Equations in Standard Form

$$y = a\sin[b(x-h)] + k$$

a = ***Amplitude*** → influences how tall the sine curve is.

$b = \frac{360^\circ}{P}$ $b = \frac{2\pi}{P}$ → influences how often the pattern repeats. $P = \frac{360^\circ}{b}$ or $\frac{2\pi}{b}$

h = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift.

- If C is positive → Shift Left
- If C is negative → Shift Right

k = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If d is positive → Shift Up
- If d is negative → Shift Down

Sketching Sinusoidal Functions using Mapping

Development of a standard form for sinusoidal functions...

Standard Form $\longrightarrow$ $y = a\sin\left[b(x-h)\right]+k$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.

2. Amplitude: The amplitude of the graph will be equal to | a | . Amp = |a|

3. Period: The period of the graph will be equal to $\frac{360°}{b}$

4. Horizontal Phase Shift: The graph will shift "h" units to the right. (c)
(Translation)

5. Vertical Translation: The graph will shift "k" units up. d

The Mapping Rule: $(x,y) \rightarrow \left[\frac{x}{b}+h, ay+k\right]$

$$(x,y) \rightarrow \left[\frac{1}{b}x+h, ay+k\right]$$

State ***a, b, c, d, and P*** from the following sinusoidal equations:

$$2y+6=4\sin(4x+\frac{\pi}{2})-2$$

a = b = c = d =

P =

Use Mapping to Graph

$$\frac{1}{2}(y+1) = 3\sin\left(\frac{1}{2}\theta - 90°\right) + 2$$

$y+1 = 6\sin[\frac{1}{2}(\theta-180°)]+4$

$y = \underline{6}\sin[\underline{\frac{1}{2}}(\theta-\underline{180°})]+3$

Remember...Put in standard form first!!

a = 6   amp = 6

b = $\frac{1}{2}$ P = 360° ÷ $\frac{1}{2}$ = 720°

h = 180°

k = 3 → equal to your sinusoidal axis

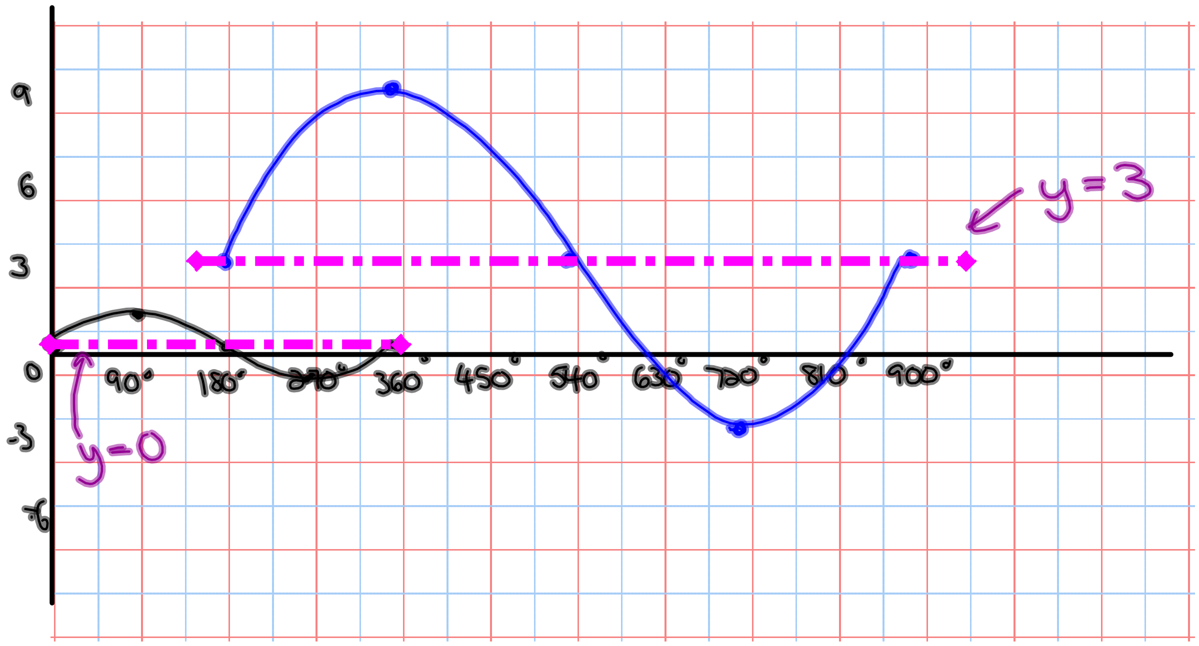
$y = \sin\theta$

θ	y
0	0
90	1
180	0
270	-1
360	0

$(x,y) \rightarrow [2x+180°, 6y+3]$

New points after mapping

θ	y
180°	3
360°	9
540°	3
720°	-3
900°	3



Use Mapping to Graph

$$\frac{3y}{3} = \frac{-6}{3}\cos(3x - \pi) - \frac{9}{3}$$

$$y = -2\cos\left[\boxed{3}\left(x - \frac{\pi}{3}\right)\right] - 3$$

a = 2 b = 3 c = $\frac{\pi}{3}$ d = −3

P = $\frac{2\pi}{3}$

$y = \cos\theta$

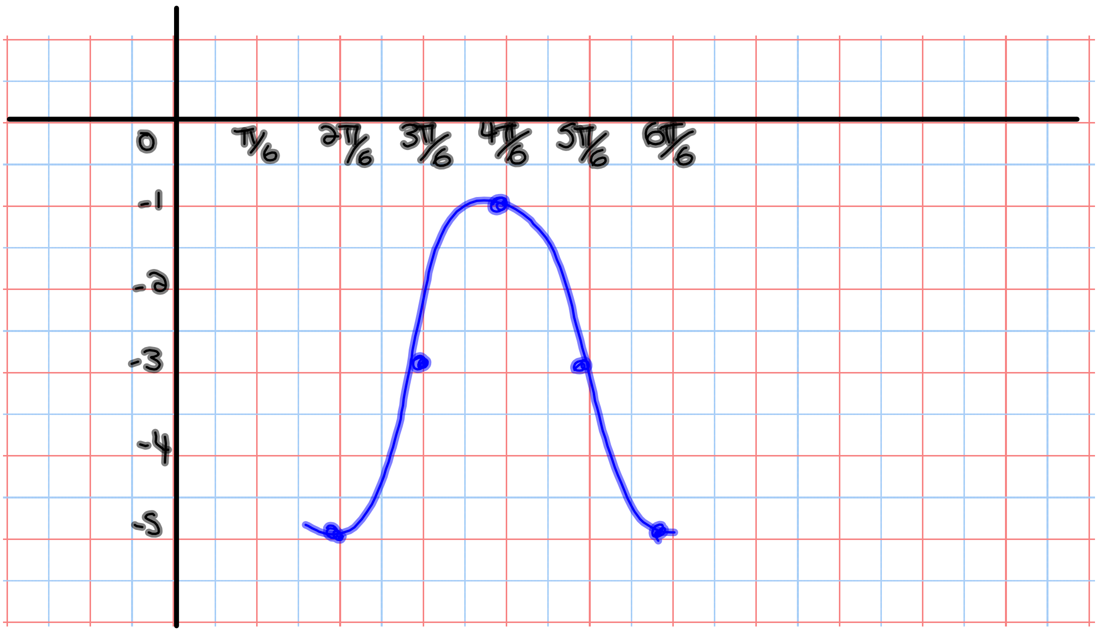
θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$$(x, y) \rightarrow \left[\frac{x}{b} + c, ay + d\right]$$

New points after mapping

$$(x, y) \rightarrow \left[\frac{1}{3}x + \frac{\pi}{3}, -2y - 3\right]$$

	θ	y
$\left(\frac{2\pi}{6}\right)$	$\pi/3$	-5
$\left(\frac{3\pi}{6}\right)$	$\pi/2$	-3
$\left(\frac{4\pi}{6}\right)$	$2\pi/3$	-1
	$5\pi/6$	-3
$\left(\frac{6\pi}{6}\right)$	π	-5

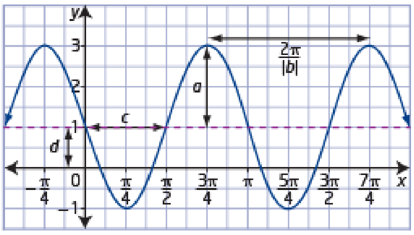


Key Ideas

- You can determine the amplitude, period, phase shift, and vertical displacement of sinusoidal functions when the equation of the function is given in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

  For: $y = a \sin b(x - c) + d$
  $y = a \cos b(x - c) + d$

  How does changing each parameter affect the graph of a function?

  Vertical stretch by a factor of $|a|$
  • changes the amplitude to $|a|$
  • reflected in the x-axis if $a < 0$

  Horizontal stretch by a factor of $\frac{1}{|b|}$
  • changes the period to $\frac{360°}{|b|}$ (in degrees) or $\frac{2\pi}{|b|}$ (in radians)
  • reflected in the y-axis if $b < 0$

  Horizontal phase shift represented by c
  • to right if $c > 0$
  • to left if $c < 0$

  Vertical displacement represented by d
  • up if $d > 0$
  • down if $d < 0$

  $$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- You can determine the equation of a sinusoidal function given its properties or its graph.

Homework

Finish worksheet

① $y = 3\sin[1(\theta - 0°)] + 2$

Solutions to the homework

① $y = 3\cos(x) - 2$

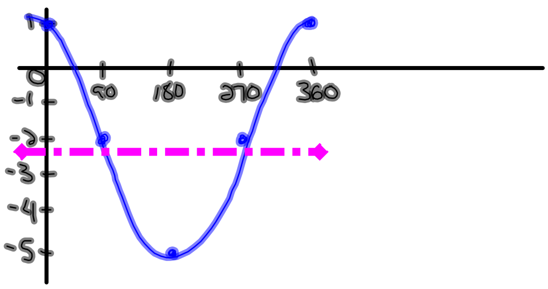
$A = 3 \quad b = 1 \quad C = 0 \quad D = -2 \quad P = 360$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

→

x	y
0	1
90	-2
180	-5
270	-2
360	1



② $y = -\sin\left(2x - \frac{\pi}{6}\right)$

$y = -\sin\left[2\left(x - \frac{\pi}{12}\right)\right]$

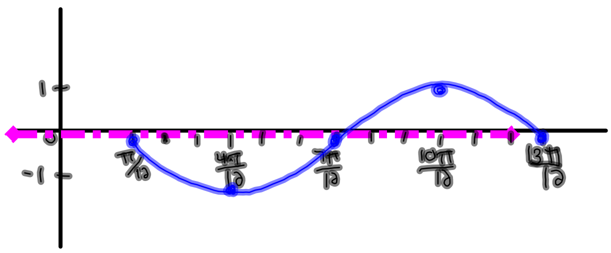
$A = 1 \quad b = 2 \quad C = \frac{\pi}{12} \quad D = 0 \quad P = \pi$

$y = \sin x$

x	y
0	0
$\pi/2$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

→

x	y
$\pi/12$	0
$\pi/3$	-1
$\frac{7\pi}{12}$	0
$5\pi/6$	1
$13\pi/12$	0



③ $y = 4\sin(3x - 180°) + 2$

$y = 4\sin[3(x - 60°)] + 2$

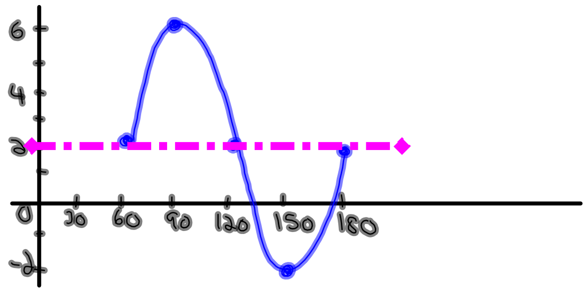
$A = 4 \quad b = 3 \quad c = 60 \quad D = 2 \quad P = 120°$

$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

→

x	y
60	2
90	6
120	2
150	-2
180	2



⑤ $2y+3 = -4\sin\left(4x-\frac{\pi}{3}\right)-3$

$2y = -4\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-6$

$y = -2\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-3$

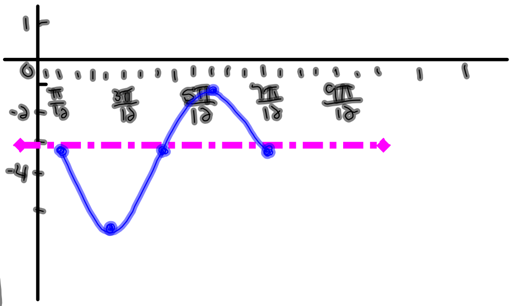
$A=2 \quad b=4 \quad C=\frac{\pi}{12} \quad D=-3 \quad P=\frac{\pi}{2}$

$y=\sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$\rightarrow$

x	y
$\pi/12$	-3
$5\pi/24$	-5
$\pi/3$	-3
$11\pi/24$	-1
$7\pi/12$	-3



⑥ $\cancel{2}\,\frac{y-1}{\cancel{2}} = {}^{2\cdot}2\cos(\theta+45^\circ) \,{}^{2}+0$

$y-1 = 4\cos(\theta+45^\circ)+0 \; +1$

$\boxed{y = 4\cos(\theta+45^\circ)+1}$

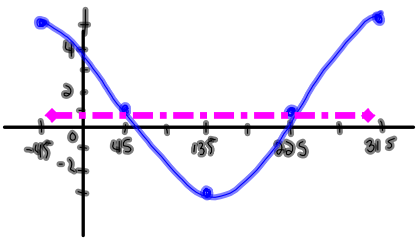
$A=4 \quad b=1 \quad C=-45 \quad D=1 \quad P=360$

$y=\cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

$\rightarrow$

x	y
-45	5
45	1
135	-3
225	1
315	5



① $\frac{1}{2}y-1 = \sin\left[2\left(x+\frac{\pi}{4}\right)\right]$

$2\cdot\ \frac{1}{2}y = 2\cdot\ \sin\left[2\left(x+\frac{\pi}{4}\right)\right]+1 \;\cdot 2$

$y = 2\sin\left[2\left(x+\frac{\pi}{4}\right)\right]+2$

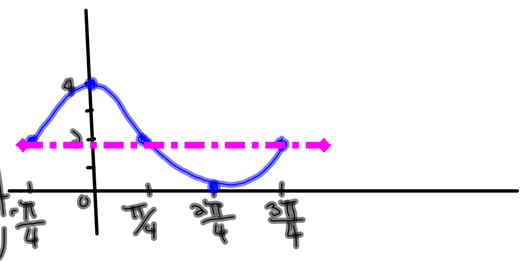
$A=2 \quad b=2 \quad C=-\frac{\pi}{4} \quad D=2 \quad P=\pi$

$y=\sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$\rightarrow$

x	y
$-\pi/4$	2
0	4
$\pi/4$	2
$\pi/2$	0
$3\pi/4$	2



⑧ $y = -4\cos(3x + 90°) - 2$

$y = -4\cos[3(x+30)] - 2$

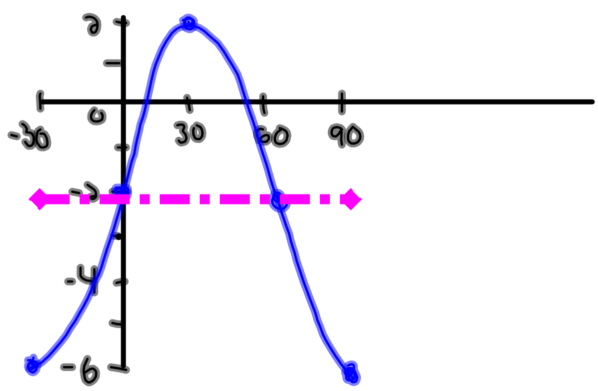
$A = 4 \quad b = 3 \quad c = -30 \quad D = -2 \quad P = 120$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

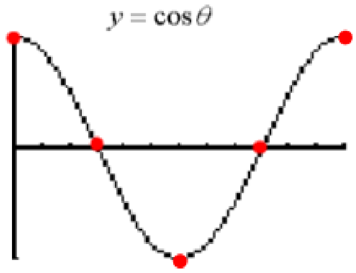
$\longrightarrow$

x	y
-30	-6
0	-2
30	2
60	-2
90	-6



Solution to Assignment

$$y = 3\cos[2(\theta - 135°)] + 2$$



Mapping:

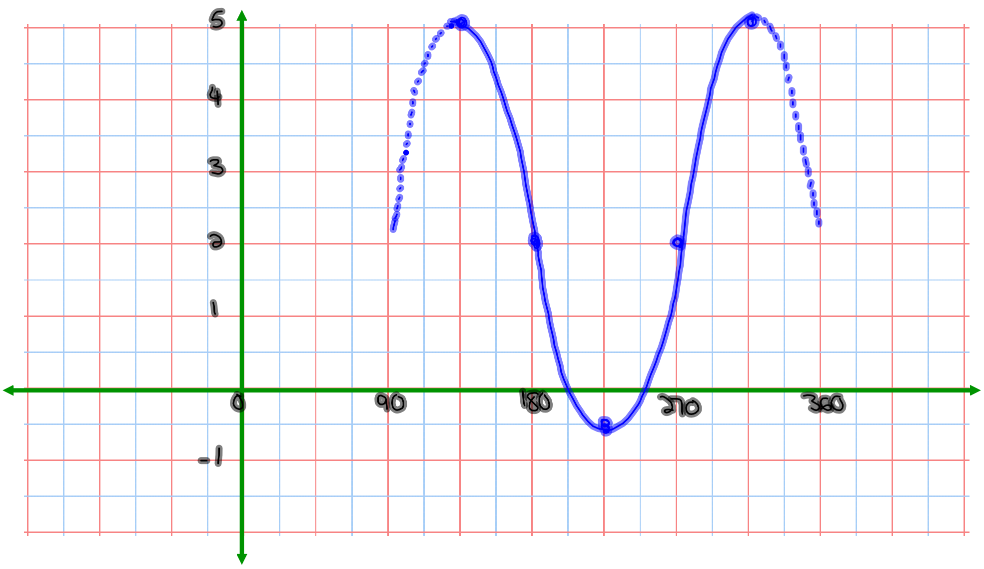
$$(x, y) \longrightarrow \left[\frac{1}{2}\theta + 135°,\ 3y + 2\right]$$

$y = \cos\theta$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
135°	5
180°	2
225°	-1
270°	2
315°	5



DOMAIN	$\{\theta \mid \theta \in R\}$
RANGE	$\{y \mid -1 \le y \le 5, y \in R\}$
AMPLITUDE	$a = 3$
PERIOD	$P = \frac{360°}{2} = 180°$
PHASE SHIFT	$c = 135°$
VERTICAL TRANSLATION	$d = 2$
EQUATION OF SINUSOIDAL AXIS	$y = 2$

Attachments

Sketching Sinusoidal Functions.pdf