Equations in Standard Form

$$y = a\sin\left[b(x-h)\right]+k$$

a = ***Amplitude*** → influences how tall the sine curve is.

$b = \frac{360°}{P}$ $b = \frac{2\pi}{P}$ → influences how often the pattern repeats. $P = \frac{360°}{b}$ or $\frac{2\pi}{b}$

h = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift.

- If C is positive → Shift Left
- If C is negative → Shift Right

k = ***Vertical Translation*** → influences how far up and down the graph will shift.

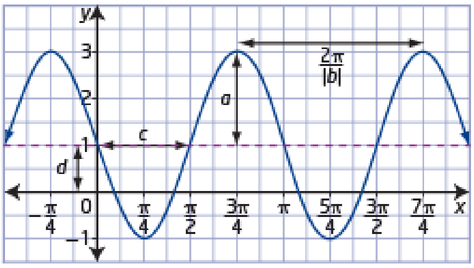
- If d is positive → Shift Up
- If d is negative → Shift Down

Key Ideas

- You can determine the amplitude, period, phase shift, and vertical displacement of sinusoidal functions when the equation of the function is given in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

For: $y = a \sin b(x - c) + d$
$y = a \cos b(x - c) + d$

How does changing each parameter affect the graph of a function?



Vertical stretch by a factor of $|a|$
- changes the amplitude to $|a|$
- reflected in the x-axis if $a < 0$

Horizontal stretch by a factor of $\frac{1}{|b|}$
- changes the period to $\frac{360°}{|b|}$ (in degrees) or $\frac{2\pi}{|b|}$ (in radians)
- reflected in the y-axis if $b < 0$

Horizontal phase shift represented by c
- to right if $c > 0$
- to left if $c < 0$

Vertical displacement represented by d
- up if $d > 0$
- down if $d < 0$

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- You can determine the equation of a sinusoidal function given its properties or its graph.

Sketching Sinusoidal Functions using Mapping

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $y = a\sin[b(x-h)]+k$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.

2. Amplitude: The amplitude of the graph will be equal to | a | . always stated as a positive

3. Period: The period of the graph will be equal to $\frac{360°}{b}$ or $\frac{2\pi}{b}$

4. Horizontal Phase Shift: The graph will shift "h" units to the right.
(Change the sign when you remove it from brackets)

5. Vertical Translation: The graph will shift "k" units up.

The Mapping Rule: $(x, y) \rightarrow \left[\frac{x}{b}+h, ay+k\right]$

Use Mapping to Graph

$$\frac{1}{2}(y+1) = 3\sin\left(\frac{1}{2}\theta - 90°\right) + 2$$

Remember...Put in standard form first!!

$y+1 = 6\sin\left(\frac{1}{2}\theta - 90°\right) + 4$

$y = 6\sin\left(\frac{1}{2}\theta - 90°\right) + 3$

(Factor)

$y = 6\sin\left[\frac{1}{2}(\theta - 180°)\right] + 3$

a = 6 b = 1/2 h = 180° k = 3

P = 720°

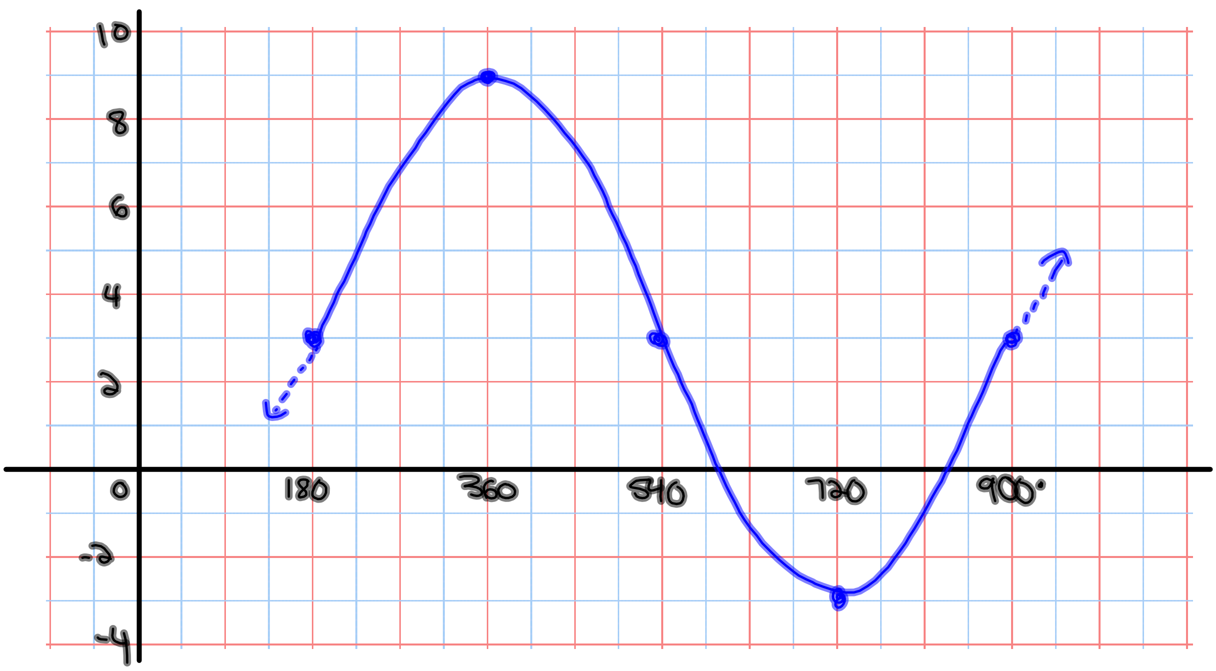
$y = \sin\theta$

θ	y
0	0
90	1
180	0
270	-1
360	0

$(x, y) \rightarrow [2x + 180°, 6y + 3]$

New points after mapping

θ	y
180°	3
360°	9
540°	3
720°	-3
900°	3



Use Mapping to Graph

$$\frac{3y}{3} = \frac{-6}{3}\cos(3x-\pi) - \frac{9}{3}$$

$$y = -2\cos\left[3\left(x - \frac{\pi}{3}\right)\right] - 3$$

a = 2 b = 3 h = $\frac{\pi}{3}$ k = -3

P = $\frac{2\pi}{3}$

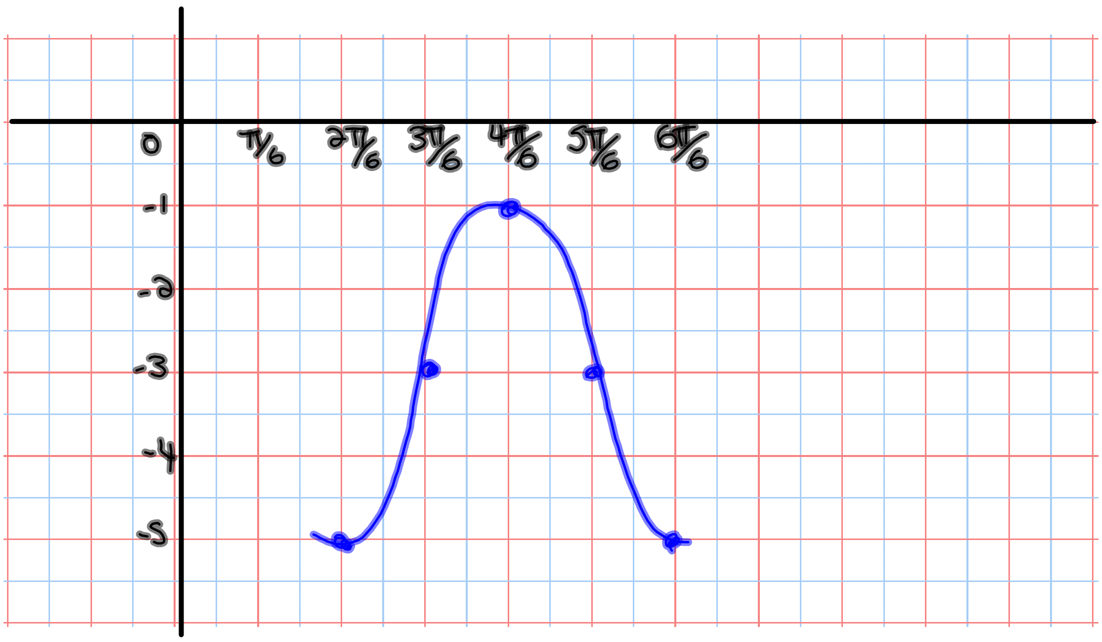
$y = \cos\theta$

θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

$(x, y) \rightarrow \left[\frac{1}{3}x + \frac{\pi}{3}, -2y - 3\right]$

New points after mapping

θ	y
$(2\pi/6)$ $\pi/3$	-5
$(3\pi/6)$ $\pi/2$	-3
$(4\pi/6)$ $2\pi/3$	-1
$5\pi/6$	-3
$(6\pi/6)$ π	-5



Questions from Homework

Example...

Graph the equation $y = -3\sin(2\theta + \pi) + 1$ using mapping notation.

$y = -3\sin\left[2\left(\theta + \frac{\pi}{2}\right)\right] + 1$

$a = -3$ $b = 2$ $h = -\frac{\pi}{2}$ $k = 1$

AMPLITUDE	3
PERIOD	$\frac{2\pi}{2} = \pi$
PHASE SHIFT	$\pi/2$ (Left)
VERTICAL TRANSLATION	1 (up)
EQUATION OF SINUSOIDAL AXIS	$y = 1$

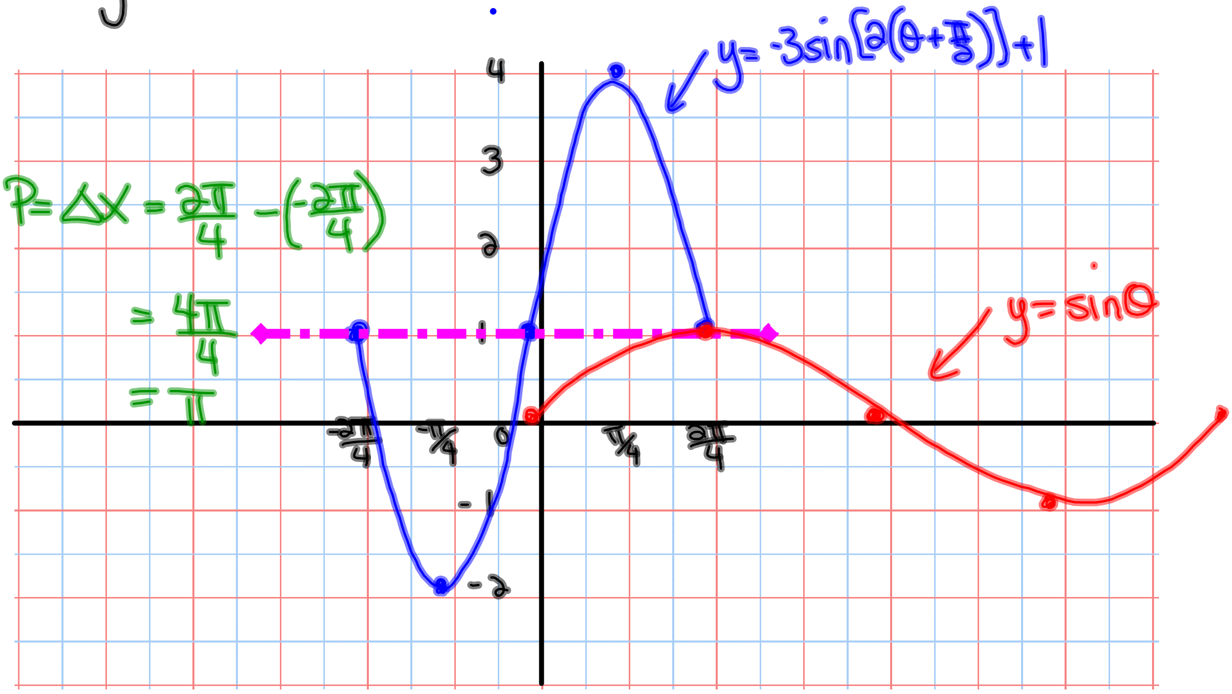
$y = \sin\theta$

θ	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

$(x, y) \rightarrow \left[\frac{1}{2}x - \frac{\pi}{2}, -3y + 1\right]$

New points after mapping

θ	y
$-\pi/2$	1
$-\pi/4$	-2
0	1
$\pi/4$	4
$\pi/2$	1



Homework

Worksheet - Sketching Trigonometric Functions.doc

Solutions to the homework

① $y = 3\cos(x) - 2$

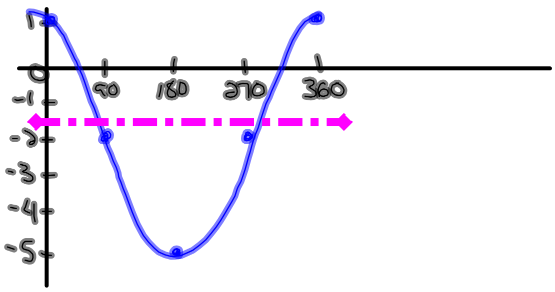
$A = 3 \quad b = 1 \quad C = 0 \quad D = -2 \quad P = 360°$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

→

x	y
0	1
90	-2
180	-5
270	-2
360	1



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② $y = -\sin(2x - \frac{\pi}{6})$

$y = -\sin[2(x - \frac{\pi}{12})]$

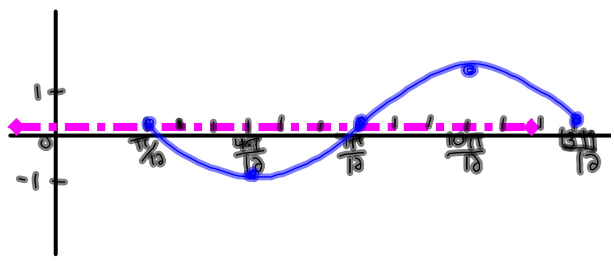
$A = 1 \quad b = 2 \quad C = \frac{\pi}{12} \quad D = 0 \quad P = \pi$

$y = \sin x$

x	y
0	0
$\pi/2$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0

→

x	y
$\pi/12$	0
$\pi/3$	-1
$\frac{7\pi}{12}$	0
$5\pi/6$	1
$13\pi/12$	0



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③ $y = 4\sin(3x - 180°) + 2$

$y = 4\sin[3(x - 60°)] + 2$

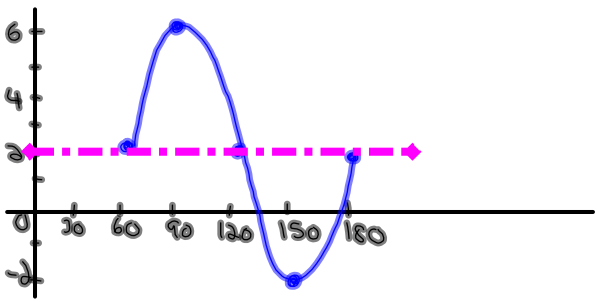
$A = 4 \quad b = 3 \quad c = 60 \quad D = 2 \quad P = 120°$

$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

→

x	y
60	2
90	6
120	2
150	-2
180	2



⑤ $2y+3=-4\sin\left(4x-\frac{\pi}{3}\right)-3$

$2y=-4\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-6$

$y=-2\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-3$

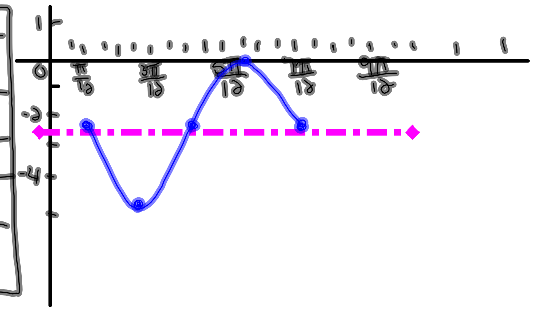
$A=2 \quad b=4 \quad C=\frac{\pi}{12} \quad D=-3 \quad P=\frac{\pi}{2}$

$y=\sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

→

x	y
$\pi/12$	-3
$5\pi/24$	-5
$\pi/3$	-3
$11\pi/24$	-1
$7\pi/12$	-3



⑥ $2\cdot\frac{y-1}{\cancel{2}}=2\cdot 2\cos(\theta+45^\circ)+0\cdot 2$

$y-1=4\cos(\theta+45^\circ)+0+1$

$\boxed{y=4\cos(\theta+45^\circ)+1}$

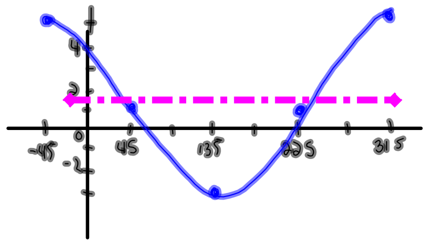
$A=4 \quad b=1 \quad C=-45 \quad D=1 \quad P=360$

$y=\cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

→

x	y
-45	5
45	1
135	-3
225	1
315	5



⑦ $\frac{1}{2}y-1=\sin\left[2\left(x+\frac{\pi}{4}\right)\right]$

$2\cdot\frac{1}{2}y=2\cdot\sin\left[2\left(x+\frac{\pi}{4}\right)\right]+1\cdot 2$

$y=2\sin\left[2\left(x+\frac{\pi}{4}\right)\right]+2$

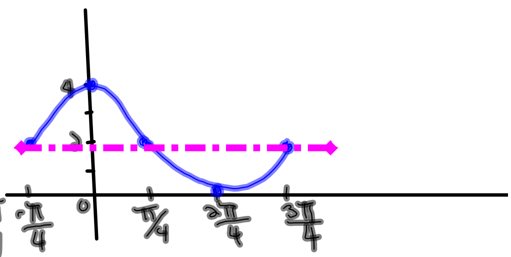
$A=2 \quad b=2 \quad C=-\frac{\pi}{4} \quad D=2 \quad P=\pi$

$y=\sin x$

x	y
0	0
$\pi/2$	1
π	0
$3\pi/2$	-1
2π	0

→

x	y
$-\pi/4$	2
0	4
$\pi/4$	2
$\pi/2$	0
$3\pi/4$	2



8) $y = -4\cos(3x + 90°) - 2$

$y = -4\cos[3(x + 30)] - 2$

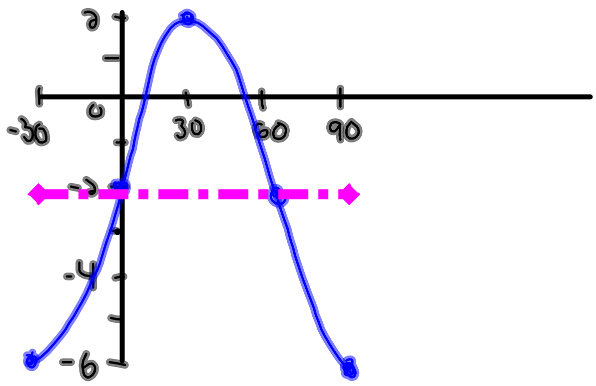
$A = 4 \quad b = 3 \quad c = -30 \quad D = -2 \quad P = 120$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1

$\longrightarrow$

x	y
-30	-6
0	-2
30	2
60	-2
90	-6



Attachments

worksheet-sketching in radian measure.doc

Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc

Sketching Sinusoidal Functions #2.pdf

Sketching Sinusoidal Functions #2.doc