

Assignment

**Complete pg. 389
Question 6 (a and d)**

***You do not have to verify
your solutions graphically**

**Complete pg. 411
Questions 11d and 12c**

Solutions

6. Solve each equation and verify your solutions graphically.

a) $|x| = x^2 + x - 3$ $x=0$ $|x| = \begin{cases} x, & x \geq 0 \\ -(x), & x < 0 \end{cases}$
 * LINEAR

Case 1

$$x = x^2 + x - 3$$

$$0 = x^2 + x - x - 3$$

$$0 = x^2 - 3$$

$$3 = x^2$$

$$\pm\sqrt{3} = x$$

* Only $x = \sqrt{3}$ satisfies the condition $x \geq 0$.

Case 2

$$-x = x^2 + x - 3$$

$$0 = x^2 + x + x - 3$$

$$0 = x^2 + 2x - 3$$

$$0 = (x+3)(x-1)$$

$$x+3=0 \text{ or } x-1=0$$

$$x = -3 \quad x = 1$$

* Only $x = -3$ satisfies the condition $x < 0$.

Solutions

$$d) |x^2 - 1| = x$$

* **QUADRATIC**

Case 1

$$x^2 - 1 = 0$$

$$(x-1)(x+1)$$

$$x=1 \text{ or } x=-1$$

$$|x^2 - 1| = \begin{cases} x^2 - 1, & x \leq -1 \text{ or } x \geq 1 \\ -(x^2 - 1), & -1 < x < 1 \end{cases}$$

$$x^2 - 1 = x$$

$$x^2 - x - 1 = 0$$

$$a=1, b=-1, c=-1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

$$x = \frac{1 + \sqrt{5}}{2} \text{ or } x = \frac{1 - \sqrt{5}}{2}$$

$$x \doteq 1.6180$$

$$x \doteq -0.6180$$

* Only $x = \frac{1 + \sqrt{5}}{2}$ satisfies the condition $x \geq 1$.

Case 2

$$-(x^2 - 1) = x$$

$$x^2 - 1 = -x$$

$$x^2 + x - 1 = 0$$

$$a=1, b=1, c=-1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = \frac{-1 + \sqrt{5}}{2} \text{ or } x = \frac{-1 - \sqrt{5}}{2}$$

$$x \doteq 0.6180$$

$$x \doteq -1.6180$$

* Only $x = \frac{-1 + \sqrt{5}}{2}$ satisfies the condition $-1 < x < 1$.

Solutions

$$11d) |m^2 - 4m| = 5$$

* QUADRATIC

$$m^2 - 4m = 0$$

$$m(m-4) = 0$$

$$m = 0 \text{ or } m - 4 = 0$$

$$m = 4$$

$$|m^2 - 4m| = \begin{cases} m^2 - 4m, & m \leq 0 \text{ or } m \geq 4 \\ -(m^2 - 4m), & 0 < m < 4 \end{cases}$$

Case 1

$$m^2 - 4m = 5$$

$$m^2 - 4m - 5 = 0$$

$$(m-5)(m+1) = 0$$

$$m = 5 \text{ or } m = -1$$

The solutions $m = 5$ and $m = -1$ both satisfy the conditions $x \leq 0$ or $x \geq 4$.

Case 2

$$-(m^2 - 4m) = 5$$

$$m^2 - 4m = -5$$

$$m^2 - 4m + 5 = 0$$

$$a = 1, b = -4, c = 5$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(5)}}{2(1)}$$

$$m = \frac{4 \pm \sqrt{16 - 20}}{2}$$

$$m = \frac{4 \pm \sqrt{-4}}{2}$$

* Stop here (No REAL Solution).

Solutions

$$12c) |x^2 - 6x| = x \quad \left\{ \begin{array}{l} x^2 - 6x = 0 \\ x(x-6) = 0 \\ x = 0 \text{ or } x-6 = 0 \\ x = 6 \end{array} \right. \quad |x^2 - 6x| = \begin{cases} x^2 - 6x, & x \leq 0 \text{ or } x \geq 6 \\ -(x^2 - 6x), & 0 < x < 6 \end{cases}$$

* QUADRATIC

Case 1

$$\begin{aligned} x^2 - 6x &= x \\ x^2 - 6x - x &= 0 \\ x^2 - 7x &= 0 \\ x(x-7) &= 0 \\ x &= 0 \text{ or } x = 7 \end{aligned}$$

The solutions $x=0$ and $x=7$ both satisfy the conditions $x \leq 0$ or $x \geq 6$.

Case 2

$$\begin{aligned} -(x^2 - 6x) &= x \\ x^2 - 6x &= -x \\ x^2 - 6x + x &= 0 \\ x^2 - 5x &= 0 \\ x(x-5) &= 0 \\ x &= 0 \text{ or } x = 5 \end{aligned}$$

↓
Extraneous

* Only $x=5$, satisfies the condition $0 < x < 6$.