Trigonometric Identities

Prerequisite Skills...

Factor:

$\cos^2\theta + 10\cos\theta - 24$ (Simple Trinomial)

$(\cos\theta - 2)(\cos\theta + 12)$

(Diff of Squares)

$\sin^2\theta - \cos^2\theta$

$(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$

$1 - cot^4\vartheta$ (Diff of Squares)

$(1 + \cot^2\theta)(1 - \cot^2\theta)$

$(1 + \cot^2\theta)(1 + \cot\theta)(1 - \cot\theta)$

(Trinomial Decomp)

$9tan^4x - 6tan^2x + 1$

$(9\tan^4 x - 3\tan^2 x)(3\tan^2 x + 1)$

$3\tan^2 x(3\tan^2 x - 1) - 1(3\tan^2 x - 1)$

$(3\tan^2 x - 1)^2$

Simplify the following expression:

$$\frac{\tan^2\theta(\cos^2\theta - 1)}{\tan\theta\cos\theta + \tan\theta}$$

Diff of Sq.

Common

$$\frac{\tan^{\cancel{2}}\theta(\cos\theta - 1)\cancel{(\cos\theta + 1)}}{\cancel{\tan\theta}(\cancel{\cos\theta + 1})}$$

$\tan\theta(\cos\theta - 1)$

Find a common denominator for each of the following:

$$\frac{3}{5a} - \frac{5}{4b}$$

$$\frac{3(4b)}{(5a)(4b)} - \frac{5(5a)}{(5a)(4b)}$$

$$\frac{12b}{20ab} - \frac{25a}{20ab}$$

$$\frac{12b-25a}{20ab}$$

$$\frac{2}{x+3} + \frac{1}{x-6}$$

$$\frac{2(x-6)}{(x+3)(x-6)} + \frac{1(x+3)}{(x+3)(x-6)}$$

$$\frac{2x-12+x+3}{(x+3)(x-6)}$$

$$\frac{3x-9}{(x+3)(x-6)} \text{ or } \frac{3(x-3)}{(x+3)(x-6)}$$

$$\frac{1}{cosA} - \frac{4}{sinA}$$

$$\frac{tanx}{1-cosx} + \frac{sinx}{1+cosx}$$

$$\frac{tanx(1+cosx)}{(1-cosx)(1+cosx)} + \frac{sinx(1-cosx)}{(1-cosx)(1+cosx)}$$

$$\frac{tanx + tanx\,cosx + sinx - sinx\,cosx}{(1-cosx)(1+cosx)}$$

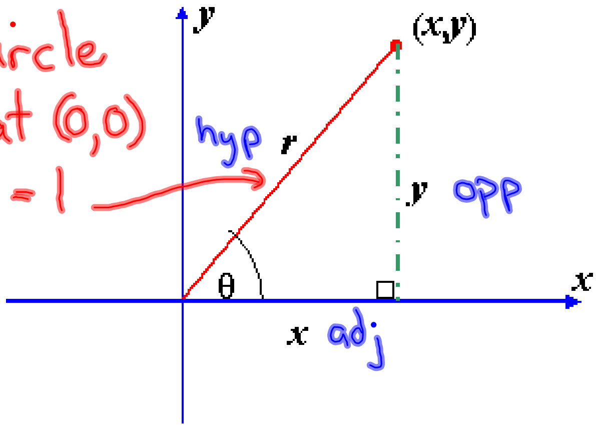
$$\text{or } \frac{tanx + tanx\,cosx + sinx - sinx\,cosx}{1-cos^2x}$$

Trig Identities

Reciprocal Identities

Unit Circle
- Center at (0,0)
- Radius = 1

$r = 1$



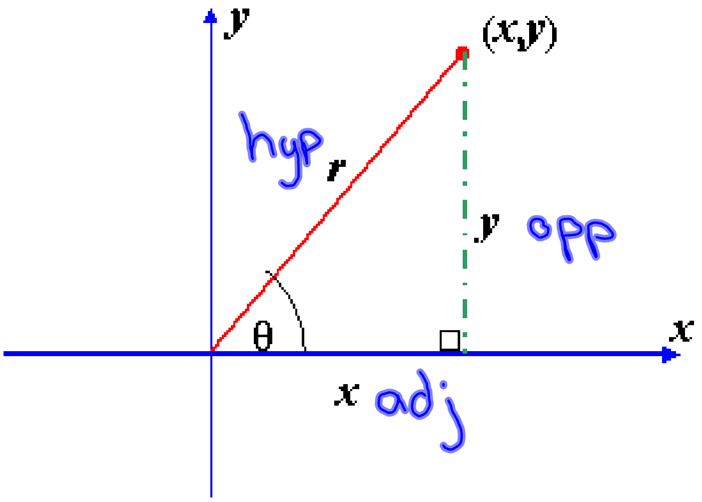
$\sin\theta = \frac{y}{r} = \frac{y}{1} = y$	$\cos\theta = \frac{x}{r} = \frac{x}{1} = x$	$\tan\theta = \frac{y}{x}$
$\csc\theta = \frac{r}{y} = \frac{1}{y}$	$\sec\theta = \frac{r}{x} = \frac{1}{x}$	$\cot\theta = \frac{x}{y}$

Reciprocal Identities

$\sin\theta = \frac{1}{\csc\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	$\tan\theta = \frac{1}{\cot\theta}$
$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

$\sin^2\theta = \frac{1}{\csc^2\theta}$	$\cos^2\theta = \frac{1}{\sec^2\theta}$	$\tan^2\theta = \frac{1}{\cot^2\theta}$
$\csc^2\theta = \frac{1}{\sin^2\theta}$	$\sec^2\theta = \frac{1}{\cos^2\theta}$	$\cot^2\theta = \frac{1}{\tan^2\theta}$

Quotient Identities



$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

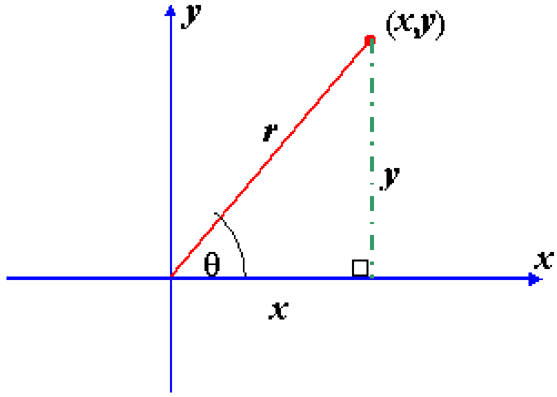
$$\cot\theta = \frac{adj}{opp} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$$

Quotient Identities

$$\boxed{\begin{array}{l} \tan\theta = \dfrac{\sin\theta}{\cos\theta} \\ \\ \cot\theta = \dfrac{\cos\theta}{\sin\theta} \end{array}}$$

$$\boxed{\begin{array}{l} \tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta} \\ \\ \cot^2\theta = \dfrac{\cos^2\theta}{\sin^2\theta} \end{array}}$$

Pythagorean Identities



$\boxed{r=1}$

$\div r^2$

$$\frac{x^2}{r^2}+\frac{y^2}{r^2}=\frac{r^2}{r^2}$$

$$x^2+y^2=1$$

$$\cos^2\theta+\sin^2\theta=1$$

$$\boxed{\sin^2\theta+\cos^2\theta=1}$$

$$\sin^2\theta=1-\cos^2\theta$$

$$\cos^2\theta=1-\sin^2\theta$$

$\div x^2$

$$\frac{x^2}{x^2}+\frac{y^2}{x^2}=\frac{r^2}{x^2}$$

$$\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

$$1+\tan^2\theta=\sec^2\theta$$

$$\boxed{\tan^2\theta+1=\sec^2\theta}$$

$$\tan^2\theta=\sec^2\theta-1$$

$$1=\sec^2\theta-\tan^2\theta$$

$\div y^2$

$$\frac{x^2}{y^2}+\frac{y^2}{y^2}=\frac{r^2}{y^2}$$

$$\frac{\cos^2\theta}{\sin^2\theta}+\frac{\sin^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\cot^2\theta+1=\csc^2\theta$$

$$\boxed{\cot^2\theta+1=\csc^2\theta}$$

$$\cot^2\theta=\csc^2\theta-1$$

$$1=\csc^2\theta-\cot^2\theta$$

Trigonometric Identities

You must know these!

Quotient	Pythagorean
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\sin^2\theta + \cos^2\theta = 1$
$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	$\tan^2\theta + 1 = \sec^2\theta$
	$1 + \cot^2\theta = \csc^2\theta$

Reciprocal

$\csc\theta = \dfrac{1}{\sin\theta}$	$\sin\theta = \dfrac{1}{\csc\theta}$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\cos\theta = \dfrac{1}{\sec\theta}$
$\cot\theta = \dfrac{1}{\tan\theta}$	$\tan\theta = \dfrac{1}{\cot\theta}$

Strategies for Proving Identities:

- Work on the most complex side and simplify so it has the same form as the simpler side
- Methods used in simplifying: direct substitution, factoring, finding a common denominator, multiplying by the conjugate → $1+\cos\theta$ → $1-\cos\theta$

today

* $\sin\theta$ + $\cos\theta$ = good guys.

Prove the following:

$$\frac{\boxed{\tan x}}{\sin x} = \boxed{\sec x}$$

$$\frac{\frac{\sin x}{\cos x}}{\sin x} \qquad \boxed{\frac{1}{\cos x}}$$

$$\frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cancel{\sin x}}$$

$$\boxed{\frac{1}{\cos x}}$$

$$\cos\theta \bullet \boxed{\sec\theta} = \boxed{1}$$

$$\cancel{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}}$$

$$\boxed{1}$$

Prove the following:

$$\boxed{\cot\theta}\bullet\sin\theta=\boxed{\cos\theta}$$

$$\frac{\cos\theta}{\cancel{\sin\theta}}\cdot\cancel{\sin\theta}$$

$$\boxed{\cos\theta}$$

$$\frac{\cos x}{\boxed{\tan x}}=\frac{\boxed{1-\sin^2 x}}{\sin x}$$

$$\frac{\cos x}{\frac{\sin x}{\cos x}}$$

$$\boxed{\frac{\cos^2 x}{\sin x}}$$

$$\cos x\cdot\frac{\cos x}{\sin x}$$

$$\boxed{\frac{\cos^2 x}{\sin x}}$$

Factor

Ex. Prove that $\sin y + \sin y \cot^2 y = \csc y$

$\sin y\left(\underline{1+\cot^2 y}\right)$ | $\boxed{\dfrac{1}{\sin y}}$

$\sin y\left(\csc^2 y\right)$

$\cancel{\sin y}\left(\dfrac{1}{\sin^{\cancel{2}} y}\right)$

$\boxed{\dfrac{1}{\sin y}}$

Homework

1. $\tan\theta\cos\theta = \sin\theta$

2. $\cot\theta\sec\theta = \csc\theta$

3. $\dfrac{1+\cot^2\theta}{\csc^2\theta} = 1$

4. $\dfrac{\tan^2\theta}{1+\tan^2\theta} = \sin^2\theta$

5. $\dfrac{\tan^2\theta}{\sin^2\theta} = 1+\tan^2\theta$