# Differential and Integral Calculus 120

$\int$ Substitution Rule! $\int$

# Substitution Rule

It often arises that we need to integrate a function which does not follow one of our basic integral formulas therefore other techniques are necessary.

One such technique is called Substitution that involves a change of variable, which permits us to rewrite an integrand in a form to which we can apply a basic integration rule.

**Substitution Rule for Indefinite Integrals**

If $u = g(x)$ , then $\int f(g(x))g'(x)dx = \int f(u)du$

Substitution Rule for Integration corresponds... to the Chain Rule for Differentiation.

Let's do an example...

Find $\int (x^2-5)^8 2xdx$

In using the substitution rule, the idea is to replace a complicated integral by a simpler integral by changing to a new variable u.

In thinking of the appropriate substitution, we try to choose u to be some function in the integrand whose differential du also occurs. (ignoring any constants)

So using the above example we have...

$$\int (x^2-5)^8 2xdx$$

Let:

$u = x^2 - 5$

$du = 2xdx$

$$\int (u)^8 du$$

$$= \frac{u^9}{9} + C$$

$$= \frac{(x^2-5)^9}{9} + C$$

Evaluate each of the following:

$$\int \frac{x^2}{\sqrt{1-x^3}}dx$$

$u = 1-x^3$

$du = -3x^2 dx$

$-\frac{1}{3}du = x^2 dx$

$= \int \frac{x^2}{(1-x^3)^{1/2}} dx$

$= \int (u)^{-1/2} \cdot \left(-\frac{1}{3}du\right)$

$= -\frac{1}{3}\int (u)^{-1/2} du$

$= -\frac{1}{3} \cdot 2u^{1/2} + C$

$= -\frac{2}{3}(1-x^3)^{1/2} + C$

$= -\frac{2}{3}(1-x^3)^{1/2} + C$

$$\int \sin 4x dx$$

$u = 4x$

$du = 4dx$

$\frac{1}{4}du = dx$

$= \int \sin u \cdot \left(\frac{1}{4}du\right)$

$= \frac{1}{4}\int \sin(u)\, du$

$= \frac{1}{4} \cdot -\cos u + C$

$= \frac{-1}{4}\cos u + C$

$= -\frac{1}{4}\cos 4x + C$

$= -\frac{1}{4}\cos 4x + C$

Evaluate each of the following:

$$\int \frac{\ln x}{x}\,dx = \int \ln x \cdot \frac{1}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \int u\,du$$

$$= \frac{u^2}{2} + C$$

$$= \frac{(\ln x)^2}{2} + C$$

$$= \frac{1}{2}(\ln x)^2 + C$$

$$\int (2+\sin x)^{10}\cos x\,dx$$

$u = 2+\sin x$

$du = \cos x\,dx$

$$= \int u^{10}\,du$$

$$= \frac{u^{11}}{11} + C$$

$$= \frac{(2+\sin x)^{11}}{11} + C$$

$$= \frac{1}{11}(2+\sin x)^{11} + C$$

Evaluate each of the following:

$$\int 2x\sqrt{1+x^2}\,dx$$

$$=\frac{2}{3}(x^2+1)^{3/2}+C$$

$$\int x^3\cos(x^4+2)\,dx$$

$$=\frac{1}{4}\sin(x^4+2)+C$$

Evaluate each of the following:

$$\int \sqrt{2x+1}\,dx$$

$$=\frac{1}{3}(2x+1)^{3/2}+C$$

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx$$

$$=-\frac{1}{4}\sqrt{1-4x^2}+C$$

Evaluate the following:

$$\int e^{5x}\,dx$$

$$=\frac{1}{5}e^{5x}+C$$

Evaluate: $\int \sqrt{1+x^2}\, x^5 dx$

appropriate substitution becomes more obvious if we factor $x^5$ as $x^4$ x

Homework - Exercise 11.3 - pp. 511-512 - Q. 1 - 6

**RED BOOK**