

## **Applications of Sinusoidal Relations**

- Strategy: (1) Translate ALL key pieces of information from the problem.  
(2) Draw a sketch with ALL key points identified.  
(3) Develop an equation that models the problem.  
(4) Answer the question(s) being asked.

CHECK??? Do the numbers make sense?

## Applications of Sinusoidal Functions

A carnival Ferris wheel with a radius of 14 m makes one complete revolution every 16 seconds. The bottom of the wheel is 1.5 m above the ground. If a person is at the top of the wheel when a stop watch is started, determine how high above the ground that person will be after 1 minute and 7 seconds? Sketch one period of this function.

$$a = 14 \quad P = 16s$$

$$\text{min} = 1.5 \text{ m}$$

$$\text{sin axis: } 15.5 \text{ m}$$

$$b = \frac{360}{16} = 22.5$$

$$\begin{aligned} \text{max} &= \text{min} + \text{diameter} \\ &= 1.5 + 28 \\ &= 29.5 \text{ m} \end{aligned}$$

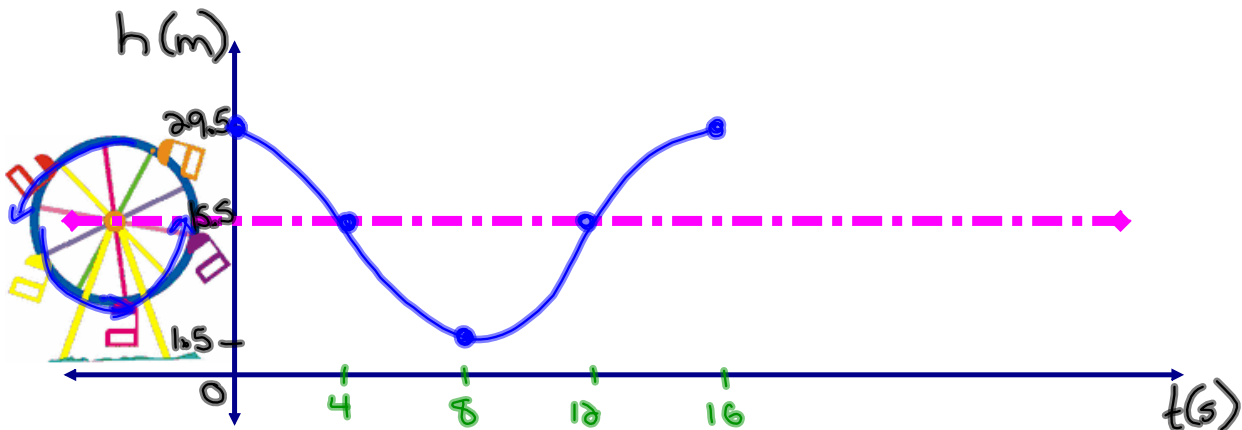
$$d = 28$$

$$\text{equation: } y = 14 \cos[22.5(x + 0)] + 15.5$$

$$y = 14 \cos[22.5(x)] + 15.5$$

$$y = 14 \cos[22.5(67)] + 15.5$$

$$y = 20.86 \text{ m}$$

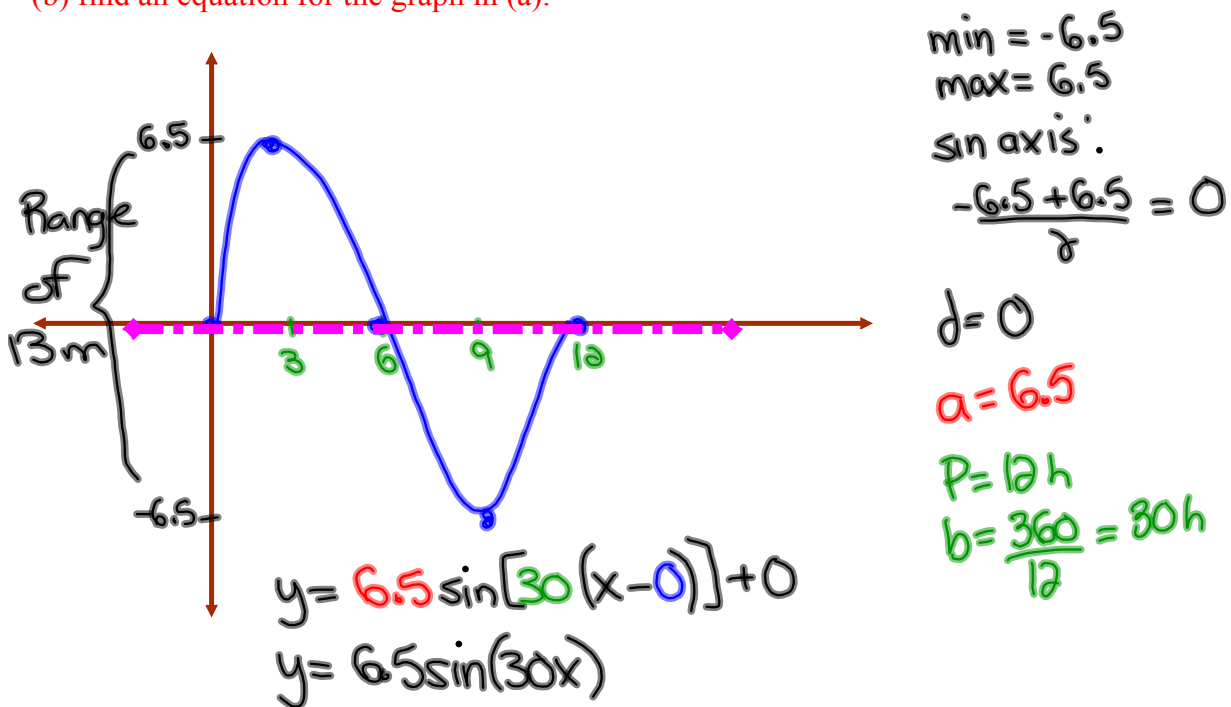


$$\frac{P}{4} = \frac{16}{4} = 4s$$

# Ocean Tides

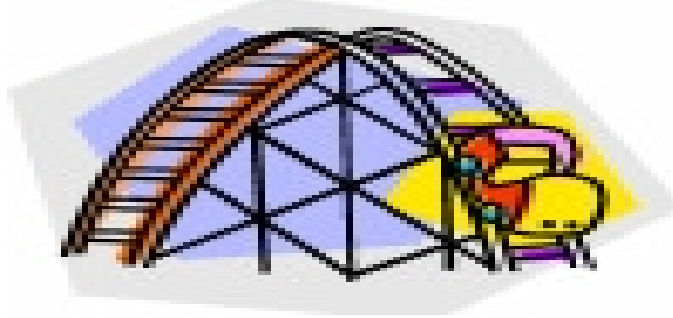
The alternating half-daily cycles of the rise and fall of the ocean are called tides. Tides in one section of the Bay of Fundy caused the water level to rise 6.5m above mean sea-level and to drop 6.5m below. The tide completes one cycle every 12 h. Assuming the height of water with respect to mean sea-level to be modelled by a sine function,

- (a) draw the graph for a the motion of the tides for one complete day;  
 (b) find an equation for the graph in (a).



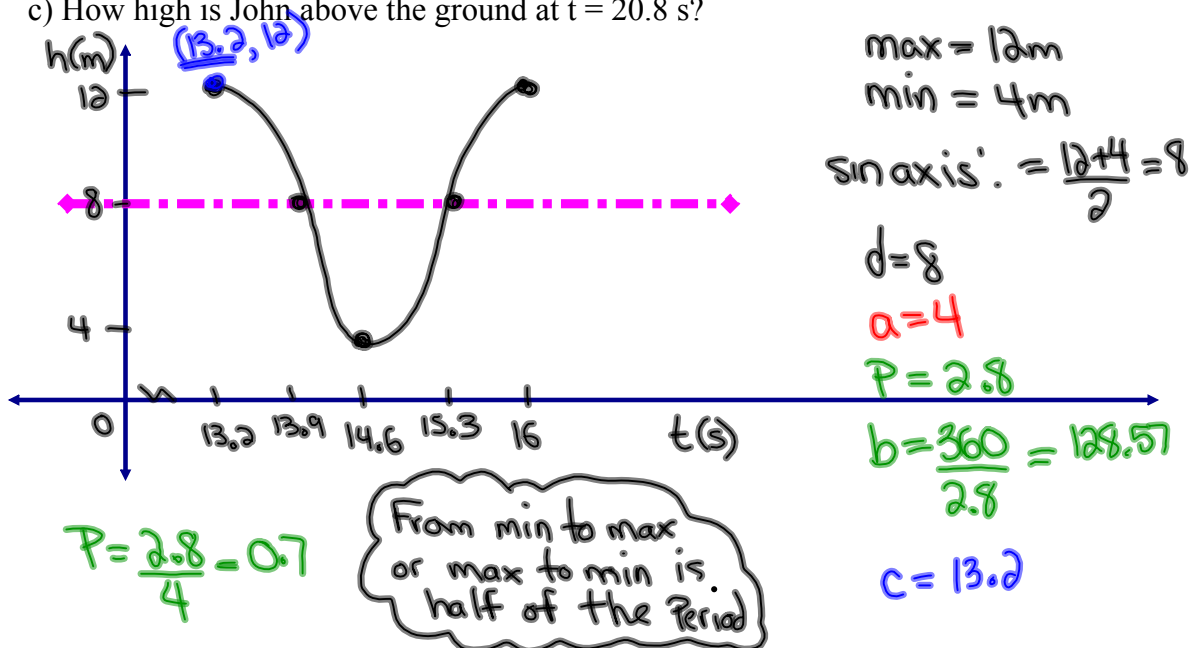
The alternating half-daily cycles of the rise and fall of the ocean are called tides. Tides in one section of the Bay of Fundy caused the water level to rise 6.5m above mean sea-level and to drop 6.5m below. The tide completes one cycle every 6 h. Assume the height of water with respect to mean sea-level to be modelled by a sinusoidal relationship. If it is high tide at 8:00 AM, determine where the water level would be at 1:47 PM.

# Roller Coaster



John climbs on a roller coaster at Six Flags Amusement Park. An observer starts a stopwatch and observes that John is at a maximum height of 12 m at  $t = 13.2$  s. At  $t = 14.6$  s, John reaches a minimum height of 4 m.

- Sketch a graph of the function.
- Find an equation that expresses John's height in terms of time.
- How high is John above the ground at  $t = 20.8$  s?



$$b) \quad y = 4 \cos[128.57(x - 13.2)] + 8$$

$$c) \quad y = 4 \cos[128.57(20.8 - 13.2)] + 8$$

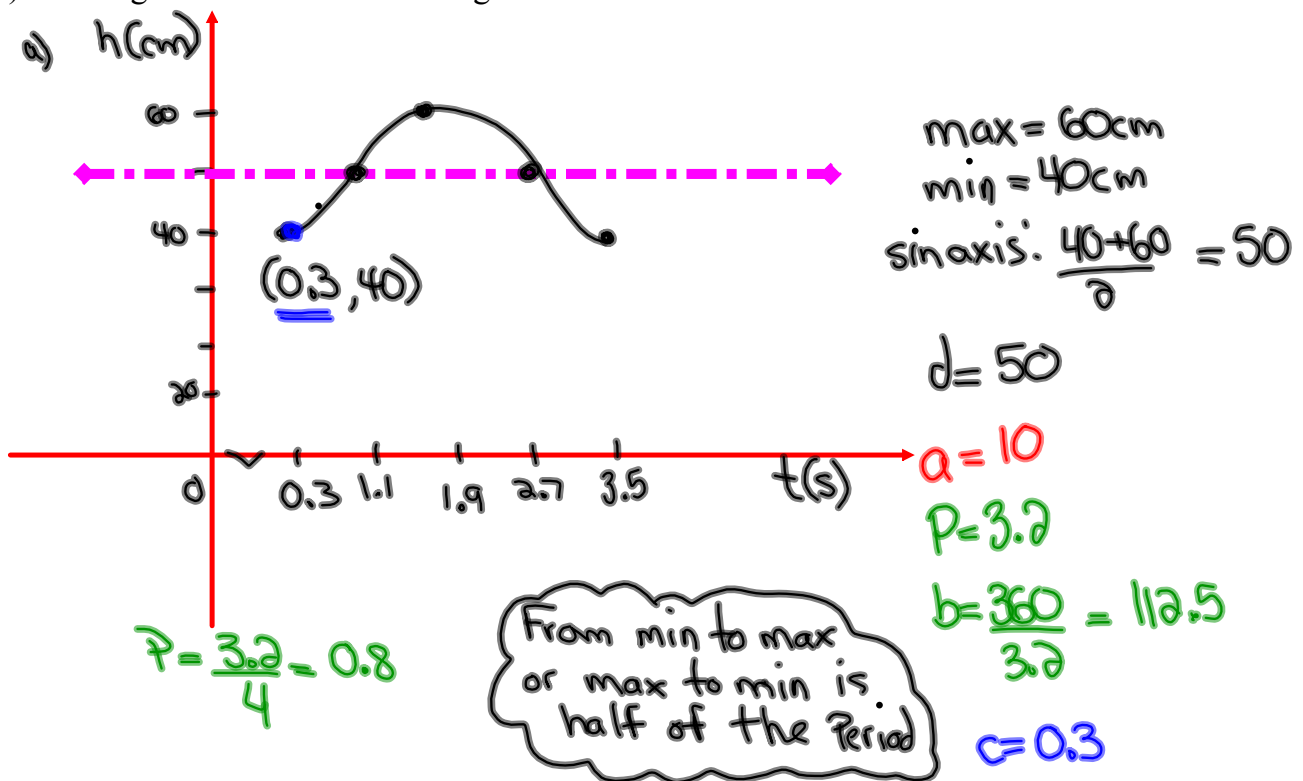
$$y = 4 \cos[128.57(7.6)] + 8$$

$$y = 7.1m$$

# Spring Problem

A weight attached to a long spring is being bounced up and down by an electric motor. As it bounces, its distance from the floor varies periodically with time. When the stopwatch reads 0.3 seconds, the weight reaches its first low point 40 cm above the ground. The next high point, 60 cm above the ground, occurs at 1.9 seconds.

- Sketch a graph of the function.
- Write an equation expressing the distance above the ground in terms of the numbers of seconds the stopwatch reads.
- How high is the mass above the ground after 17.2 seconds?



$$b) \quad y = -10 \cos [112.5(x - 0.3)] + 50$$

$$c) \quad y = -10 \cos [112.5(17.2 - 0.3)] + 50$$

$$y = -10 \cos [112.5(16.9)] + 50$$

$$y = 52 \text{ cm}$$

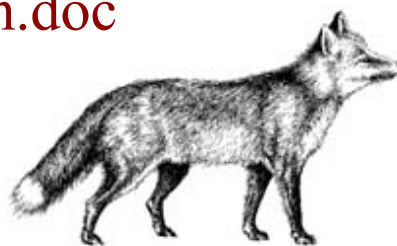
# Biology!

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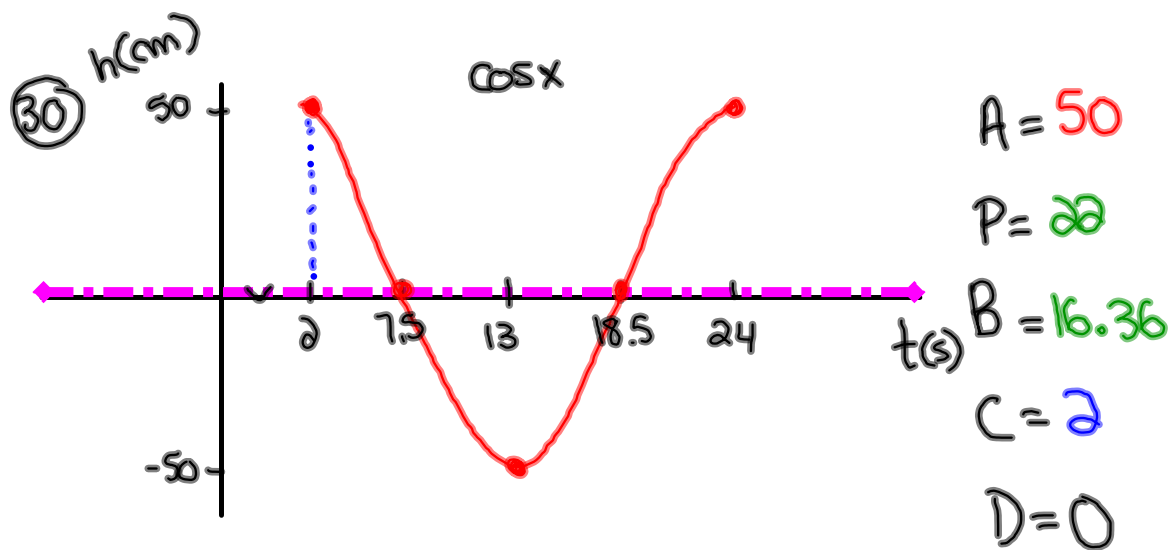
Naturalists find that the populations of some animals varies periodically with time. Records started being taken at  $t = 0$  years. A minimum number, 200 foxes, occurred when  $t = 2.9$  years. The next maximum, 800 foxes, occurred at  $t = 5.1$  years.

**Give two different times at which the fox population is 625.**

**Bonus Soln - Fox Population.doc**



## Questions from Homework



$$a) y = 50 \cos[16.36(x-2)]$$

$$b) y = \underline{15} \cos[16.36(x-2)] \quad * 50 \times 0.3 = 15$$

## Questions from Homework

31)

$$a = 4.5$$

$$b = 360 \div \frac{1}{60}$$

$$= 360 \times 60$$

$$= 21600$$

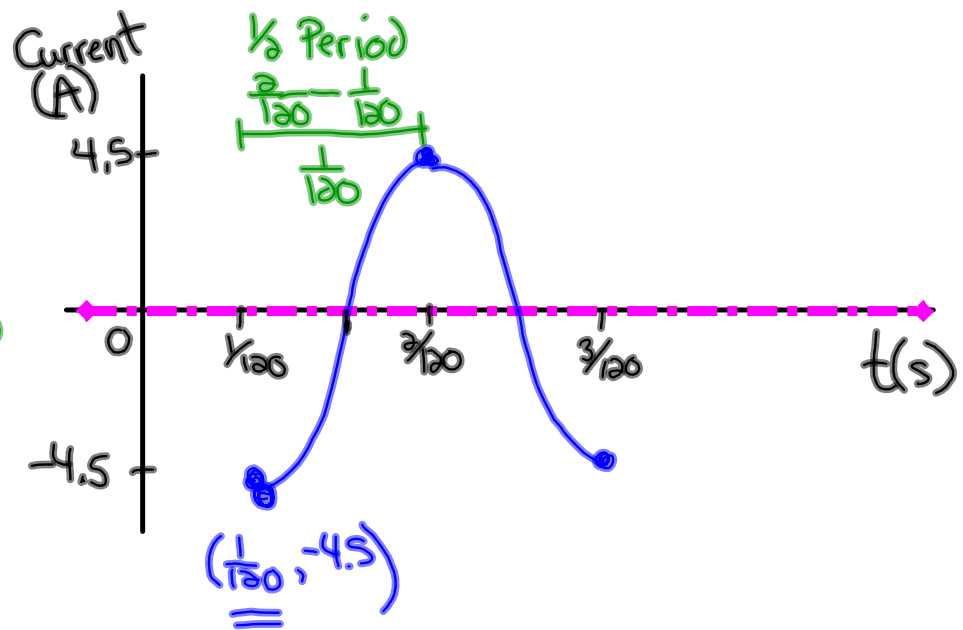
$$c = \frac{1}{120}$$

$$d = 0$$

$$P = \frac{1}{120} \times \frac{2}{1}$$

$$= \frac{2}{120}$$

$$= \frac{1}{60}$$



$$y = -4.5 \cos\left[21600\left(x - \frac{1}{120}\right)\right]$$

$$y = -4.5 \cos\left[21600\left(4 - \frac{1}{120}\right)\right]$$

$$\boxed{y = 4.5 \text{ A}}$$



# Warm Up

1. Determine the range of the trigonometric function  $\frac{1}{5}(y+2) = \sin(2\theta + 60^\circ)$

$$y+2 = 5\sin[2(\theta+30^\circ)]$$

$$y = 5\sin[2(\theta+30^\circ)] - 2$$

sin axis:  $y = -2$   
 max: 3  
 min: -7

[A]  $1 \leq y \leq 3$

[B]  $7 \leq y \leq 3$

[C]  $3 \leq y \leq 7$

[D]  $5 \leq y \leq 5$

2. The graph of  $y = \cos x$  is transformed to a new image according to the mapping rule  $(x, y) \rightarrow \left(\frac{2}{3}x + 30^\circ, 5y - 2\right)$   
 What is the period of this transformation?

$$P = \frac{360^\circ}{\frac{2}{3}} = 360^\circ \div \frac{2}{3} = 360^\circ \times \frac{3}{2} = 540^\circ$$

[A]  $240^\circ$

[B]  $540^\circ$

[C]  $72^\circ$

[D]  $1.5^\circ$

Check-Up...

Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let  $t$  be the number of seconds the stopwatch reads and let  $y$  be the number of feet Tarzan is from the riverbank. Assume that  $y$  varies sinusoidally with  $t$ , and that  $y$  is positive when Tarzan is over water and negative when he is over land. Jane finds that when  $t = 2.8$  seconds, Tarzan is at one end of his swing, 23 feet from the riverbank, over the water. She finds when  $t = 6.3$  seconds he reaches the other end of his swing and is situated 17 feet from the riverbank, however this time over land.

- (a) Where was Tarzan when Jane started the stopwatch? **at time zero ( $t=0$ )**  
 (b) Provide three instances when Tarzan was located at a position 14 feet from the riverbank, over the water.

Given:  
 max: 23 ft  
 min: -17 ft  
 sin axis:  $y = \frac{23 + (-17)}{2}$   
 $y = 3$   
 $d = 3$   
 $a = 20$   
 $p = 2(6.3 - 2.8) = 7$   
 $b = \frac{360}{7} = 51.43$   
 $c = 2.8$

$$y = 20 \cos[51.43(t - 2.8)] + 3$$

a)  $y = 20 \cos[51.43(0 - 2.8)] + 3$   
 $y = 20 \cos[51.43(-2.8)] + 3$   
 $y = -13.18$  ft

Tarzan was 13.18 ft from the riverbank, over land.

b)  $14 = 20 \cos[51.43(t - 2.8)] + 3$   
 $\frac{11}{20} = \frac{20 \cos(51.43t - 144)}{20}$

$$0.55 = \cos(51.43t - 144)$$

$$\cos^{-1}(0.55) = 51.43t - 144$$

$$56.63 = 51.43t - 144$$

$$\frac{200.63}{51.43} = \frac{51.43t}{51.43}$$

$$3.9 \text{ s} = t$$

$$3.9 + 7 = 10.9 \text{ s}$$

$$3.9 + 14 = 17.9 \text{ s}$$

Tarzan is 14 ft over water @ 3.9s, 10.9s and 17.9s

Let's look at the detailed solutions...

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 [Worksheet Solns - Applications of Sinusoidal Relations.doc](#)

What about graphs of other  
trigonometric functions ???

### Graph the Tangent Function

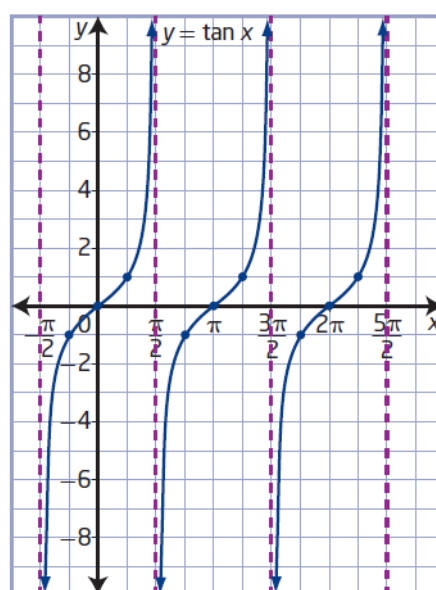
Graph the function  $y = \tan \theta$  for  $-2\pi \leq \theta \leq 2\pi$ . Describe its characteristics.

Angle Measure	0°	45°	90°	135°	180°	225°	270°	315°	360°
y-coordinate on Tangent Line	0	1	und.	-1	0	1	und.	-1	0

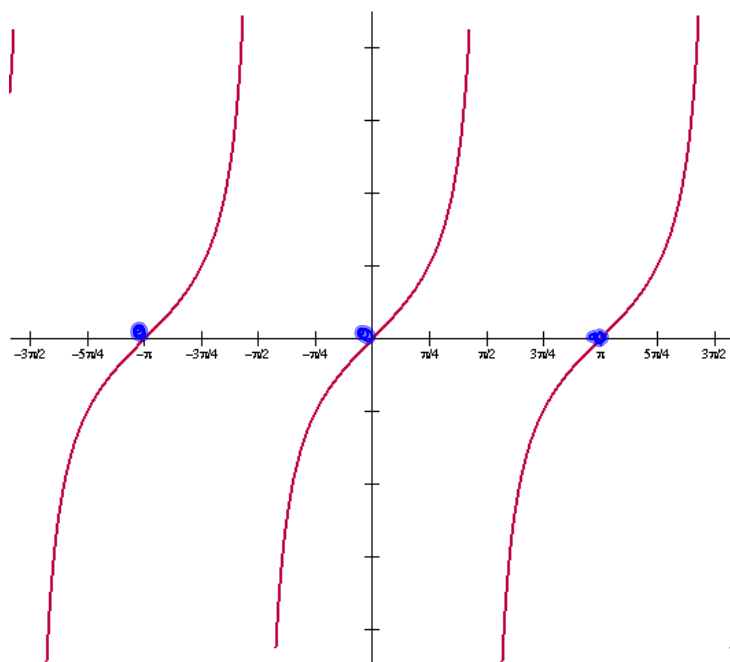
## Key Ideas

- You can use asymptotes and three points to sketch one cycle of a tangent function. To graph  $y = \tan x$ , draw one asymptote; draw the points where  $y = -1$ ,  $y = 0$ , and  $y = 1$ ; and then draw another asymptote.
- The tangent function  $y = \tan x$  has the following characteristics:
  - The period is  $\pi$ .
  - The graph has no maximum or minimum values.
  - The range is  $\{y \mid y \in \mathbb{R}\}$ .
  - Vertical asymptotes occur at  $x = \frac{\pi}{2} + n\pi$ ,  $n \in \mathbb{I}$ .
  - The domain is  $\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in \mathbb{R}, n \in \mathbb{I}\}$ .
  - The  $x$ -intercepts occur at  $x = n\pi$ ,  $n \in \mathbb{I}$ .
  - The  $y$ -intercept is 0.

How can you determine the location of the asymptotes for the function  $y = \tan x$ ?



$$y = \tan \theta$$



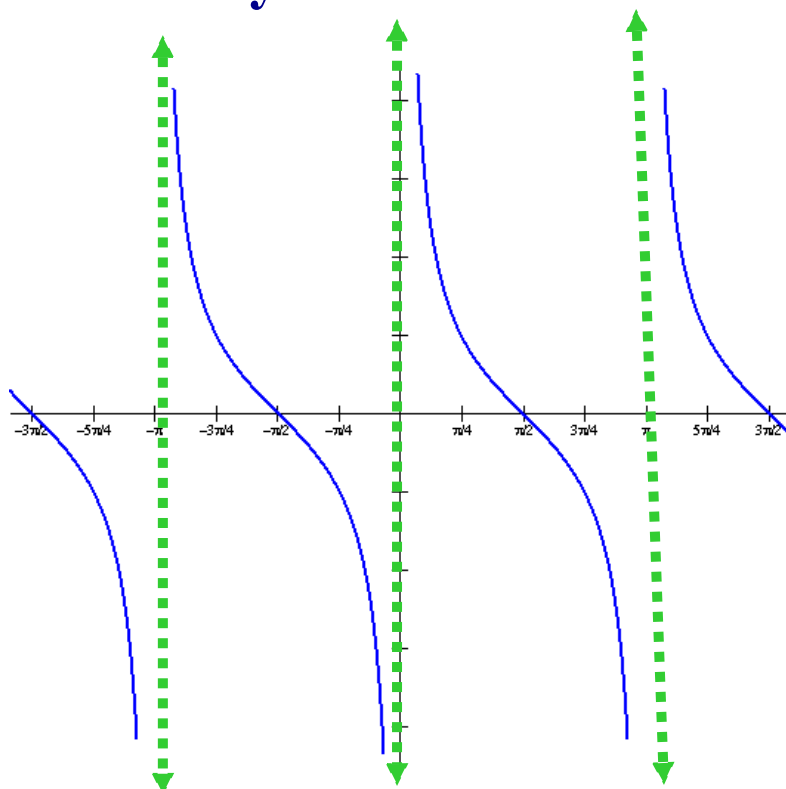
What would the graph of  $\cot \theta$  look like?

**REMEMBER:**

$$\tan x = \frac{1}{\cot x}$$

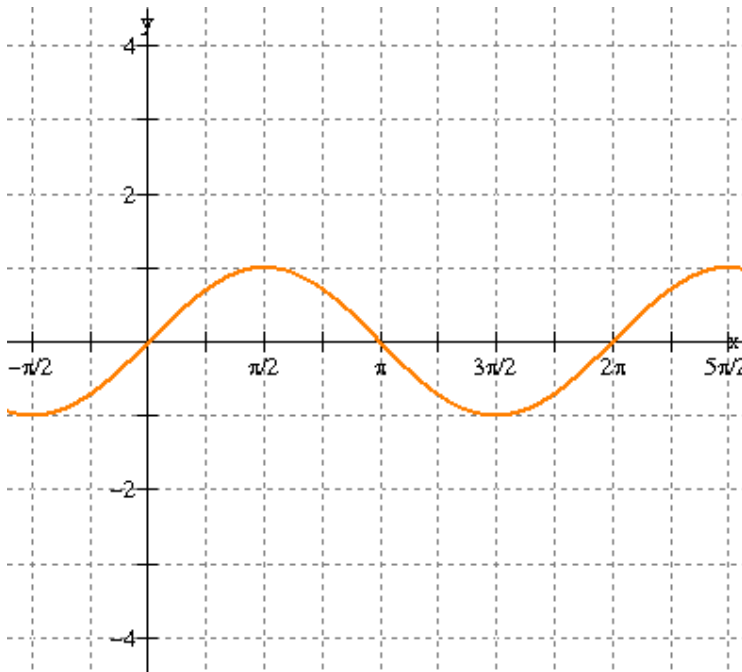
where  $\tan x = 0$ ,  
 $\cot x$  is undefined

$$y = \cot \theta$$



# Graphs of Other Trigonometric Functions

$$y = \sin \theta$$



What would the graph of  $\csc \theta$  look like?

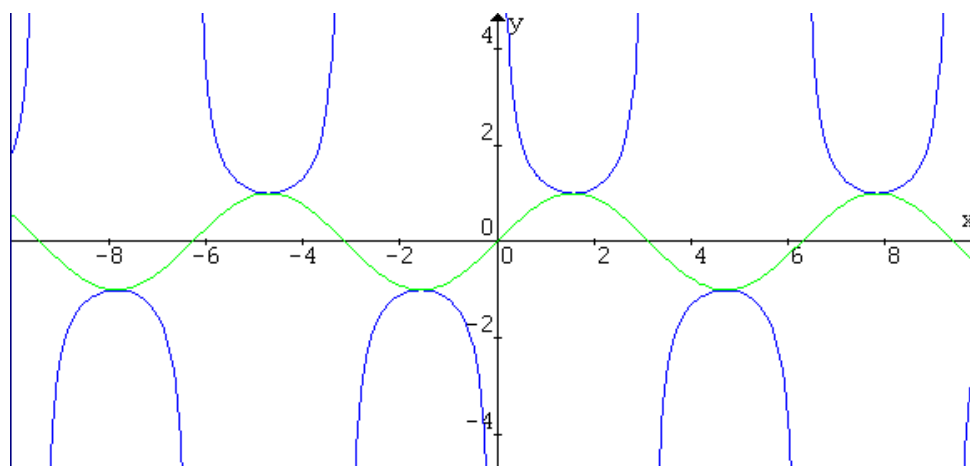
**REMEMBER:**

$$\csc \theta = \frac{1}{\sin \theta}$$

where  $\sin x = 0$ ,  
 $\csc x$  is undefined

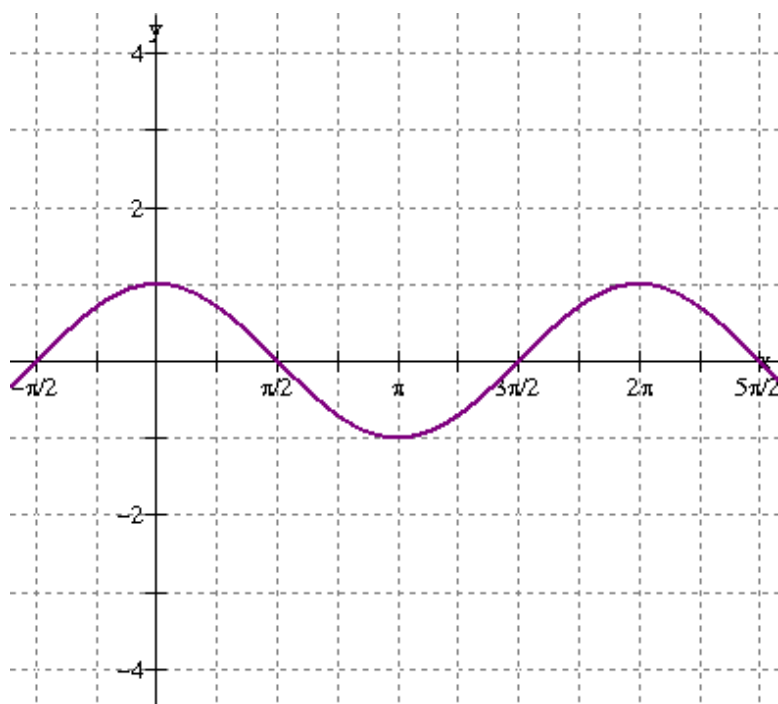
$$y = \sin x$$

$$y = \csc x$$





$$y = \cos \theta$$



What would the graph of  $\sec \theta$  look like?

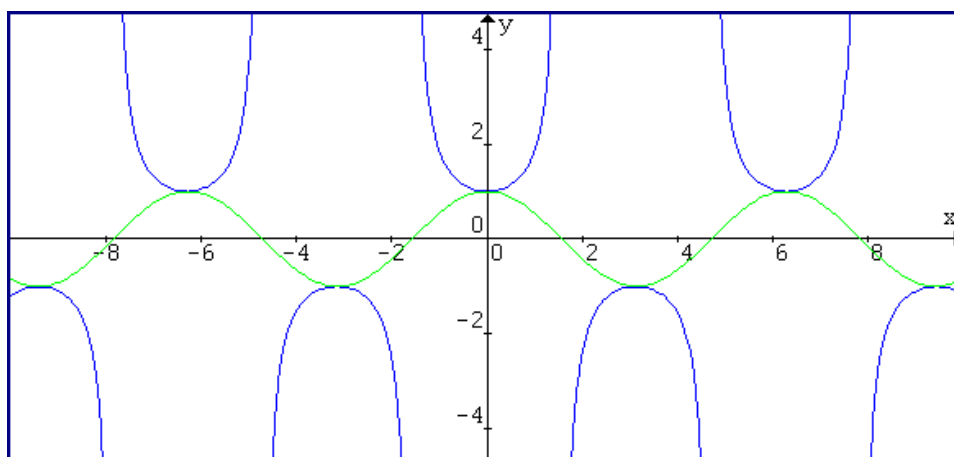
**REMEMBER:**

$$\sec \theta = \frac{1}{\cos \theta}$$

where  $\cos x = 0$ ,  
 $\sec x$  is undefined

$$y = \cos x$$

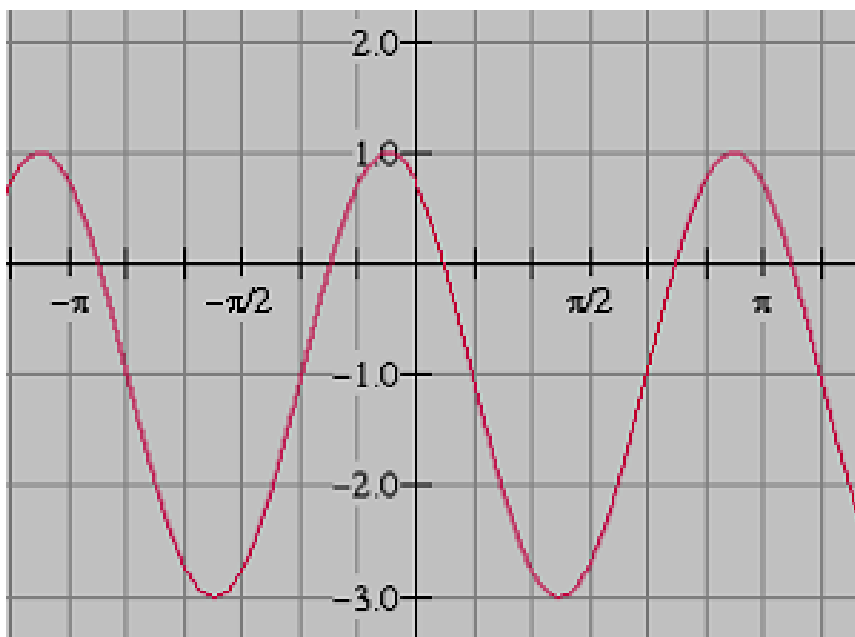
$$y = \sec x$$



## REVIEW - Sketching Trigonometric Functions

- sinusoidal functions
  - properties: domain/range, amplitude, period, phase shift, vertical translation, eq'n of sinusoidal axis, mapping notation.
  - sketching equation in standard form.
- finding the function (both a sine/cosine) given a graph
- solving trigonometric equations where period is not 360
- applications of sinusoidal functions.
  - sketch
  - develop a function
  - use function to answer question
- sketches of all SIX trigonometric ratios

Write both a cosine and sine function to describe the graph shown



Complete the chart shown below and sketch one full cycle of this function

$$-\frac{1}{2}(y+2) = \sin\left(3\theta + \frac{\pi}{8}\right) - 2$$

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

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The Canadian National Historic Windpower Centre, at Etzikom, Alberta, has various styles of windmills on display. The tip of the blade of one windmill reaches its minimum height of 8 m above the ground at a time of 2 s. Its maximum height is 22 m above the ground. The tip of the blade rotates 12 times per minute.

- a) Write a sine or a cosine function to model the rotation of the tip of the blade.
- b) What is the height of the tip of the blade after 4 s?
- c) For how long is the tip of the blade above a height of 17 m in the first 10 s?



## PRACTICE TIME...

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 Review - Practice Test for Sinusoidal Functions.doc

# Practice Test Solutions

**Part A: Multiple Choice**

- |       |                     |
|-------|---------------------|
| 1. A  | 11. A (second hand) |
| 2. D  | 12. C               |
| 3. A  | 13. A               |
| 4. C  | 14. C               |
| 5. B  | 15. D               |
| 6. D  | 16. D               |
| 7. A  | 17. B               |
| 8. D  | 18. D               |
| 9. B  | 19. A               |
| 10. A | 20. A               |

**Part B: Open Response**

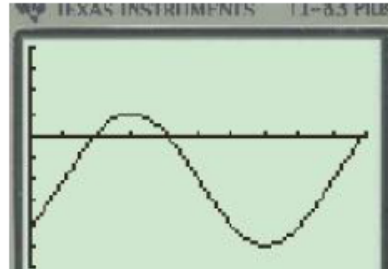
1.  $-\frac{5}{4}$

2. (i)  $y = 3 \sin \frac{3}{2}(x - 160^\circ) - 6$

$y = 3 \cos \frac{3}{2}(x + 20^\circ) - 6$

(ii)  $(x, y) \rightarrow \left( \frac{2}{3}x + 160^\circ, 3y - 6 \right)$

3.



X	Y1
15	1.2
45	1
75	0.2
105	-0.8
135	-1.2
165	-1
195	0.2

X=195

4. 10.28 m

# MORE PRACTICE???

Review - Trigonometric Functions.doc

## SOLUTIONS

1. (a)  $39^\circ$

(b)  $53^\circ$

2. (a)  $-2$

(b)  $\frac{7-2\sqrt{3}}{4}$

3. (a) II

(b) II

4. (a)  $-1.2799$

(b)  $-1.0864$

c)  $1.2690$

(d)  $39^\circ$

(e)  $-5$

(f)  $25^\circ$

5.  $\sin \theta = \frac{-\sqrt{5}}{5}$

$\csc \theta = -\sqrt{5}$

$\cos \theta = \frac{-2\sqrt{5}}{5}$

$\sec \theta = \frac{\sqrt{5}}{2}$

$\tan \theta = \frac{1}{2}$

$\cot \theta = 2$

6.  $\frac{-\sqrt{10}}{2}$

8. Amp = 3

(b) Amp = 2

Period =  $180^\circ$

Period =  $120^\circ$

V.T. = Up 2

V.T. = Down 2

P.S. = none

P.S. =  $60^\circ$  left

Domain:  $0^\circ \leq \theta \leq 360^\circ$

Domain:  $\mathbb{R}$

(c) Amp = 2

(d) Amp = 6

Period =  $720^\circ$

Period =  $360^\circ$

V.T. = Up 5

V.T. = None

P.S. = none

P.S. =  $90^\circ$  right

Domain:  $-90 \leq \theta \leq 360^\circ$

Domain:  $\theta \in \mathbb{R}$

Range:  $-3 \leq y \leq 7$

Range:  $-6 \leq y \leq 6$

10. 11.9 m

11. 46.2 cm





## Attachments

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Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc

Sketching Sinusoidal Functions #2.pdf

Sketching Sinusoidal Functions #2.doc

Sketching Sinusoidal Functions #3 (Solutions).doc

worksheet-sketching in radian measure.doc