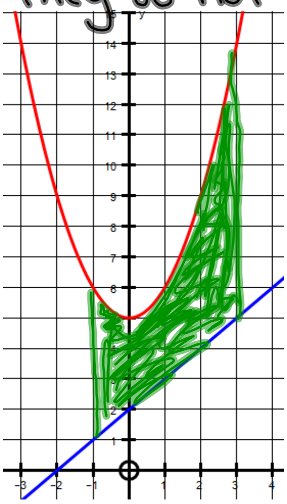
Find the area bounded between the curves defined by:

$y = x^2 + 5$ $\quad$ $y = x + 2$ $\quad$ $x = -1$ $\quad$ and $\quad$ $x = 3$

$f(x)$ $\quad$ $g(x)$

where do they intersect??

↳ they do not



$$A = \int_{-1}^{3} x^2 - x + 3\,dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + 3x \Big]_{-1}^{3}$$

$$= \frac{(3)^3}{3} - \frac{(3)^2}{2} + 3(3) - \left[\frac{(-1)^3}{3} - \frac{(-1)^2}{2} + 3(-1)\right]$$

$$= 9 - \frac{9}{2} + 9 + \frac{1}{3} + \frac{1}{2} + 3$$

$$= \frac{126}{6} - \frac{27}{6} + \frac{2}{6} + \frac{3}{6}$$

$$= \frac{104}{6}$$

$$= \boxed{\frac{52}{3}}$$

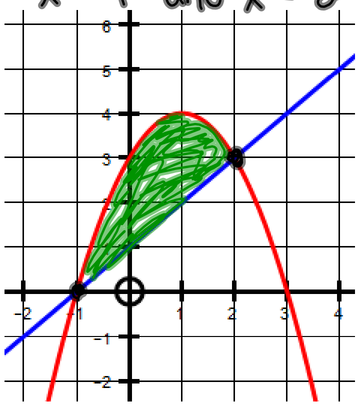
Find the area bounded between the curves defined by:

$y = -x^2 + 2x + 3$ and $y = x + 1$

$f(x)$ $g(x)$

where do they intersect??

$x = -1$ and $x = 2$



$$A = \int_{-1}^{2} -x^2 + x + 2\,dx$$

$$= -\frac{x^3}{3} + \frac{x^2}{2} + 2x \Big]_{-1}^{2}$$

$$= -\frac{(2)^3}{3} + \frac{(2)^2}{2} + 2(2) - \left[-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + 2(-1)\right]$$

$$= \frac{-8}{3} + 2 + 4 - \frac{1}{3} - \frac{1}{2} + 2$$

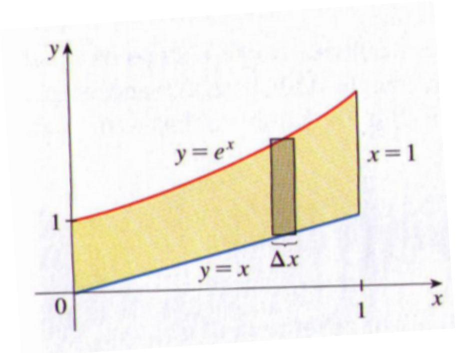
$$= \frac{48}{6} - \frac{16}{6} - \frac{2}{6} - \frac{3}{6}$$

$$= \boxed{-\frac{27}{6}}$$

Find the area of the region bounded by:

$y = e^x$, $y = x$, $x = 0$, and $x = 1$

$f(x)$ $g(x)$



$$A = \int_0^1 e^x - x\,dx$$

$$= e^x - \frac{x^2}{2}\Big]_0^1$$

$$= e^1 - \frac{(1)^2}{2} - \left[e^0 - \frac{(0)^2}{2}\right]$$

$$= e - \frac{1}{2} - 1$$

$$= e - \frac{1}{2} - \frac{2}{2}$$

$$\boxed{= e - \frac{3}{2}}$$

WARM-UPS

Compute the area under each of the following curves in [a,b] using the Fundamental Theorem of Calculus!

$$A=\int_{-1}^{2}(-x^2+x+2)dx$$

$$= -\frac{x^3}{3}+\frac{x^2}{2}+2x\Big]_{-1}^{2}$$

$$= -\frac{8}{3}+2+4-\left[\frac{1}{3}+\frac{1}{2}-2\right]$$

$$= 8-\frac{8}{3}-\frac{1}{3}-\frac{1}{2} \qquad A=\frac{9}{2}$$

$$A=\int_{1}^{4}(3x^2-2x)dx \qquad A=48$$

$$A=\int_{1}^{4}\sqrt{x}\,dx \qquad A=\frac{14}{3}$$

$$A=\int_{0}^{3}(x-1)dx \qquad A=\frac{3}{2}$$

We also have indefinite integrals! (we were asked to find in Ex. 11.2)
(just integrate and put a + C at the end)

Find:

$$\int(6x^2+\csc^2 x)dx \qquad =2x^3-\cot x+C$$

$$\int(10x^4-2\sec^2 x)dx \qquad =2x^5-2\tan x+C$$

$$\int\frac{\cos\theta}{\sin^2\theta}d\theta$$

- isn't immediately apparent, so perhaps change using a trig identity first

$$=-\csc x+C$$

$$\int\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}d\theta$$

$$\int\csc\theta\cot\theta\,d\theta=-\csc\theta+C$$

In the previous examples, the curves i.e. $y = x^2 + 2$ and $y = -x$ do not intersect, and the values of a and b were explicitly given.

A more common type of problem involves the area of the region bounded by two intersecting curves, therefore the value of a and b must be calculated first before determining the area between the intersecting curves.

Example: Find the area of the region bounded by:

$y = 2 - x^2$, $\quad y = x$

First, find the points of intersection. (set the two functions equal to each other and solve for x)

$2 - x^2 = x$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x + 2 = 0 \quad | \quad x - 1 = 0$

$x = -2 \quad | \quad x = 1$

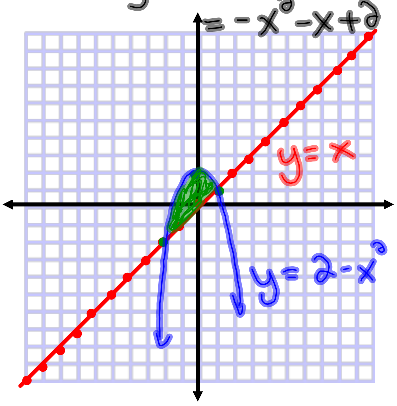
lower limit upper limit

Second, integrate to compute the area between the curves.

Let's do a quick sketch first

$f(x) - g(x) = 2 - x^2 - x$

$= -x^2 - x + 2$



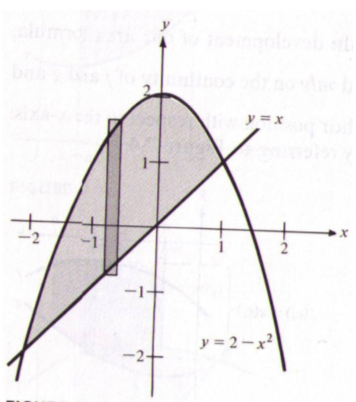


FIGURE 7.6

$$A = \int_{-2}^{1} -x^2 - x + 2\,dx$$

$$= -\frac{x^3}{3} - \frac{x^2}{2} + 2x \Big]_{-2}^{1}$$

$$= -\frac{(1)^3}{3} - \frac{(1)^2}{2} + 2(1) - \left[-\frac{(-2)^3}{3} - \frac{(-2)^2}{2} + 2(-2)\right]$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 - \frac{8}{3} + 2 + 4$$

$$= 8 - \frac{1}{3} - \frac{1}{2} - \frac{8}{3}$$

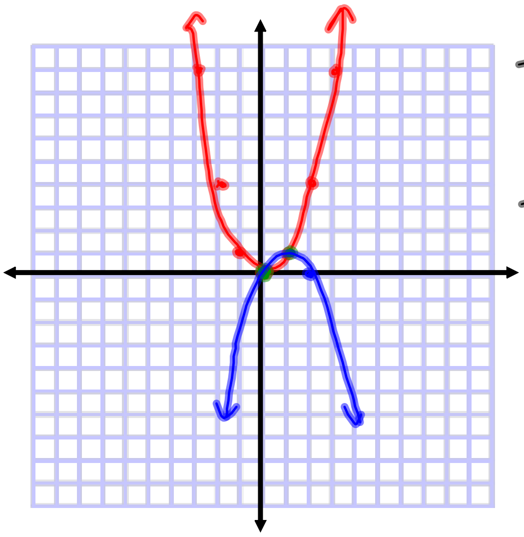
$$= \frac{48}{6} - \frac{2}{6} - \frac{3}{6} - \frac{16}{6}$$

$$= \frac{27}{6}$$

$$A = \frac{9}{2}$$

Example: Find the area of the region bounded by:

$y = x^2$, and $y = 2x - x^2 = x(2-x)$



Intersection:

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

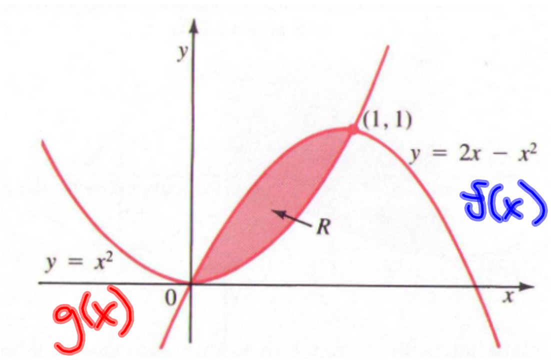
$2x = 0 \mid x - 1 = 0$

$x = 0 \mid x = 1$

Since $y = 2x - x^2$ is above $y = x^2$ on $[0, 1]$ we must integrate:

$f(x) - g(x) = 2x - x^2 - x^2$

$= 2x - 2x^2$



$A = \int_0^1 2x - 2x^2\,dx = x^2 - \frac{2}{3}x^3 \Big]_0^1$

$A = (1)^2 - \frac{2(1)^3}{3} - \left[(0)^2 - \frac{2(0)^3}{3}\right]$

$A = 1 - \frac{2}{3} - 0$

$A = \frac{1}{3}$

Evaluate each of the following and interpret the result in terms of areas.

(we did this one using Riemman sum)

$$A=\int_0^3 (x^3-6x)dx \qquad A=-\frac{27}{4}$$

$$= \frac{x^4}{4}-3x^2\Big]_0^3 = \frac{81}{4}-27-0 = \frac{81-108}{4} = -\frac{27}{4}$$

$$A=\int_0^2\left(2x^3-6x+\frac{3}{x^2+1}\right)dx \qquad A=-4+3\tan^{-1}2\approx -0.67855$$

$$= \frac{x^4}{2}-3x^2+3\tan^{-1}x\Big]_0^2 = 8-12+3\tan^{-1}2-0$$

$$= -4+3\tan^{-1}2$$

$$A=\int_1^9\left(\frac{2t^2+t^2\sqrt{t}-1}{t^2}\right)dt \qquad A=\frac{292}{9}\approx 32.444$$

$$A=\int_1^9 2+t^{1/2}-t^{-2}\,dt = 2t+\frac{2}{3}t^{3/2}+t^{-1}\Big]_1^9$$

$$= 18+\frac{54}{3}+\frac{1}{9}-2-\frac{2}{3}-1$$

$$= 15+\frac{54}{3}+\frac{1}{9}-\frac{2}{3}$$

$$= \frac{135+162+1-6}{9}$$

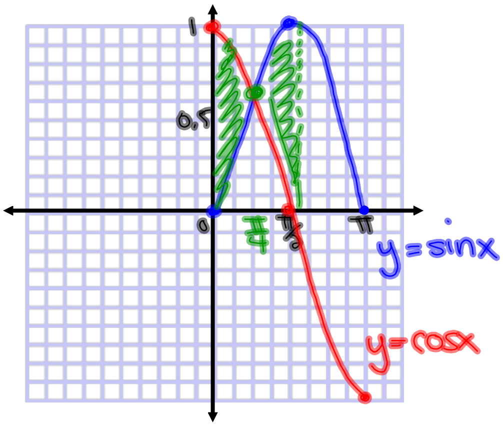
$$= \frac{292}{9}$$

Find the area of the region bounded by:

$y = \sin x$, $y = \cos x$, $x = 0$, and $x = \pi/2$

be careful... which curve is above the other and when???

Let's do a quick sketch first



On $[0, \frac{\pi}{4}]$

$$A = \int_0^{\pi/4} \cos x - \sin x \, dx$$

$$= \sin x + \cos x \Big]_0^{\pi/4}$$

$$= \sin\frac{\pi}{4} + \cos\frac{\pi}{4} - [\sin 0 + \cos 0]$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1$$

$$= \sqrt{2} - 1$$

On $[\frac{\pi}{4}, \frac{\pi}{2}]$

$$A = \int_{\pi/4}^{\pi/2} \sin x - \cos x \, dx$$

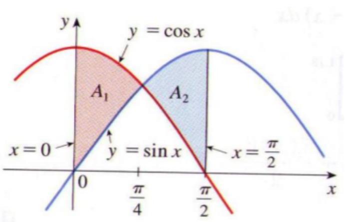
$$= -\cos x - \sin x \Big]_{\pi/4}^{\pi/2}$$

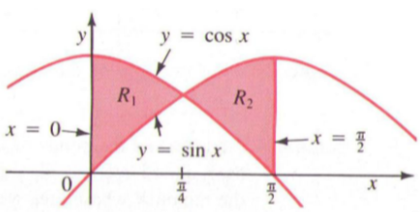
$$= -\cos\frac{\pi}{2} - \sin\frac{\pi}{2} - [-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}]$$

$$= 0 - 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$= \sqrt{2} - 1$$

$$\boxed{\begin{gathered} A = \sqrt{2} - 1 + \sqrt{2} - 1 \\ A = 2\sqrt{2} - 2 \end{gathered}}$$



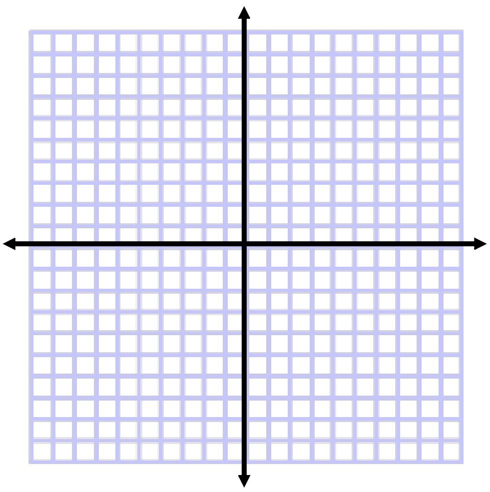


<u>Example:</u> Find the area of the region bounded by:

$y = x^2 - 3x - 4$ and the x-axis, $(y = 0)$

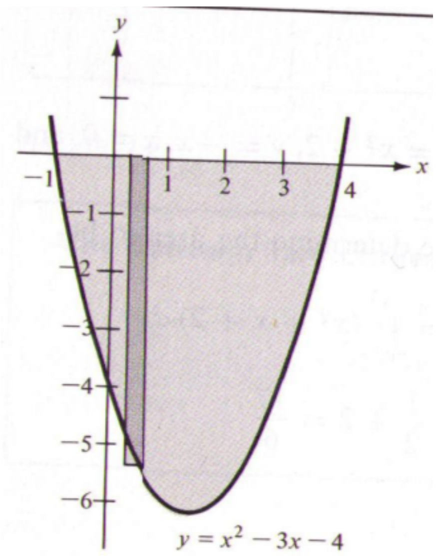
be careful... which curve is above the other???

Let's do a quick sketch first

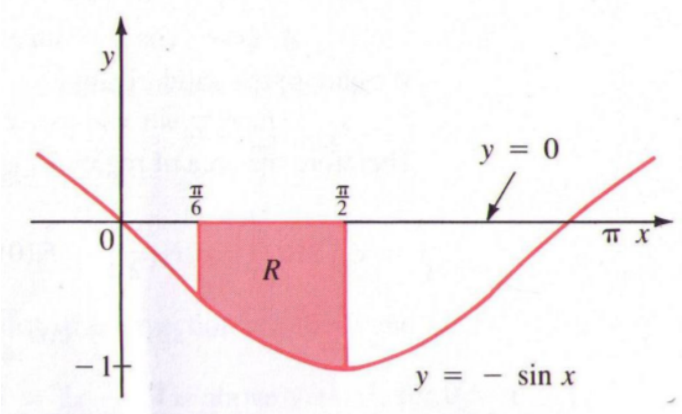


$$A = \int_{-1}^{4} (-x^2 + 3x + 4)dx$$

$$A = \frac{125}{6}$$



Example 4 Find the area between $y = -\sin x$ and the x-axis from $\frac{\pi}{6}$ to $\frac{\pi}{2}$.

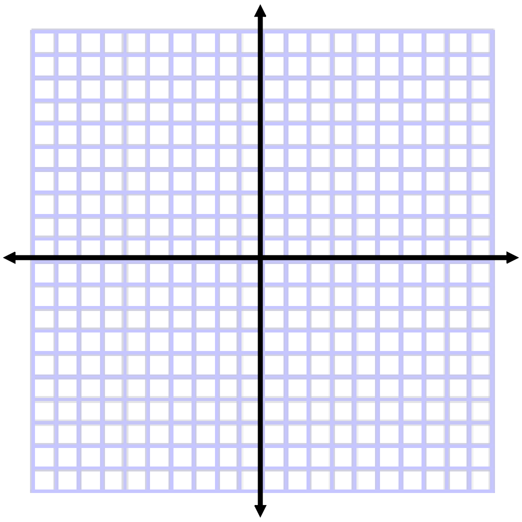


$A = \frac{\sqrt{3}}{2}$

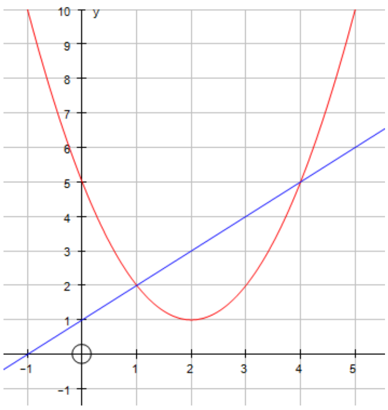
Determine the area between the curves:

$y = x^2 - 4x + 5$ and $y = x + 1$

First, were do they intersect??



A = 9/2

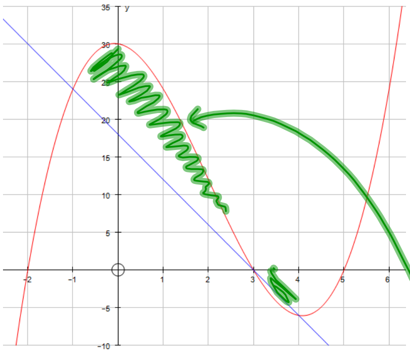


a) Determine the area between the curves:

$y = x^3 - 6x^2 - x + 30$ and $y = -6x + 18$ $A = 32$

between $x = -1$ to $x = 3$

$x^3 - 6x^2 - x + 30 \quad + 6x - 18$



$$A = \int_{-1}^{3} x^3 - 6x^2 + 5x + 12 \, dx$$

$$= \frac{x^4}{4} - 2x^3 + \frac{5x^2}{2} + 12x \Big]_{-1}^{3}$$

$$= \frac{81}{4} - 54 + \frac{45}{2} + 36 - \frac{1}{4} - 2 - \frac{5}{2} + 12$$

$$= 20 + 20 - 54 + 36 - 2 + 12$$

$$\boxed{= 32}$$

b) Determine the area between the curves from $x = 3$ to $x = 4$.

$-6x + 18 - x^3 + 6x^2 + x - 30$

$$A = \int_{3}^{4} -x^3 + 6x^2 - 5x - 12 \, dx$$

$$= -\frac{x^4}{4} + 2x^3 - \frac{5x^2}{2} - 12x \Big]_{3}^{4}$$

$$= -64 + 128 - 40 - 48 + \frac{81}{4} - 54 + \frac{45}{2} + 36$$

$$= -42 + \frac{81}{4} + \frac{90}{4}$$

$$= \frac{-168 + 81 + 90}{4}$$

$$\boxed{= \frac{3}{4}}$$

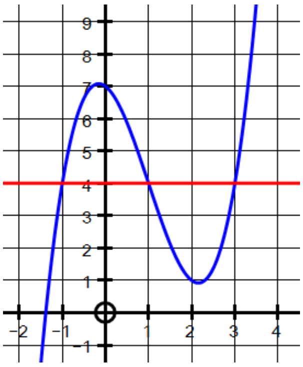
Total Area: $= 32 + \frac{3}{4} = \boxed{\frac{131}{4}}$

b) Determine the area between the curves:

A = 8

$y = x^3 - 3x^2 - x + 7$ and $y = 4$

between x = - 1 to x = 3

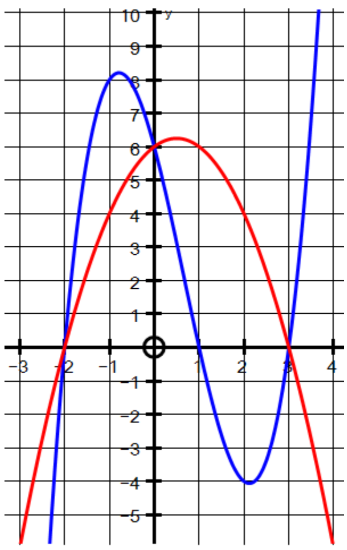


c) Determine the area between the curves:

A = 253/12

$y = x^3 - 2x^2 - 5x + 6$ and $y = - x^2 + x + 6$

between x = - 2 to x = 3

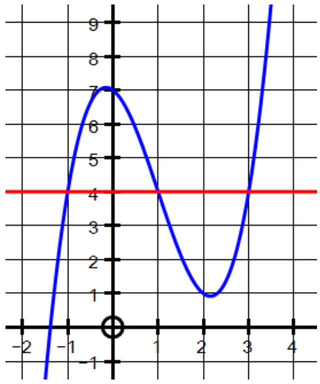


b) Determine the area between the curves:

$y = x^3 - 3x^2 - x + 7$ and $y = 4$

between $x = -1$ to $x = 3$

$A = 8$

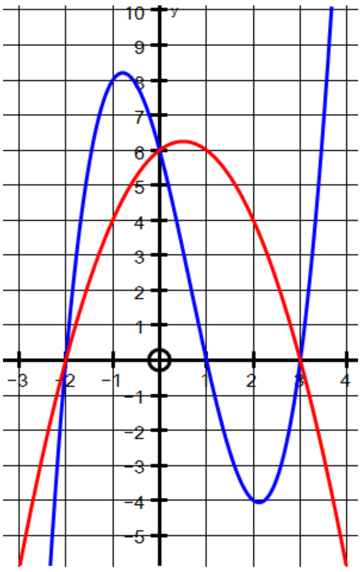


c) Determine the area between the curves:

$y = x^3 - 2x^2 - 5x + 6$ and $y = -x^2 + x + 6$

between $x = -2$ to $x = 3$

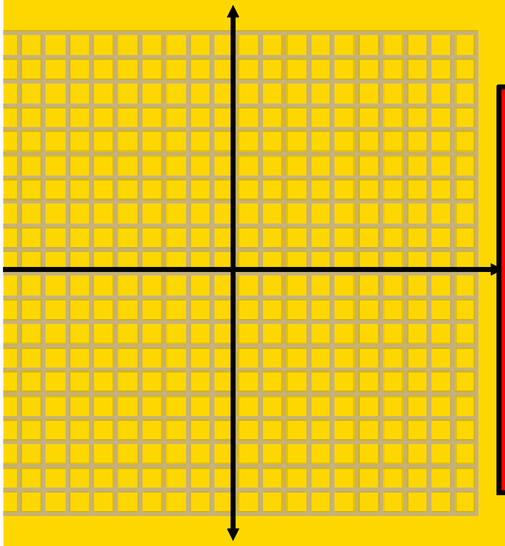
$A = 253/12$



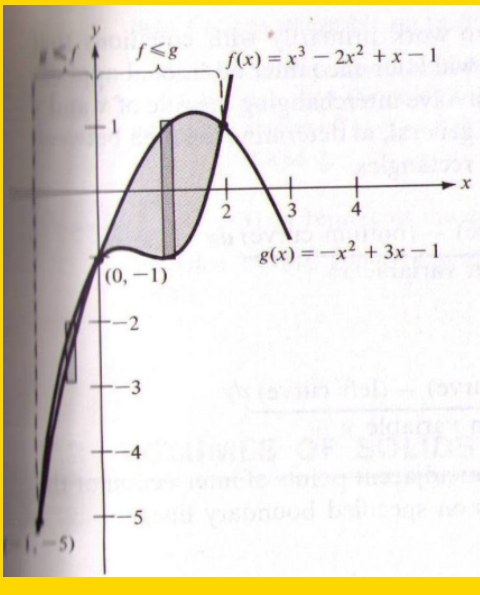
Occasionally, two curves intersect in more than two points. In determining the area of the region between two such curves, we must find all points of intersection and check to see which curve is above the other in each interval determined by these points.

<u>Example:</u> Find the area of the region between the curves:

$f(x) = x^3 - 2x^2 + x - 1$ and $g(x) = -x^2 + 3x - 1$



$$A = \int_{-1}^{0}[f(x) - g(x)]dx + \int_{0}^{2}[g(x) - f(x)]dx$$



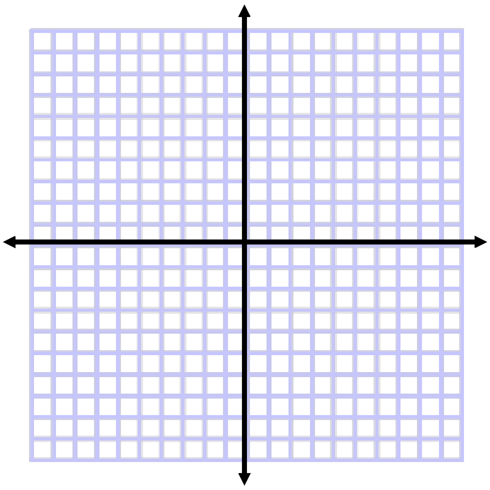
<u>Example:</u> Find the area of the region bounded by:

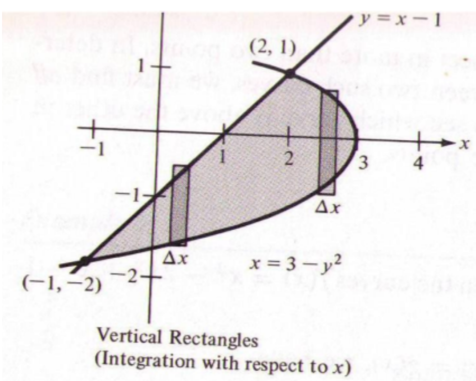
$x = 3 - y^2$ and $x = y + 1$

First find points of intersection....

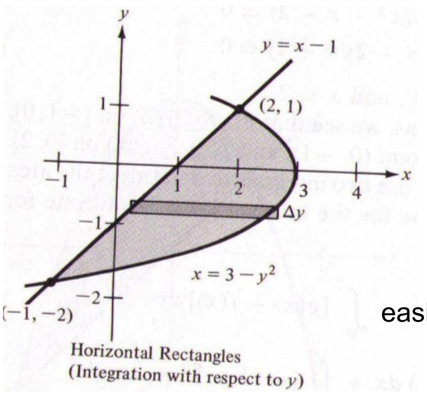
This will be easier to integrate with respect to y, ie. horizontal rectangles.... so....

Let's do a quick sketch first





Vertical Rectangles
(Integration with respect to x)



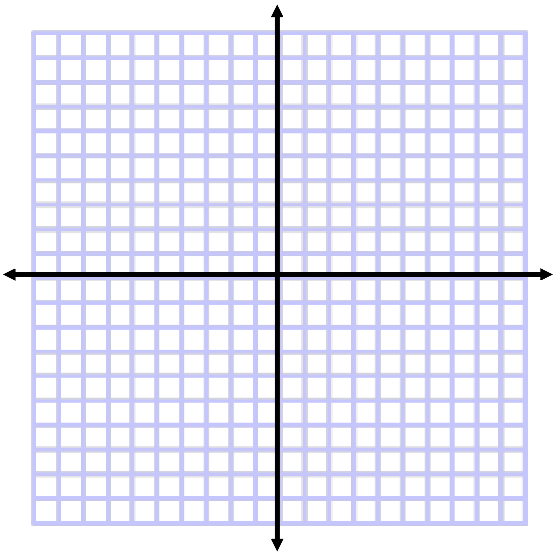
Horizontal Rectangles
(Integration with respect to y)

easier

<u>Example:</u> Find the area of the region bounded by:

$y^2 = 2x + 6$ and $y = x - 1$

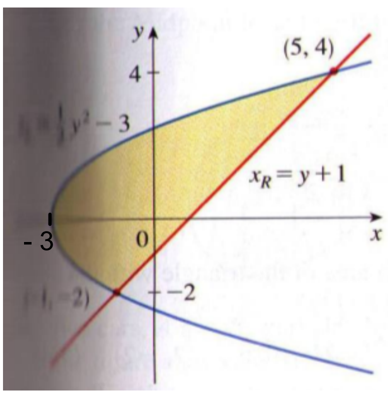
Let's do a quick sketch first



First find points of intersection....

This will be easier to integrate with respect to y,

ie. horizontal rectangles.... so....



Ex. 10.2 - Red Book - find the areas using integrals

Stewart - Ex. 6.1 - worksheet

Larson - Ex. 7.1 - worksheet

Example: Find the area of the region bounded by:

$x = 3 - y^2$ and $x = y + 1$

$y^2 = 3 - x$

$y = \sqrt{3-x}$

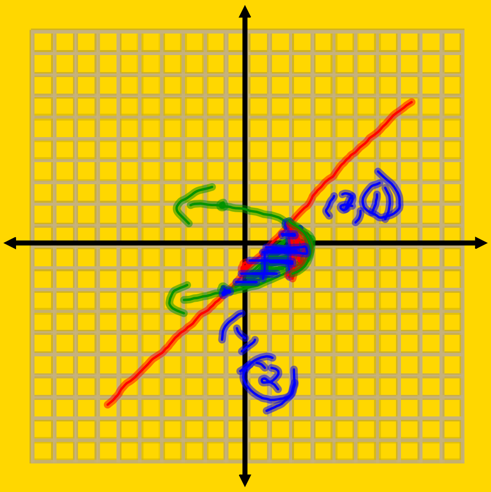
$\boxed{y = x - 1}$

First find points of intersection....

This will be easier to in

ie. horizontal rec

Let's do a quick sketch first



$x = 3 - (x-1)^2$

$x = 3 - (x^2 - 2x + 1)$

$$A = \int_{-2}^{1} \left[-y^2 - y + 2\right] dy$$

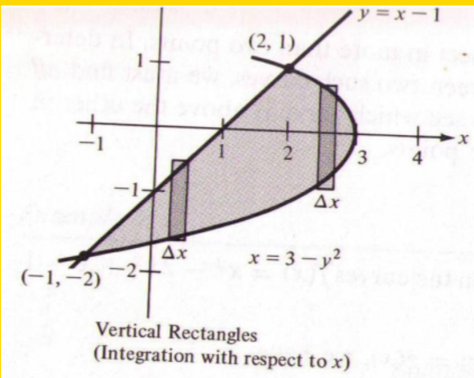
$x = 3 - x^2 + 2x - 1$

$x^2 - x - 2 = 0$

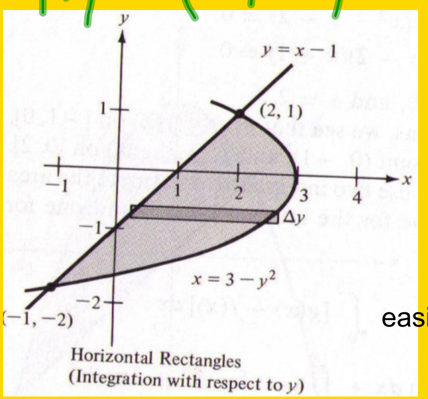
$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$(2,1)$ $(-1,-2)$



Vertical Rectangles
(Integration with respect to x)



Horizontal Rectangles
(Integration with respect to y)

easier

Example: Find the area of the region bounded by:

$\frac{y^2+6}{2}$

$y^2 = 2x + 6$ and $y = x - 1$

$y = \sqrt{2x+6}$ $\quad$ $\boxed{y + 1 = x}$

$(x-1)^2 = 2x + 6$

$x^2 - 2x + 1 = 2x + 6$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

First find points of intersection....

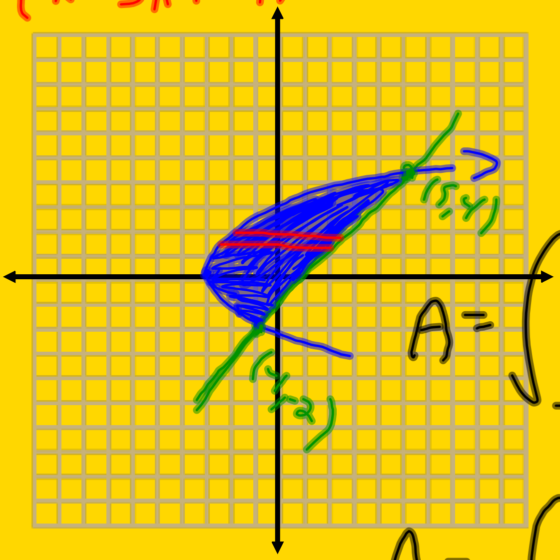
This will be easier to integrate with respect to y,

ie. horizontal rectangles.... so....

Let's do a quick sketch first

$(5,4)$

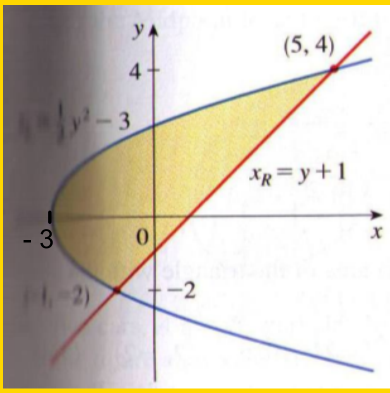
$(-1,-2)$



$$A = \int_{-2}^{4} (y+1) - \left(\frac{y^2}{2} - 3\right) dy$$

$$A = \int_{-2}^{4} -\frac{y^2}{2} + y + 4 \, dy$$

$$= -\frac{1}{6}y^3 + \frac{1}{2}y^2 + 4y \Big|_{-2}^{4}$$



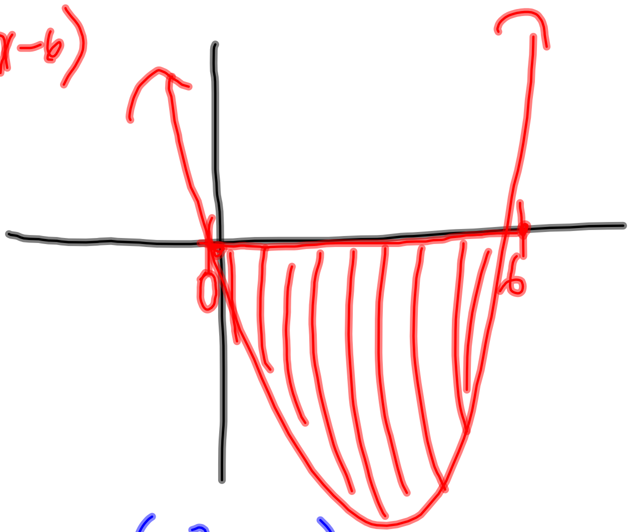
$$= \left(-\frac{64}{6} + 8 + 16\right) - \left(\frac{4}{3} + 2 - 8\right)$$

$$= -\frac{72}{6} + 30$$

$$= -12 + 30 = 18$$

$f(x)\ x^2-6x = x(x-6)$

$g(x) = 0$



$$A = \int_0^6 0 - (x^2-6x)\,dx$$

$$= \int_0^6 -x^2+6x\ dx$$

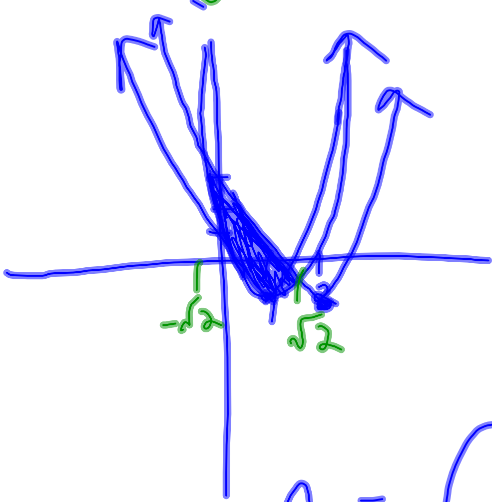
$$= -\frac{1}{3}x^3 + 3x^2 \Big|_0^6$$

$\frac{36}{6}$

$$-72+108 = \boxed{36}$$

7 $f(x) = 2x^2 - 4x + 1$ $4x - 4 = 0$, $x = 1$

$2x - 4 = 0$ $g(x) = x^2 - 4x + 3$



$2x^2 - 4x + 1 = x^2 - 4x + 3$

$x^2 - 2 = 0$

$x = \pm\sqrt{2}$

$$A = \int_{-\sqrt{2}}^{\sqrt{2}} -x^2 + 2 \, dx$$

$$= -\frac{1}{3}x^3 + 2x \Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= -x\left(\frac{1}{3}x^2 - 2\right)\Big|_{-\sqrt{2}}^{\sqrt{2}}$$

$$= -\sqrt{2}\left(\frac{2}{3} - 2\right) - (\sqrt{2})\left(\frac{2}{3} - 2\right)$$

$$-\frac{4\sqrt{2}}{3} + \frac{12\sqrt{2}}{3} = \boxed{\frac{8\sqrt{2}}{3}}$$